Belirsiz İntegral (Basit Kesirlere Ayırma Yöntemi)

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Belirsiz İntegraller...

BELİRSİZ İNTEGRAL

(Basit Kesirlere Ayırma)

Basit Kesirlere Ayırma

$P(x)$ ve $Q(x)$ x in birer polinomu olsunlar. $P(x)$ polinomunun derecesi $Q(x)$ polinomunun derecesinden büyük ise $\frac{P(x)}{Q(x)} = K(x) + \frac{R(x)}{Q(x)}$ yazılabilir. Burada $R(x)$ ve $K(x)$ birer polinomdur. $R(x)$ 'in derecesi $Q(x)$ 'in derecesinden küçüktür. Bu durumda $\frac{P(x)}{Q(x)}$ şeklinde bir rasyonel fonksiyonun integrallenmesi payının derecesi paydasının derecesinden küçük olan $\frac{R(x)}{Q(x)}$ şeklinde bir rasyonel fonksiyonun integrallenmesine indirgenir. Bu durumdaki integraller aşağıdaki örneklerdeki gibi hesaplanır.

ÖRNEK $\int \frac{2x+5}{x^2+4x+13}dx = ?$

$\Delta = b^2-4ac = 16-4.1.13 = -36 < 0$, Kök yok , Çarpanlarına ayrılmaz.

$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C$

$(x^2+4x+13)' = 2x+4$

$$= \underbrace{\int \frac{2x+4}{x^2+4x+13}dx}_{*} + \int \frac{dx}{\underbrace{x^2+4x+13}_{(x+2)^2+9}} \ * \Rightarrow \int \frac{dx}{(x+2)^2+3^2} = \int \frac{du}{u^2+3^2}$$

$x^2+4x+13 = t$

$(2x+4)dx = dt$

$\int \frac{dt}{t} = \ln(t)+C$

$= \ln(x^2+4x+13)+C$

$x+2 = u$

$dx = du$

$= \frac{1}{3} \operatorname{arctg} \frac{u}{3} + C$

$= \frac{1}{3} \operatorname{arctg} \frac{x+2}{3} + C$

$= \ln(x^2+4x+13) + \frac{1}{3} \operatorname{arctg}\left(\frac{x+2}{3}\right) + C$

ÖRNEK $\int \frac{4}{x^2-4}dx = ?$

$(x-2)(x+2)$

$$\frac{4}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$(x+2)$ $(x-2)$

$$\frac{4}{(x-2)(x+2)} = \frac{A(x+2)+B(x-2)}{(x-2)(x+2)}$$

$A(x+2)+B(x-2)=4$

$x=2,$ $4A=4$, $A=1$

$x=-2,$ $-4B=4$, $B=-1$

$$\int \frac{4}{(x-2)(x+2)}dx = \int \frac{dx}{x-2} - \int \frac{dx}{x+2} = \ln(x-2) - \ln(x+2) + C = \ln\left(\frac{x-2}{x+2}\right) + C$$

$x-2=t$

$dx=dt$

$= \int \frac{dt}{t} = \ln(t)$

ÖRNEK $\int \frac{4x-3}{(x+4)^2}\,dx = ?$

$(x+4)(x+4)$

Tekrarlayan kök

$$\frac{\quad}{(x+4)^3} = \frac{\quad}{x+4} + \frac{\quad}{(x+4)^2} + \frac{\quad}{(x+4)^3}$$

$$\frac{4x-3}{(x+4)(x+4)} = \frac{A}{x+4} + \frac{B}{(x+4)^2}$$

$(x+4)$ $\quad$ (1)

$$\frac{4x-3}{(x+4)^2} = \frac{A(x+4)+B}{(x+4)^2},$$

$$A(x+4)+B = 4x-3$$

$$Ax+4A+B = 4x-3$$

$$A=4, \qquad 4A+B=-3$$

$$B=-19$$

$$\int \frac{4x-3}{(x+4)^2}\,dx = 4\int \frac{dx}{x+4} - 19\int \frac{dx}{(x+4)^2}$$

$$x+4=u, \quad dx=du, \qquad \int \frac{du}{u^2} = -\frac{1}{u}+C$$

$$= -\frac{1}{x+4}+C$$

$$= 4\ln(x+4) + \frac{19}{x+4} + C$$

ÖRNEK $\int \frac{-2x+4}{(1+x^2)(x-1)^2}\,dx = ?$

$$\frac{1}{(1+x^2)^2} = \frac{Ax+B}{1+x^2} + \frac{Cx+D}{(1+x^2)^2}$$

$$\frac{-2x+4}{(1+x^2)(x-1)^2} = \frac{Ax+B}{1+x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$(x-1)(x-1)$ $\quad (x-1)^2 \quad (x-1)(1+x^2) \quad (1+x^2)$

$$-2x+4 = (Ax+B)(x-1)^2 + C(x-1)(1+x^2) + D(1+x^2)$$

$$= (Ax+B)(x^2-2x+1) + C(x+x^3-1-x^2) + D(1+x^2)$$

$$= Ax^3 - 2Ax^2 + Ax + Bx^2 - 2Bx + B + Cx^3 - Cx^2 + Cx - C + Dx^2 + D$$

$$-2x+4 = (A+C)x^3 + (-2A+B-C+D)x^2 + (A-2B+C)x + (B-C+D)$$

$A+C=0 \quad \Rightarrow C=-2$

$-2A+B-C+D=0 \quad \Rightarrow -2A=-4 \quad \Rightarrow A=2$

$A-2B+C=-2 \quad \Rightarrow \quad B=1$

$B-C+D=4 \quad \Rightarrow \quad D=1$

$$\int \frac{-2x+4}{(x-1)^2(1+x^2)}\,dx = \int \frac{2x+1}{1+x^2}\,dx - 2\int\frac{dx}{x-1} + \int\frac{dx}{(x-1)^2}$$

$$= \int \frac{2x\,dx}{1+x^2} + \int\frac{dx}{1+x^2} - 2\int\frac{dx}{x-1} + \int\frac{dx}{(x-1)^2}$$

$1+x^2=t$ $\quad$ ✓ $\quad x-1=u \quad x-1=k$

$2x\,dx=dt \quad dx=du \quad dx=dk$

$\int\frac{dt}{t}=\ln(t) \quad \int\frac{du}{u}=\ln u \quad \int\frac{dk}{k^2}=-\frac{1}{k}$

$$= \ln(1+x^2) + \arctan x - 2\ln(x-1) - \frac{1}{x-1} + C$$

ÖRNEK $\int \frac{2x^4-6x^3+2x^2-2x-2}{x^3-3x^2+3x-1}dx = ?$

$$\begin{array}{r|l} 2x^4-6x^3+2x^2-2x-2 & x^3-3x^2+3x-1 \\ -\ (2x^4-6x^3+6x^2-2x) & 2x \\ \hline -4x^2-2 & \end{array}$$

$$, \quad \frac{2x^4-6x^3+2x^2-2x-2}{(x-1)^3} = 2x - \frac{4x^2+2}{(x-1)^3}$$

$$= \underbrace{\int 2x\,dx}_{*} - \underbrace{\int \frac{4x^2+2}{(x-1)^3}dx}_{*}$$

$$\int \frac{4x^2+2}{(x-1)^3}dx \Rightarrow \quad \frac{4x^2+2}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$$

$(x-1)^2 \quad (x-1) \quad 1$

$$4x^2+2 = A(x-1)^2 + B(x-1) + C$$
$$= A(x^2-2x+1) + B(x-1) + C$$
$$4x^2+2 = Ax^2 - 2Ax + A + Bx - B + C$$

$A = 4, \quad -2A+B = 0, \quad A-B+C = 2$

$B = 8 \qquad c = 6$

$$\int \frac{4x^2+2}{(x-1)^3}dx = 4\int \frac{dx}{x-1} + 8\int \frac{dx}{(x-1)^2} + 6\int \frac{dx}{(x-1)^3}$$

$x-1=t$, $dx=dt$, $\int \frac{dt}{t} = \ln t + c$

$x-1=u$, $dx=du$, $\int \frac{du}{u^2} = -\frac{1}{u} + c$

$x-1=k$, $dx=dk$, $\int \frac{dk}{k^3} = -\frac{1}{2k^2} + c$

$$= 4\ln(x-1) - \frac{8}{x-1} - \frac{3}{(x-1)^2} + c$$

Cevap $= \int 2x\,dx - \int \frac{4x^2+2}{(x-1)^3}dx = \underline{\underline{x^2 - 4\ln(x-1) + \frac{8}{x-1} + \frac{3}{(x-1)^2} + c}}$ //