CALCULUS I

AUTUMN 2015 - HOMEWORK 1

1. Find the domain and graph of the function $g = \sqrt{|x|}$.

2. Find the range of the below function:

$$y = 2 + \frac{x^2}{x^2 + 4}$$

3. Graph the function

$$f(x) = \begin{cases} 4 - x^2 & , \quad x \leq 1 \\ x^2 + 2x & , \quad x > 1 \end{cases}$$

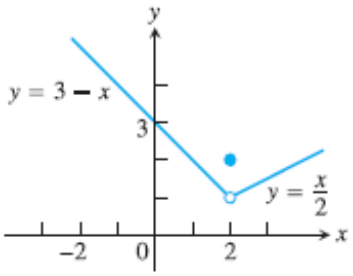
4. Graph the function $y = -4\sqrt{x}$. What symmetries, if any, do the graph have? Specify the intervals over which the function is increasing and the intervals where it is decreasing.

5. Find the below limit:

$$\lim_{x \to -2} \frac{x + 2}{\sqrt{x^2 + 5} - 3}$$

6. Let

$$f(x) = \begin{cases} 3 - x & , \quad x < 2 \\ 2 & , \quad x = 2 \\ \frac{x}{2} & , \quad x > 2 \end{cases} .$$



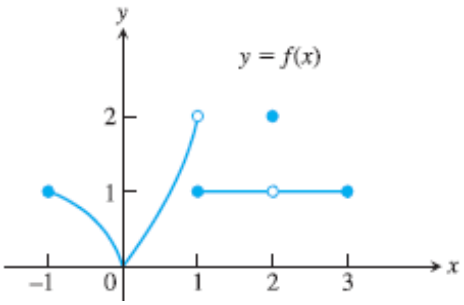
a) Find $\lim_{x \to 2^+} f(x)$, $\lim_{x \to 2^-} f(x)$ and $f(2)$.
b) Does $\lim_{x \to 2} f(x)$ exist? If so, what is it? If no, why not?
c) Find $\lim_{x \to 1^+} f(x)$ and $\lim_{x \to 1^-} f(x)$.
d) Does $\lim_{x \to 1} f(x)$ exist? If so, what is it? If no, why not?

7. Graph the function

$$f(x) = \begin{cases} x & , \quad -1 \leq x < 0 \text{ or } 0 < x \leq 1 \\ 1 & , \quad x = 0 \\ 0 & , \quad x < -1 \text{ or } x > 1 \end{cases} .$$

a) What is the domain and range of f?
b) At what points c, if any, does $\lim_{x\to c} f(x)$ exist?
c) At what points does only the left-hand limit exist?
d) At what points does only the right-hand limit exist?

8. At which points does the below function fail to be continuous? At which points, if any, are the discontinuities removable? Not removable? Give reasons for your answers.



9. At what points is the below function continuous?

$$f(x) = \begin{cases} \frac{x^3-8}{x^2-4} & , \quad x \neq 2, x \neq -2 \\ 3 & , \quad x = 2 \\ 4 & , \quad x = -2 \end{cases}.$$

10. Define $f(1)$ in a way that extends $f(x) = (x^3 - 1)/(x^2 - 1)$ to be continuous at $x = 1$.