Eskişehir Osmangazi University - Electrical Engineering Department
Advanced Calculus
2nd Midterm Examination - Fall 2011

!!! For each problem put the answer next to it !!!
1. Evaluate

$$\int_C (z+1)dz$$

where $C:\ 2e^{it},\ 0 \le t \le 3\frac{\pi}{4}$
Solution The integrand is entire, therefore, only the initial and final points matter! So

$$\int_C (z+1)dz = \left[\frac{z^2}{2}+z\right]_{2e^{i0}}^{2e^{3\pi i/4}} = \left[\frac{z^2}{2}+z\right]_{2}^{-1.41+i1.41} = -5.41 - i0.58$$

Of course, one may also solve it by parametrizing and using the definition of the integral.
2. Evaluate:

$$\int_D \frac{dz}{(z-1)(z-2)(z+2)}$$

where D is the positively oriented circle centered at $z=1$ with radius 2.
Solution Two poles are inside the contour, therefore, we need the residues at $z=2$ and $z=2$. At $z=1$ it is $\left[\frac{1}{(z-2)(z+2)}\right]_{z=1} = \frac{-1}{3}$ and at $z=2$ it is $\left[\frac{1}{(z-1)(z+2)}\right]_{z=2} = \frac{1}{4}$. The answer is $2\pi i(\frac{-1}{3}+\frac{1}{4}) = \frac{-\pi i}{6}$
3. Solve

$$\frac{dy}{dx} + 6y = 3e^x$$

Solution First order linear d.e., by the closed form formula solution is
$\frac{3}{7}e^x + Ce^{-6x}$ where C is an arbitrary constant.
4. Solve

$$2xdx + 4ydy = 0,\ y(1) = 2$$

Solution First order separable d.e. Integrate throughout: $x^2+2y^2=c$. Use the initial conditions: $1^2+2\times 2^2 = c \to c = 9$. Thus the solution is: $x^2+2y^2=9$.
Good Luck!
A. Karamancıoğlu