T.C.
ESKİŞEHİR OSMANGAZİ ÜNİVERSİTESİ
MÜHENDİSLİK MİMARLIK FAKÜLTESİ

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ÖĞRETİM ÜYESİNİN
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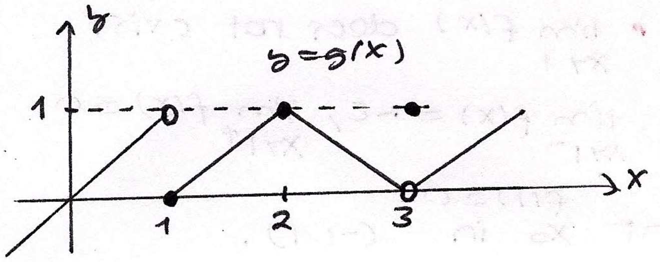
İmzası :

DİKKAT: Cep telefonunuzu görünmeyen bir yerde ve kapalı tutunuz.

Limits from graphs

①



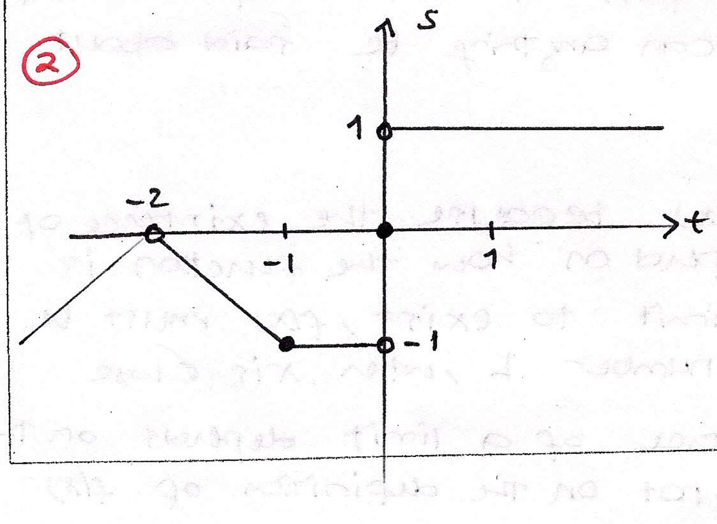
- $\lim_{x\to1} g(x)$ does not exist. $\lim_{x\to1^+} g(x) = 0$, $\lim_{x\to1^-} g(x) = 1$

  There is no single number L that all the values g(x) get arbitrarily close to as $x\to1$.

- $\lim_{x\to2} g(x) = 1$. $\lim_{x\to2^+} g(x) = 1$ and $\lim_{x\to2^-} f(x) = 1$. $f(2) = 1$

- $\lim_{x\to3} g(x) = 0$ $\lim_{x\to3^+} g(x) = 0$, $\lim_{x\to3^-} g(x) = 0$ $g(3) = 1$.

②



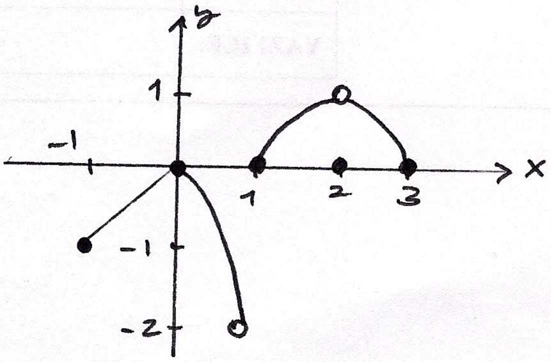
- $\lim_{t\to -2} f(t) = 0$, $\lim_{t\to -2^-} f(t) = 0$, $\lim_{t\to -2^+} f(t) = 0$ $f(-2)$ does not exist.

- $\lim_{t\to -1} f(t) = -1$, $\lim_{t\to -1^-} f(t) = -1$, $\lim_{t\to -1^+} f(t) = -1$ $f(-1) = -1$

- $\lim_{t\to 0} f(t)$ does not exist. $\lim_{t\to 0^-} f(t) = -1$, $\lim_{t\to 0^+} f(t) = 1$

- $\lim_{t\to -0.5} f(t) = -1$

③



- $\lim_{x\to 2} f(x) = 1$, $\lim_{x\to 2^-} f(x) = 1 = \lim_{x\to 2^+} f(x)$

  $f(2) = 0.$

- $\lim_{x\to 1} f(x)$ does not exist.

  $\lim_{x\to 1^-} f(x) = -2$, $\lim_{x\to 1^+} f(x) = 0$

  $f(1) = 0$

- $\lim_{x\to x_0} f(x)$ exists at every point x_0 in $(-1,1)$.

④ $\lim_{x\to 1} \frac{1}{x-1} = ?$

As x approaches 1 from the left, the values of $\frac{1}{x-1}$ become increasingly large and negative. As x approaches 1 from the right, the values become increasingly large and positive. There is no one number L that all the function values get arbitrarily close to as $x\to 1$, so $\lim_{x\to 1} \frac{1}{x-1}$ does not exist.

⑤ req // Suppose that a function $f(x)$ is defined for all real values of x except $x = x_0$. Can anything be said about the existence of $\lim_{x\to x_0} f(x)$?

Nothing can be said about $f(x)$ because the existence of a limit as $x\to x_0$ does not depend on how the function is defined at x_0. In order for a limit to exist, $f(x)$ must be arbitrarily close to a single real number L, when x is close enough to x_0. That is, the existence of a limit depends on the values of $f(x)$ for x near x_0, not on the definition of $f(x)$ at x_0 itself.

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ÖĞRETİM ÜYESİNİN
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İmzası :

DİKKAT: Cep telefonunuzu görünmeyen bir yerde ve kapalı tutunuz.

⑥ $\lim_{x\to -2} (x^3 - 2x^2 + 4x + 8) = (-2)^3 - 2(-2)^2 + 4(-2) + 8 = -16.$

⑦ $\lim_{x\to 1} \frac{x^4-1}{x^3-1} = \lim_{x\to 1} \frac{(x^2+1)(x+1)(x-1)}{(x^2+x+1)(x-1)} = \lim_{x\to 1} \frac{(x^2+1)(x+1)}{(x^2+x+1)} = \frac{4}{3}.$

⑧ $\lim_{x\to 4} \frac{4-x}{5-\sqrt{x^2+9}} = \lim_{x\to 4} \frac{(4-x)(5+\sqrt{x^2-9})}{(5-\sqrt{x^2+9})(5+\sqrt{x^2-9})}$

$= \lim_{x\to 4} \frac{(4-x)(5+\sqrt{x^2-9})}{25-(x^2+9)} = \lim_{x\to 4} \frac{(4-x)(5+\sqrt{x^2+9})}{16-x^2}$

$= \lim_{x\to 4} \frac{(4-x)(5+\sqrt{x^2+9})}{(4-x)(4+x)} = \lim_{x\to 4} \frac{(5+\sqrt{x^2+9})}{4+x} = \frac{10}{8} = \frac{5}{4}$

⑨ $\lim_{x\to 0} \frac{1+x+\sin x}{3\cos x} = \frac{1+0+\sin 0}{3\cos 0} = \frac{1}{3}$

⑩ If $x^4 \le f(x) \le x^2$ for x in $[-1,1]$, and $x^2 \le f(x) \le x^4$ for $x<-1$ and $x>1$, at what points c $\lim_{x\to c} f(x)$?

$\lim_{x\to c} f(x)$ exists at those points c where $\lim_{x\to c} x^4 = \lim_{x\to c} x^2$.

Thus, $c^4 = c^2 \Rightarrow c^2(1-c^2) = 0 \Rightarrow c = 0, 1$ or -1.

moreover, $\lim_{x\to 0} f(x) = \lim_{x\to 0} x^2 = 0$ and

$\lim_{x\to -1} f(x) = \lim_{x\to 1} f(x) = 1.$

⑪ If $\lim_{x\to 0} \frac{f(x)}{x^2} = 1$, find

a) $\lim_{x\to 0} f(x)$ b) $\lim_{x\to 0} \frac{f(x)}{x}$

ⓐ $\underbrace{\left[\lim_{x\to 0} \frac{f(x)}{x^2}\right]}_{=1}\underbrace{\left[\lim_{x\to 0} x^2\right]}_{=0} = \left[\lim_{x\to 0} \frac{f(x)}{x^2}\right]\left[\lim_{x\to 0} x^2\right] = \lim_{x\to 0}\left[\frac{f(x)}{x^2}\cdot x^2\right]$

$= \lim_{x\to 0} f(x) = 0.$

ⓑ $\underbrace{\left[\lim_{x\to 0} \frac{f(x)}{x^2}\right]}_{=1}\underbrace{\left[\lim_{x\to 0} x\right]}_{=0} = \lim_{x\to 0}\left[\frac{f(x)}{x^2}\cdot x\right] = \lim_{x\to 0} \frac{f(x)}{x} \Rightarrow \lim_{x\to 0} \frac{f(x)}{x} = 0.$

⑫ Show that $\lim_{x\to 0} x \sin\frac{1}{x} = 0.$

$-1 \leq \sin\frac{1}{x} \leq 1$ for $x \neq 0$.

$x>0 \Rightarrow -x \leq x\sin\frac{1}{x} \leq x \Rightarrow \lim_{x\to 0} x\sin\frac{1}{x} = 0$ by the sandwich theorem.

$x<0 \Rightarrow -x \geq x\sin\frac{1}{x} \geq x \Rightarrow \lim_{x\to 0} x\sin\frac{1}{x} = 0$ by the sandwich theorem.

⑬ $\lim_{x\to 0} x^2\cos\left(\frac{1}{x^3}\right) = 0.$

$-1 \leq \cos\left(\frac{1}{x^3}\right) \leq 1$ for $x \neq 0$.

$-x^2 \leq x^2\cos\left(\frac{1}{x^3}\right) \leq x^2 \Rightarrow \lim_{x\to 0} x^2\cos\left(\frac{1}{x^3}\right) = 0$ by the sandwich theorem.

⑭ Let $f(x) = \begin{cases} x, & x<1 \\ x+1, & x>1 \end{cases}$

Let $\varepsilon = 1/2$. Show that no possible $\delta>0$ satisfies the following condition:

For all x, $0<|x-1|<\delta \Rightarrow |f(x)-2| < 1/2$.

That is, for each $\delta>0$ show that there is a value of x such that

$0<|x-1|<\delta$ and $|f(x)-2| \geq 1/2$.

③

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DİKKAT: Cep telefonunuzu görünmeyen bir yerde ve kapalı tutunuz.

$-\delta < x-1 < 0 \Rightarrow 1-\delta < x < 1 \Rightarrow f(x)=x$ ($x<1 \Rightarrow f(x)=x$ idi)

Then $|f(x)-2| = |x-2| \underset{\substack{\wedge \\ x<1 \\ \text{old.dan}}}{=} 2-x \underset{\substack{\downarrow \\ x<1 \\ \text{old.} \\ \text{dan.}}}{>} 2-1 = 1$.

That is, $|f(x)-2| \geq 1 \geq 1/2$ no matter how small δ is taken when $1-\delta < x < 1 \Rightarrow \lim_{x\to 1} f(x) \neq 2$.

⑮ Let $f(x) = \begin{cases} 3-x & , x<2 \\ \frac{x}{2}+1 & , x>2 \end{cases}$

- $\lim_{x\to 2^+} f(x) = \frac{2}{2}+1 = 2$

$\lim_{x\to 2^-} f(x) = 3-2 = 1$

$\lim_{x\to 2} f(x)$ does not exist because $\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$

- $\lim_{x\to 4^-} f(x) = \frac{4}{2}+1 = 3 = \lim_{x\to 4^+} f(x)$

$\lim_{x\to 4} f(x) = 3.$

⑯ Let $f(x) = \begin{cases} 0, & x \leq 0 \\ \sin\frac{1}{x}, & x > 0. \end{cases}$

$\lim_{x \to 0^+} f(x) = ?$ $\lim_{x \to 0^-} f(x) = ?$

$\lim_{x \to 0^+} f(x)$ does not exist since $\sin\frac{1}{x}$ does not approach any single value as x approaches 0.

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 0 = 0.$

$\lim_{x \to 0} f(x)$ does not exist.

⑰ Let $g(x) = \sqrt{x} \sin\frac{1}{x}.$

$\lim_{x \to 0^+} g(x) = ?$ $\lim_{x \to 0^-} g(x) = ?$

$\lim_{x \to 0^+} g(x) = 0$. By the sandwich theorem $-1 \leq \sin\frac{1}{x} \leq 1$ for $x \neq 0$.

$x > 0 \Rightarrow -\sqrt{x} \leq \sqrt{x} \sin\frac{1}{x} \leq \sqrt{x}$

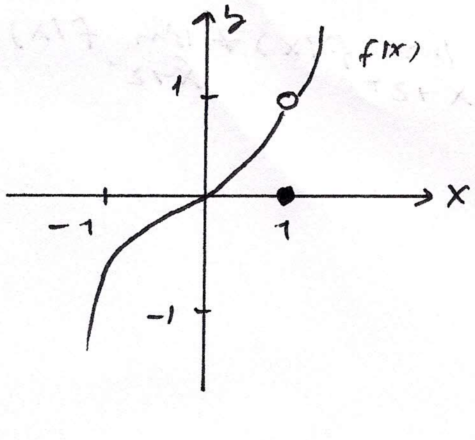
$\Rightarrow \lim_{x \to 0^+} \sqrt{x} \sin\frac{1}{x} = 0.$

$\lim_{x \to 0^-} g(x)$ does not exist since $\sqrt{x}$ is not defined for $x < 0$.

$\lim_{x \to 0} g(x)$ does not exist.

⑱ Graph $f(x) = \begin{cases} x^3, & x \neq 1 \\ 0, & x = 1 \end{cases}$

$\lim_{x \to 1^-} f(x) = ?$ $\lim_{x \to 1^+} f(x) = ?$



$\lim_{x \to 1^-} f(x) = 1 = \lim_{x \to 1^+} f(x)$

$\lim_{x \to 1} f(x) = 1.$

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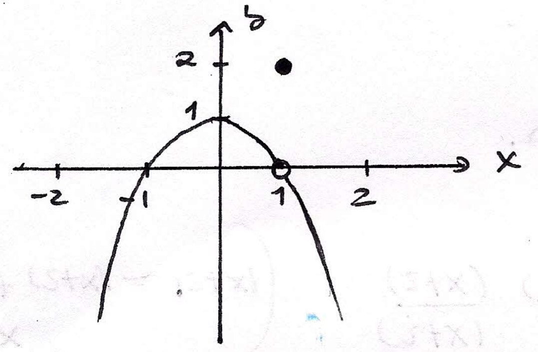
DİKKAT: Cep telefonunuzu görünmeyen bir yerde ve kapalı tutunuz.

⑲ Graph $f(x) = \begin{cases} 1-x^2 , & x \neq 1 \\ 2 , & x = 1 \end{cases}$

$\lim_{x \to 1^+} f(x) = ?$ $\lim_{x \to 1^-} f(x) = ?$

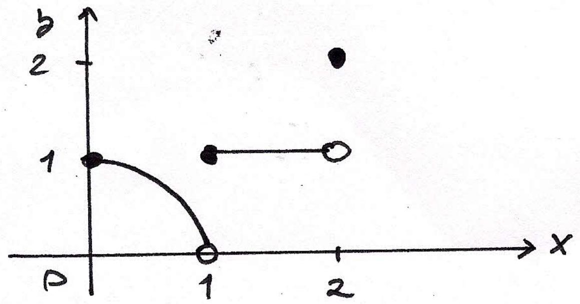


$\lim_{x \to 1^-} f(x) = 0 = \lim_{x \to 1^+} f(x)$

$\lim_{x \to 1} f(x) = 0 .$

⑳ Graph $f(x) = \begin{cases} \sqrt{1-x^2} , & 0 \leq x < 1 \\ 1 , & 1 \leq x < 2 \\ 2 , & x = 2 \end{cases}$

a) Domain ? Range ?
b) At what points c, if any, does $\lim_{x \to c} f(x)$ exist ?
c) At what points does only the left-hand limit exist ?
d) At what points " " " right-hand " "



ⓐ Domain : $0 \leq x \leq 2$

Range : $0 \leq y \leq 1$ and $y = 2$

ⓑ $\lim_{x \to c} f(x)$ exists for c belonging to $(0,1) \cup (1,2)$

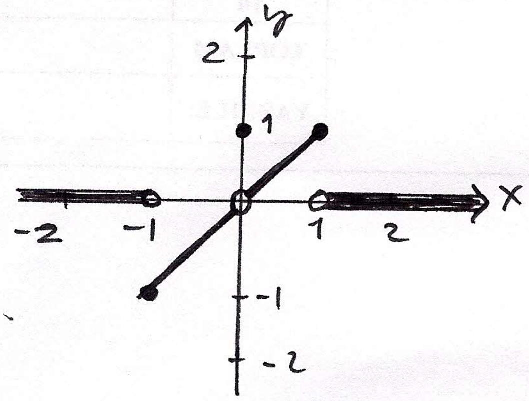
ⓒ $x = 2$

ⓓ $x = 0$

(21) Same question

$$f(x) = \begin{cases} x, & -1 \leq x < 0 \quad \text{or} \quad 0 < x \leq 1 \\ 1, & x = 0 \\ 0, & x < -1 \quad \text{or} \quad x > 1 \end{cases}$$

Gözme



a) Domain : $-\infty < x < +\infty$

Range : $-1 \leq y \leq 1$

b) $\lim_{x \to c} f(x)$ exists for c belonging to $(-\infty,-1) \cup (-1,1) \cup (1,\infty)$

c) none

d) none.

(22) $$\lim_{x \to 1^+} \sqrt{\frac{x-1}{x+2}} = \frac{\sqrt{1-1}}{\sqrt{1+2}} = \sqrt{0} = 0.$$

(23) $$\lim_{x \to -2^+} (x+3) \frac{|x+2|}{x+2} = \lim_{x \to -2^+} (x+3) \frac{(x+2)}{(x+2)}$$

$\left(|x+2| = (x+2) \text{ for } x > -2 \right)$

$$= \lim_{x \to -2^+} (x+3) = [(-2)+3] = 1$$

$$\lim_{x \to -2^-} (x+3) \frac{|x+2|}{x+2} = \lim_{x \to -2^-} (x+3) \frac{-(x+2)}{(x+2)}$$

$\left(|x+2| = -(x+2) \text{ for } x < -2 \right)$

$$= \lim_{x \to -2^-} (x+3)(-1) = -(-2+3)$$
$$= -1$$

(24) $$\lim_{x \to 3^+} \frac{\lfloor x \rfloor}{x} = \frac{3}{3} = 1$$

$$\lim_{x \to 3^-} \frac{\lfloor x \rfloor}{x} = \frac{2}{3}$$

⑤

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Use $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$

DİKKAT: Cep telefonunuzu görünmeyen bir yerde ve kapalı tutunuz.

㉕ $\lim_{t\to 0} \frac{\sin kt}{t}$ (k constant)

$$\lim_{t\to 0} \frac{\sin kt}{t} = \lim_{t\to 0} k\frac{\sin kt}{kt} = \lim_{\theta\to 0} \frac{k\sin\theta}{\theta} = k\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = k\cdot 1$$

$\theta = kt$

$$= k.$$

㉖ $\lim_{x\to 0} \frac{x + x\cos x}{\sin x\cos x} = ?$

$$\lim_{x\to 0}\left(\frac{x}{\sin x\cos x} + \frac{x\cos x}{\sin x\cos x}\right) = \lim_{x\to 0}\left(\frac{x}{\sin x}\cdot\frac{1}{\cos x}\right) + \lim_{x\to 0}\frac{x}{\sin x}$$

$$= \lim_{x\to 0}\left(\frac{1}{\frac{\sin x}{x}}\right)\lim_{x\to 0}\left(\frac{1}{\cos x}\right) + \lim_{x\to 0}\left(\frac{1}{\frac{\sin x}{x}}\right) = (1)(1) + 1 = 2$$

㉗ $\lim_{\theta\to 0}\frac{1-\cos\theta}{\sin 2\theta} = ?$

$$\lim_{\theta\to 0}\frac{(1-\cos\theta)(1+\cos\theta)}{(2\sin\theta\cos\theta)(1+\cos\theta)} = \lim_{\theta\to 0}\frac{1-\cos^2\theta}{(2\sin\theta\cos\theta)(1+\cos\theta)} = \lim_{\theta\to 0}\frac{\sin^2\theta}{(2\sin\theta\cos\theta)(1+\cos\theta)}$$

$$= \lim_{\theta\to 0}\frac{\sin\theta}{(2\cos\theta)(1+\cos\theta)} = \frac{0}{(2)(2)} = 0.$$

(28) $\lim_{x\to 0} \frac{x - x\cos x}{\sin^2 3x} = ?$

$$\lim_{x\to 0}\frac{x(1-\cos x)}{\sin^2 3x} = \lim_{x\to 0}\frac{\frac{x(1-\cos x)}{9x^2}}{\frac{\sin^2 3x}{9x^2}} = \lim_{x\to 0}\frac{\frac{1-\cos x}{9x}}{\left(\frac{\sin 3x}{3x}\right)^2}$$

$$= \frac{\frac{1}{9}\lim_{x\to 0}\left(\frac{1-\cos x}{x}\right)}{\lim_{x\to 0}\left(\frac{\sin 3x}{3x}\right)^2} = \frac{\frac{1}{9}(0)}{1^2} = 0.$$

(29) Suppose that f is an odd function of x. Does knowing that $\lim_{x\to 0^+} f(x)=3$ tell you anything about $\lim_{x\to 0^-} f(x)$?

If f is an odd function of then $f(-x)=-f(x)$.

Given $\lim_{x\to 0^+} f(x)=3$ then $\lim_{x\to 0^-} f(x)=-3$.

(30) Suppose that f is an even function of x. Does knowing that $\lim_{x\to 2^-} f(x)=7$ tell you anything about either $\lim_{x\to -2^-} f(x)$ or $\lim_{x\to -2^+} f(x)$?

ÖDEV

If f is an even function of x, then $f(-x)=f(x)$. Given $\lim_{x\to 2^-} f(x)=7$ then $\lim_{x\to -2^+} f(x)=7$. However, nothing can be said about $\lim_{x\to -2^-} f(x)$ because we don't know $\lim_{x\to 2^+} f(x)$.

yes

(31) $\lim_{x\to 2^+}\frac{x-2}{|x-2|} = 1$ Use definitions of right-hand and left-hand limits to prove.

$x\to 2^+ \Rightarrow x>2 \Rightarrow |x-2| = x-2$

$\Rightarrow \left|\frac{x-2}{|x-2|} - 1\right| = \left|\frac{x-2}{x-2} - 1\right| < \varepsilon \Rightarrow 0<\varepsilon$ which is always true so long as $x>2$. Hence we can choose any $\delta>0$, and thus $2<x<2+\delta \Rightarrow \left|\frac{x-2}{|x+2|} - 1\right| < \varepsilon$. Thus, $\lim_{x\to 2^+}\frac{x-2}{|x-2|} = 1.$

⑥

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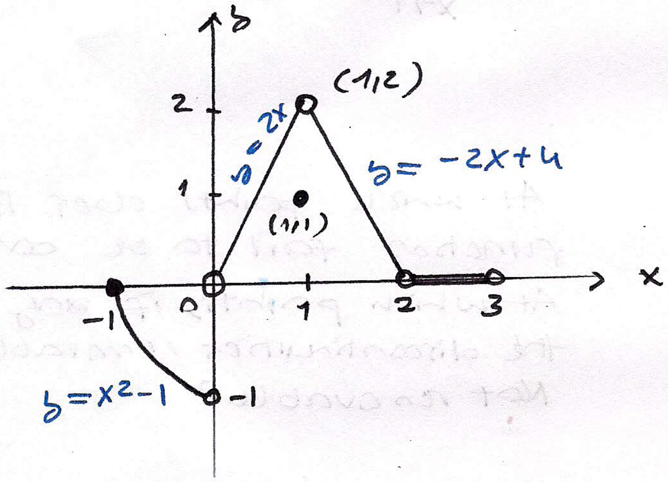
(continuity)

DİKKAT: Cep telefonunuzu görünmeyen bir yerde ve kapalı tutunuz.

㉜

$$f(x) = \begin{cases} x^2-1 & , -1 \le x < 0 \\ 2x & , 0 < x < 1 \\ 1 & , x = 1 \\ -2x+4 & , 1 < x < 2 \\ 0 & , 2 < x < 3 \end{cases}$$



a) $\lim\limits_{x \to -1^+} f(x) = 0$

$\lim\limits_{x \to -1^+} f(x) = 0 = f(-1)$

f is continuous at x=-1.

($\lim\limits_{x \to a^+} f(x) = f(a) \Rightarrow$ sürekli idi
burada da $\lim\limits_{x \to -1^+} f(x) = f(-1)$)

b) $\lim\limits_{x \to 1} f(x) = 2$

$f(1) = 1$

$\lim\limits_{x \to 1} f(x) \neq f(1) \Rightarrow$ f is discontinuous at x=1.

c) Is f defined at $x=2$?

No.

f is not continuous at $x=2$.

$\lim_{x\to 2} f(x)=0$. ~~but~~ $f(2)$ does not exist.

d) At what values of x is f continuous?

$[-1,0)\cup(0,1)\cup(1,2)\cup(2,3)$

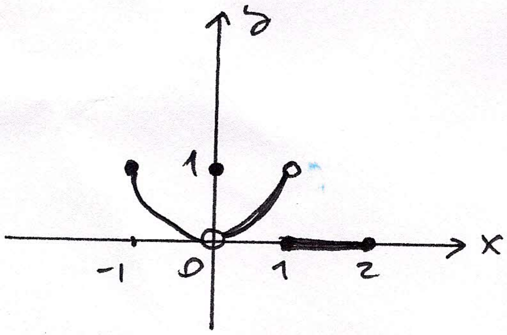
e) What value should be assigned to $f(2)$ to make the extended function continuous at $x=2$?

$f(2)=0$ since $\lim_{x\to 2^-} f(x)=0=\lim_{x\to 2^+} f(x)$

f) To what new value should $f(1)$ be changed to remove the discontinuity?

$f(1)$ should be changed to $2=\lim_{x\to 1} f(x)$

33



At which points does the function fail to be continuous? At which points, if any, are the discontinuities removable? Not removable?

$x=0$, $x=1$.

Nonremovable discontinuity at $x=1$ because $\lim_{x\to 1} f(x)$ fails to exist. ($\lim_{x\to 1^-} f(x)=1$ and $\lim_{x\to 1^+} f(x)=0$)

Removable discontinuity at $x=0$ by assigning the number $\lim_{x\to 0} f(x)=0$ to be the value of $f(0)$ rather than $f(0)=1$.

⑦

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DİKKAT: Cep telefonunuzu görünmeyen bir yerde ve kapalı tutunuz.

㉞ At what points are the functions continuous?

$y = \frac{1}{(x+2)^2} + 4$

↓

discontinuous only when

$(x+2)^2 = 0 \Rightarrow x = -2$

$y = \frac{x+3}{x^2-3x-10}.$

↓

discontinuous only when

$x^2-3x-10=0 \Rightarrow (x-5)(x+2)=0$

$x=5$ or $x=-2$

㉟ a) $y = |x-1| + \sin x$

b) $y = \frac{1}{|x|+1} - \frac{x^2}{2}$

a) continuous everywhere. $|x-1|+\sin x$ defined for all x ; limits exist and are equal to function values.

b) continuous everywhere. $|x|+1 \neq 0$ for all x. limits exist and are equal to function values.

㊱ a) $y = \frac{x+2}{\cos x}$

b) $y = \frac{\sqrt{x^4+1}}{1+\sin^2 x}.$

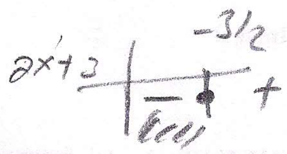
a) Discontinuous at odd integer multiples of $\frac{\pi}{2}$, i.e.,
$x = (2n-1)\frac{\pi}{2}$, n an integer, but continuous at all other x.

b) continuous everywhere since $x^4+1 \geq 1$ and
$-1 \leq \sin x \leq 1 \Rightarrow 0 \leq \sin^2 x \leq 1 \Rightarrow 1+\sin^2 x \geq 1$ ⇒ limits exist
and are equal to the function values.

(37) a) $y = \sqrt{2x+3}$

b) $y = \sqrt[4]{3x-1}$

(a) Discontinuous when $2x+3 < 0$ or $x < -3/2$



⇒ continuous on the interval $[-3/2, \infty)$.

(b) Discontinuous when $3x-1 < 0$ or $x < 1/3$

⇒ continuous on the interval $[1/3, \infty)$

(38) $g(x) = \begin{cases} \dfrac{x^2-x-6}{x-3}, & x \neq 3 \\ 5, & x = 3 \end{cases}$

continuous everywhere since $\lim\limits_{x\to 3} \dfrac{x^2-x-6}{x-3} = \lim\limits_{x\to 3} \dfrac{(x-3)(x+2)}{(x-3)} = 5 = g(3)$

(39) Are the functions continuous at the point being approached?

a) $\lim\limits_{x\to\pi} \sin(x-\sin x)$. b) $\lim\limits_{x\to 0} \tan\left(\frac{\pi}{4}\cos(\sin x^{1/3})\right)$

(a) $\lim\limits_{x\to\pi} \sin(x-\sin x) = \sin(\pi - \sin\pi) = \sin(\pi - 0) = \sin\pi = 0$, and
function continuous at $t=0$.

(b) $\lim\limits_{x\to 0} \tan\left(\frac{\pi}{4}\cos(\sin x^{1/3})\right) = \tan\left(\frac{\pi}{4}\cos(\sin 0)\right) = \tan\left(\frac{\pi}{4}\cos 0\right)$
$= \tan\pi/4 = 1$

and function continuous at $x=0$.

⑪

$\sqrt{x^2} \to |x|$

$x\to-\infty$ $-x$

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$$\frac{x^{1/3}-x^{1/5}}{x^{1/3}+x^{1/5}} = \frac{1-x^{1/5}/x^{1/3}}{1+x^{1/5}/x^{1/3}}.$$

payı ve paydayı en yüksek mertebelerine bölelim.

DİKKAT: Cep telefonunuzu görünmeyen bir yerde ve kapalı tutunuz.

⑥ $$\lim_{x\to-\infty}\frac{\sqrt[3]{x}-\sqrt[5]{x}}{\sqrt[3]{x}+\sqrt[5]{x}} = \lim_{x\to-\infty}\frac{1-x^{(1/5)-(1/3)}}{1+x^{(1/5)-(1/3)}}$$

$$=\lim_{x\to-\infty}\frac{1-\frac{1}{x^{2/15}}}{1+\frac{1}{x^{2/15}}} = 1$$

→ [illegible]

⑦ $$\lim_{x\to-\infty}\frac{\sqrt[3]{x}-5x+3}{2x+x^{2/3}-4} = \lim_{x\to-\infty}\frac{1/x^{2/3}-5+3/x}{2+\frac{1}{x^{1/3}}-\frac{4}{x}} = -\frac{5}{2}$$

⑧ $$\lim_{x\to-\infty}\frac{\sqrt{x^2+1}}{x+1} = \lim_{x\to-\infty}\frac{\sqrt{x^2+1}\,/\sqrt{x^2}}{(x+1)\,/\sqrt{x^2}} = \lim_{x\to-\infty}\frac{\sqrt{(x^2+1)/x^2}}{(x+1)/(-x)}$$

$\sqrt{x^2}=|x|=-x$
$x\to-\infty$ için

$$=\lim_{x\to-\infty}\frac{\sqrt{1+\frac{1}{x^2}}}{-1-\frac{1}{x}} = \frac{\sqrt{1+0}}{(-1-0)} = -1.$$

⑨ $$\lim_{x\to7}\frac{1}{(x-7)^2} = \infty.$$ (sağ sol aynı.)

⑩ $$\lim_{x\to0}\frac{-1}{x^2(x+1)} = -\infty$$

↓
positive

↳ 0'a soldan da yaklaşsan sağdan da yaklaşsan positive.

⑪ $\lim\limits_{x\to 3^+} \frac{1}{x-3} = \infty$ $\frac{\text{positive}}{\text{positive}}$

⑫ $\lim \left(\frac{x^2}{2} - \frac{1}{x}\right)$ as

a) $x\to 0^+$

b) $x\to 0^-$

c) $x\to -1$

$\lim\limits_{x\to 0^+}\left(\frac{x^2}{2}-\frac{1}{x}\right) = 0 + \lim\limits_{x\to 0^+}\frac{1}{-x} = -\infty.$

$\lim\limits_{x\to 0^-}\left(\frac{x^2}{2}-\frac{1}{x}\right) = 0 + \lim\limits_{x\to 0^-}\frac{1}{-x} = \infty.$

$\lim\limits_{x\to -1}\left(\frac{x^2}{2}-\frac{1}{x}\right) = \frac{1}{2} - \left(\frac{1}{-1}\right) = \frac{3}{2}$

⑬ $\lim\limits_{x\to 0^+}\left(2 - \frac{3}{x^{1/3}}\right) = -\infty$ $\lim\limits_{x\to 0^-}\left(2-\frac{3}{x^{1/3}}\right) = \infty.$

⑭ $\lim\limits_{x\to\infty}\left(\sqrt{x+9} - \sqrt{x+4}\right) = ?$

$\lim\limits_{x\to\infty}\left(\sqrt{x+9}-\sqrt{x+4}\right)\left[\frac{\sqrt{x+9}+\sqrt{x+4}}{\sqrt{x+9}+\sqrt{x+4}}\right] = \lim\limits_{x\to\infty}\frac{(x+9)-(x+4)}{\sqrt{x+9}+\sqrt{x+4}}$

$= \lim\limits_{x\to\infty}\frac{5}{\sqrt{x+9}+\sqrt{x+4}} = \lim\limits_{x\to\infty}\frac{5/\sqrt{x}}{\sqrt{1+\frac{9}{x}}+\sqrt{1+\frac{4}{x}}} = \frac{0}{1+1} = 0.$

⑮ $\lim\limits_{x\to-\infty}\left(\sqrt{x^2+3}+x\right) = ?$

$\lim\limits_{x\to-\infty}\left[\sqrt{x^2+3}+x\right]\left[\frac{\sqrt{x^2+3}-x}{\sqrt{x^2+3}-x}\right] = \lim\limits_{x\to-\infty}\frac{(x^2+3)-x^2}{\sqrt{x^2+3}-x}.$

$\sqrt{x^2} = |x|$
$x\to-\infty$ olduğundan
$-x$ dışa çıkar.

$= \lim\limits_{x\to-\infty}\frac{3}{\sqrt{x^2+3}-x} = \lim\limits_{x\to-\infty}\frac{3/\sqrt{x^2}}{\sqrt{1+\frac{3}{x^2}} - \frac{x}{\sqrt{x^2}}} = \lim\limits_{x\to-\infty}\frac{-3/x}{\sqrt{1+\frac{3}{x^2}}+1}.$

$= \frac{0}{1+1} = 0.$

(12)

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DİKKAT: Cep telefonunuzu görünmeyen bir yerde ve kapalı tutunuz.

(16) Graph the rational function. Include the graph and equations of the asymptotes.

$y = \frac{x^2}{x-1}$

degree of numerator is 1 greater then the degree of denominator.

$$\begin{array}{l|l} x^2 & x-1 \\ \underline{-x^2 \overset{+}{-} x} & x+1 \\ \quad x & \\ \underline{\; -x \overset{+}{-} 1} & \\ \quad 1 & \end{array}$$

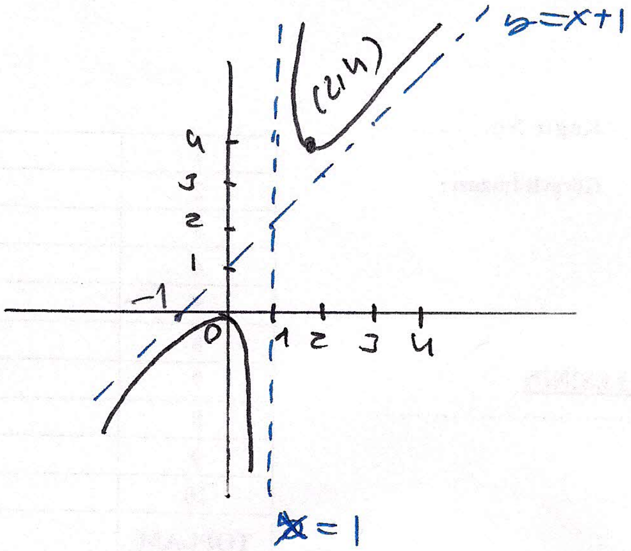
$$\frac{x^2}{x-1} = \underbrace{x+1}_{\substack{\text{linear} \\ g(x)}} + \underbrace{\frac{1}{x-1}}_{\text{remainder}}$$

$g(x) = x+1$ is the oblique asymptote.

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{x^2}{x-1} = \infty$$

$$\lim_{x\to 1^-} \frac{x^2}{x-1} = -\infty.$$

$x = 1$ is a vertical asymptote.



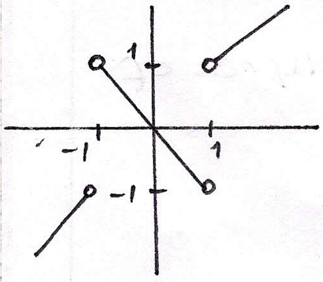
$x \to 1^-$ $y = -\infty$
$x \to 1^+$ $y = +\infty$.

⑰ Can $f(x) = x(x^2-1) / |x^2-1|$ be extended to be continuous at $x=1$ or -1 ? Give reasons for your answer.

At $x=-1$: • $\lim\limits_{x\to -1^-} f(x) = \lim\limits_{x\to -1^-} \dfrac{x(x^2-1)}{|x^2-1|} = \lim\limits_{x\to -1^-} \dfrac{x(x^2-1)}{x^2-1}$

$\downarrow$
(−2) koy

$= \lim\limits_{x\to -1^-} x = -1$



• $\lim\limits_{x\to -1^+} f(x) = \lim\limits_{x\to -1^+} \dfrac{x(x^2-1)}{|x^2-1|} = \lim\limits_{x\to -1^+} \dfrac{x(x^2-1)}{-(x^2-1)}$

$\downarrow$
0 koy

$= \lim\limits_{x\to -1^+} (-x) = -(-1) = 1$

Since $\lim\limits_{x\to -1^-} f(x) \neq \lim\limits_{x\to -1^+} f(x) \Rightarrow \lim\limits_{x\to -1} f(x)$ does not exist;

the function cannot be extended to a continuous function at $x=-1$.

At $x=1$: $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} \dfrac{x(x^2-1)}{|x^2-1|} = \lim\limits_{x\to 1^-} \dfrac{x(x^2-1)}{-(x^2-1)} = \lim\limits_{x\to 1^-} (-x) = -1$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} \dfrac{x(x^2-1)}{|x^2-1|} = \lim\limits_{x\to 1^+} \dfrac{x(x^2-1)}{x^2-1} = \lim\limits_{x\to 1^+} x = 1$

$\Rightarrow \lim\limits_{x\to 1} f(x)$ does not exist. So f can not be extended to a continuous function at $x=1$ either.