2. Operational Amplifiers

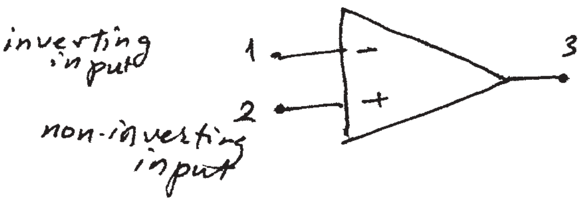
An integrated ckt (IC) amplifier. It is used in countless circuits.

Early amplifiers: Vacuum tubes, discrete transistors

Mid 60s: The first IC opamp $\mu A709$ expensive

Today: high-quality opamp – dirt cheap!

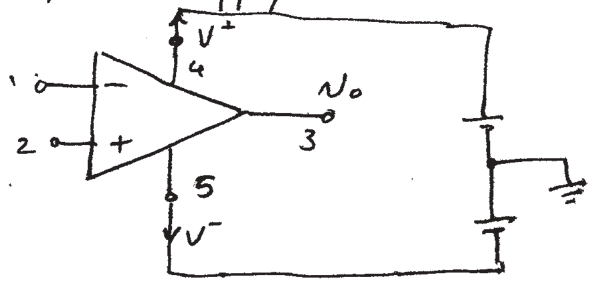
2.1 The OPAMP Terminals



1, 2 : input terminals

3 : Output terminal

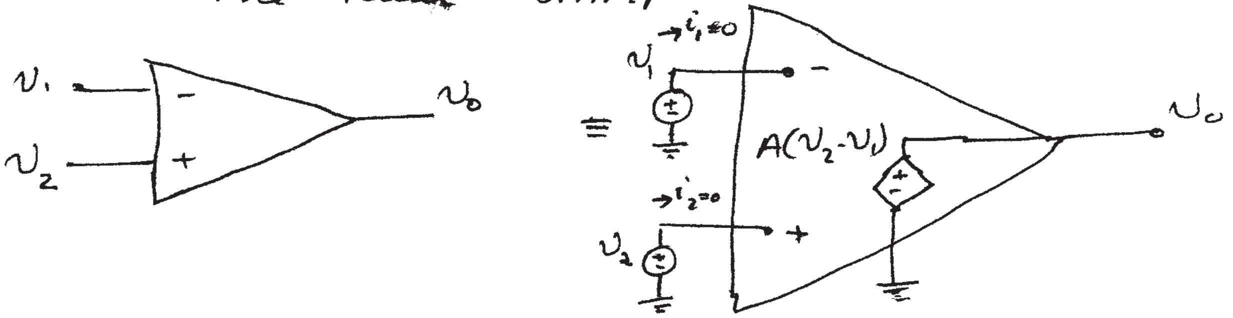
Because it needs DC power, sometimes the powersupply leads are also shown:



← This is the voltage-reference for inputs and output

There are other terminals for specific purposes: e.g., frequency compensation, offset nulling.

2.2 The ideal OPAMP



No currents goes in at the input

no limit on the output current

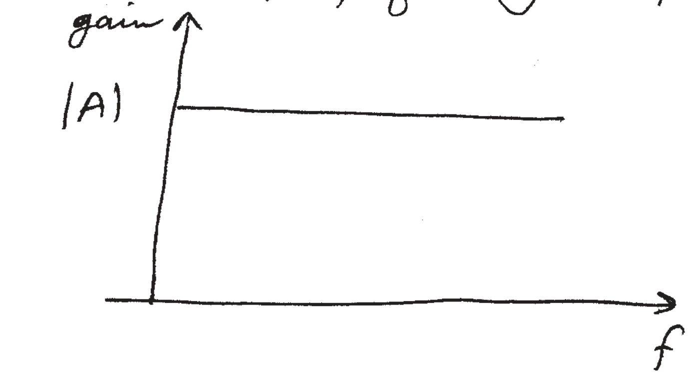
$v_o = A(v_2 - v_1)$, $A \to \infty$ $\Rightarrow$ needs negative f.b.

Opamp is a differential-input, single-ended amplifier.

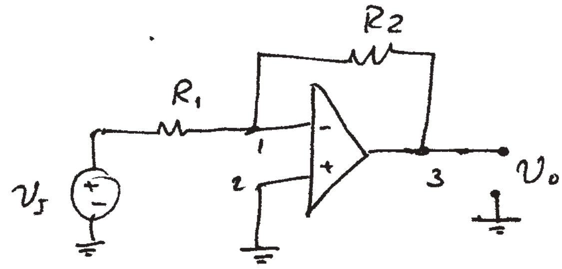
A: differential gain, Open-loop gain

Overall gain of the amplifier circuit $\longrightarrow$ closed-loop gain

ideal Opamp frequency response



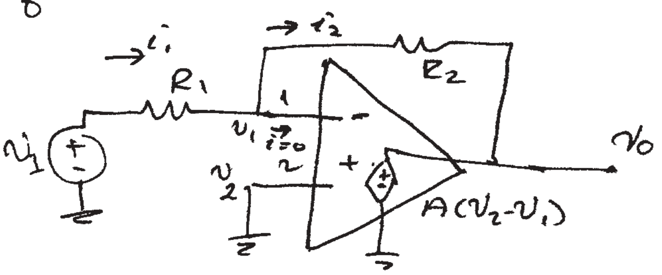
2.3 OPAMP in Inverting Configuration



R_2 provides negative feedback because a portion of the output voltage is brought to the inverting input.

Closed-loop gain (overall circuit gain) $G \equiv \frac{v_o}{v_I}$

Equivalent ckt for the amplifier



ideal opamp, finite output → $v_o = A(v_2 - v_1)$

$$v_2 - v_1 = \frac{v_o}{A} \cong 0$$

Since v_2 is connected to the ground, v_1 has the same potential → virtual ground. No current goes into the Opamp at the input terminals.

$$i_1 = \frac{v_I - v_1}{R_1} = \frac{v_I}{R_1}, \quad i_2 = i_1, \quad v_o = v_1 - i_2 R_2$$

$$v_o = -i_2 R_2 = -i_1 R_2$$

$$v_o = -\frac{R_2}{R_1} v_I$$

$$\boxed{G = \frac{v_o}{v_I} = -\frac{R_2}{R_1}}$$

example: $R_1 = 1\,k\Omega$
$R_2 = 10\,k\Omega$
$G = -10$

$A \to \infty, \quad G = -10$

Finite open-loop gain → $v_2 - v_1 = v_o/A$

$$v_2 = 0 \Rightarrow v_1 = -\frac{v_o}{A}$$

$$i_1 = \frac{v_I + \frac{v_o}{A}}{R_1}, \quad i_2 = i_1, \quad v_o = -\frac{v_o}{A} - \left(\frac{v_I + v_o/A}{R_1}\right) R_2$$

$$G = \frac{v_o}{v_I} = \frac{-R_2/R_1}{1 + \left(1 + \frac{R_2}{R_1}\right)/A} \qquad A \to \infty \Rightarrow G = -R_2/R_1$$

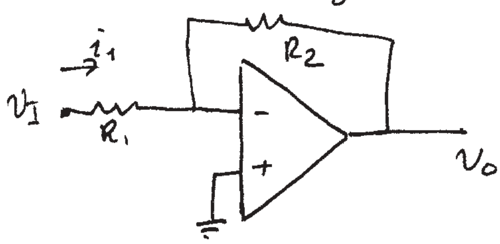
$\left(1 + \frac{R_2}{R_1}\right)/A$ in the denominator should be made ~~as~~ small so that variations in A does not affect the overall gain.

$$1 \gg \frac{1 + R_2/R_1}{A}$$

Example $R_1 = 1\,k\Omega$, $R_2 = 100\,k\Omega$

A =	10^3	10^4	10^5	∞
G =	90.83	99.00	99.90	100

input resistance of the amplifier:



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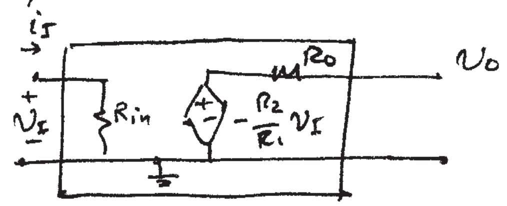


$$R_{in} = \frac{v_1}{i_1} = R_1 \qquad\qquad R_o = 0$$

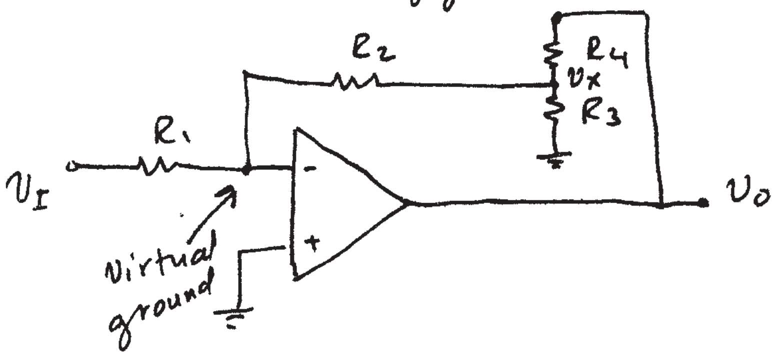
Design problem: An inverting amplifier is to be designed. Specs: $R_{in} = 1\ M\Omega$, $|G| = 100$.

Solution : $R_1 = 1 M\Omega$ $\quad R_2 = \underline{\underline{100}}\ M\Omega$.

↑ this is very large!

In practice we keep the resistors in $k\Omega$ range preferable not to exceed $1 M\Omega$.

Here is a configuration:



R_3 and R_4 work almost like a voltage divider.

$$v_x = v_o \frac{R_2 \| R_3}{R_4 + R_2 \| R_3}$$

$$v_x = -\frac{R_2}{R_1} v_I$$

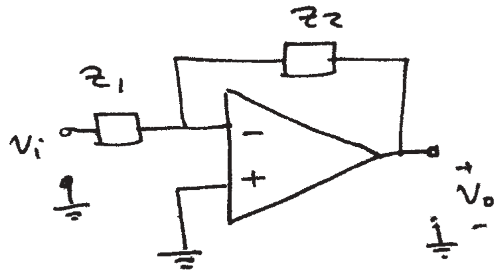
$$\therefore \quad \frac{v_o}{v_I} = -\frac{R_2}{R_1}\left(1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right) \quad \text{is obtained.}$$

$R_{in} = R_1 = 1 M\Omega$, $\quad R_2 = 1 M\Omega$, $\quad R_4 = 1\ M\Omega \Rightarrow$ ~~$R_3 = \ldots$~~

$R_3 = 10.2\ k\Omega$

2.4 Other Inverting Amp Configurations

In general



$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$$

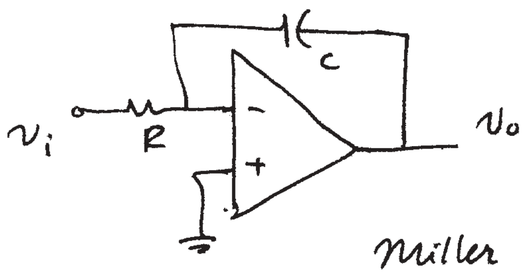
Specific examples: $Z_1 = R$, $Z_2 = \frac{1}{sC}$

$$\frac{V_o}{V_i} = -\frac{1}{sCR} \quad \text{or} \quad \frac{V_o}{V_i} = -\frac{1}{j\omega CR}$$

in time domain

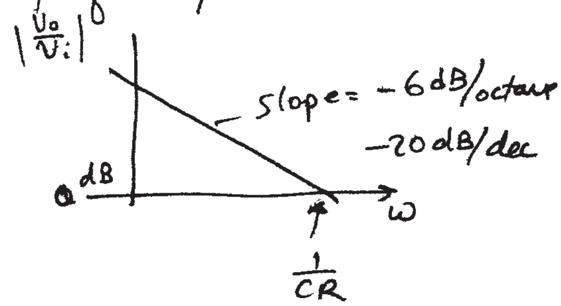
$$i_1 = \frac{V_I(t)}{R}, \qquad V_o = -\frac{1}{C}\int_0^t \frac{V_I(t)}{R}\,dt + V_C(0)$$

$$= -\frac{1}{RC}\int_0^t V_I(t)\,dt + V_C(0)$$

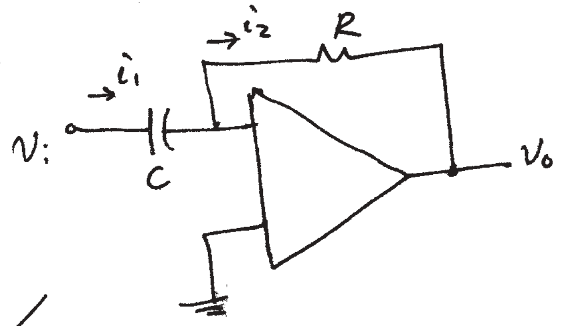


Miller integrator

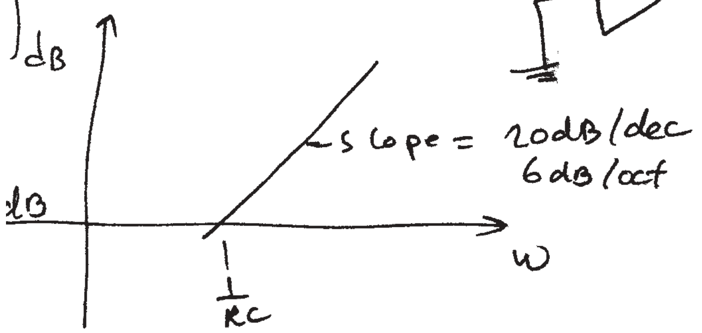
freq. response



Differentiator:



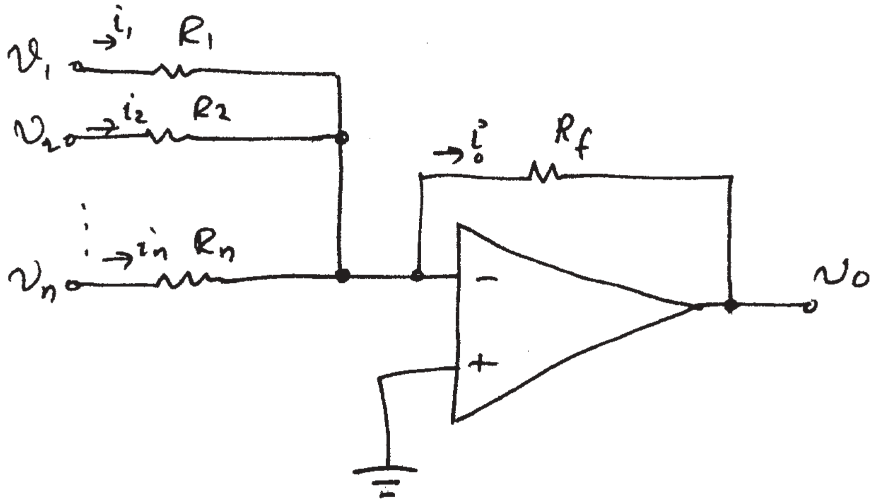
$$i_1 = C\frac{dV_i}{dt}$$



$$V_o = -i_2 R$$

$$V_o = -RC\frac{dV_i}{dt}$$

Summer (weighted summer)



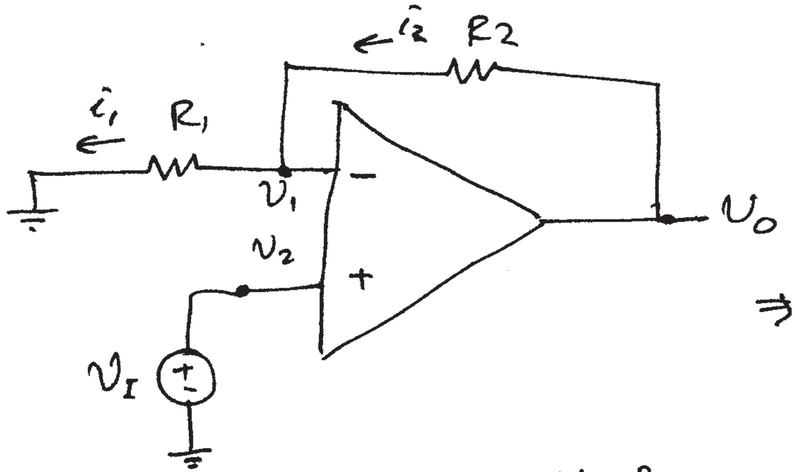
$$i_o = i_1 + i_2 + \cdots + i_n$$

$$= \frac{v_1}{R_1} + \frac{v_2}{R_2} + \cdots + \frac{v_n}{R_n} \quad \Rightarrow \quad v_o = -R_f\left(\frac{v_1}{R_1} + \frac{v_2}{R_2} + \cdots + \frac{v_n}{R_n}\right)$$

if $R_1 = R_2 = \cdots = R_n = R_f \quad \Rightarrow \quad v_o = -(v_1 + v_2 + \cdots + v_n)$

.5 Non-inverting Configuration

– still need negative feedback for linear operation



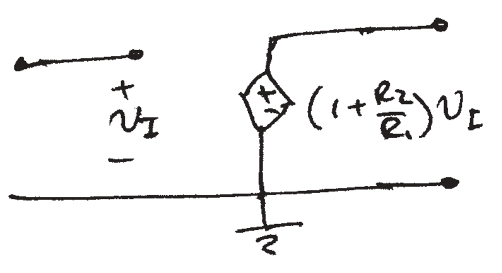
Negative feedback → $v_1 = v_2$

voltage divider

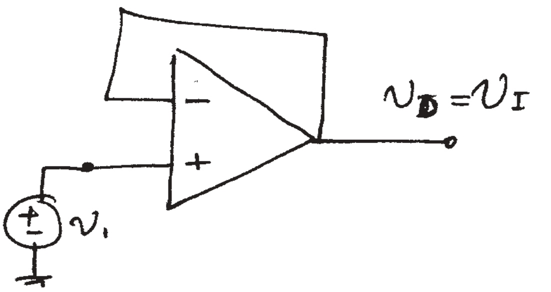
$$\Rightarrow v_1 = v_o \frac{R_1}{R_1 + R} = v_2 = v_1$$

$$v_1 = v_o \frac{R_1}{R_1 + R_2} \quad \Rightarrow \quad \frac{v_o}{v_1} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

eq. ckt:

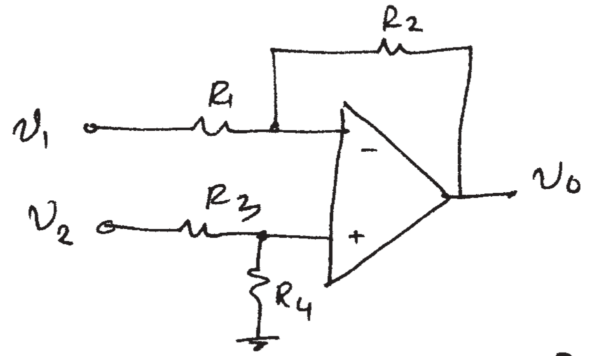


useful as buffer amplifier



voltage follower.

2.6 Other Examples: of Op Amp applications



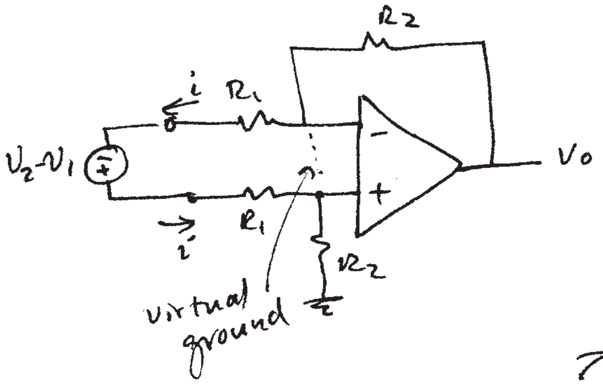
Difference amplifier

$$v_O = -\frac{R_2}{R_1} v_1 + \frac{1 + R_2/R_1}{1 + R_3/R_4} v_2$$

$$\text{if} \quad \frac{R_4}{R_3} = \frac{R_2}{R_1} \Rightarrow v_O = \frac{R_2}{R_1}(v_2 - v_1)$$

Since there are two inputs, a "differential" input resistance is defined:

$$R_{in} \equiv \frac{v_2 - v_1}{i}$$

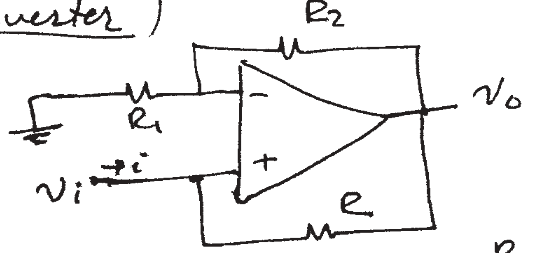


$$R_{in} = R_1 + R_1 = 2R_1$$

(instrumentation amplifier)

To improve the input resistance (i.e. to increase it) buffers may be placed before the inputs.

(Negative impedance converter)

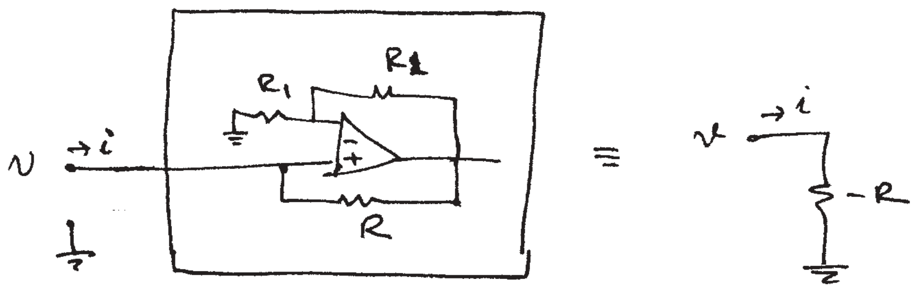


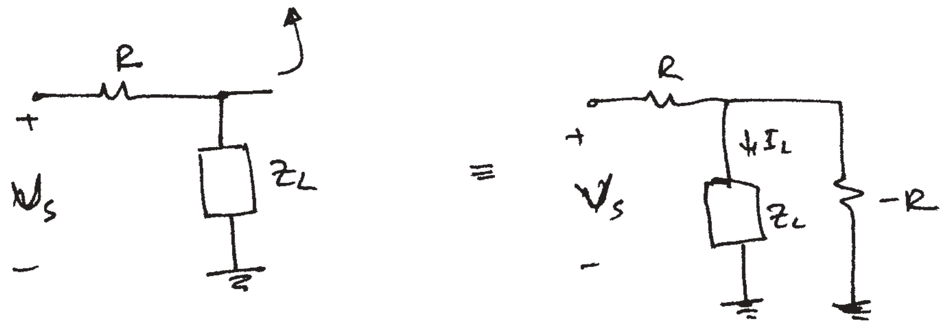
$$i = -\frac{(v_O - v_i)}{R} = -\frac{v\left(\cancel{1}+\frac{R_2}{R_1}\right)}{R}, \quad \frac{v}{i} = \frac{-R}{\left(\cancel{1}\,\frac{R_2}{R_1}\right)} = -R\frac{R_1}{R_2}$$

if $R_1 = R_2 \Rightarrow R_{in} = \frac{v_i}{i} = -R$

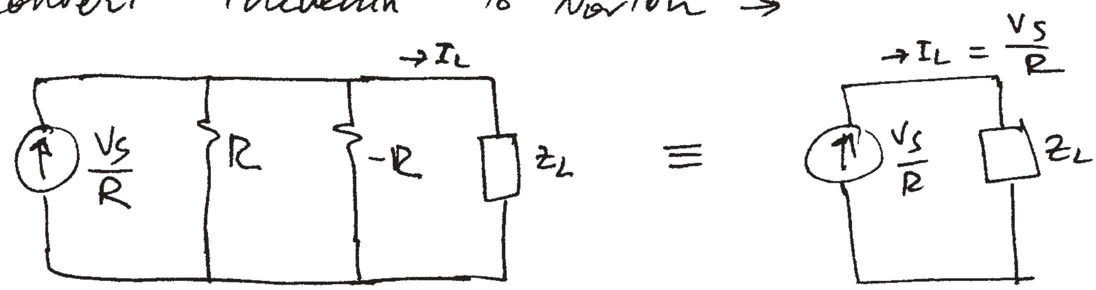
That means the ckt above is equivalent to a negative resistance. (The resistance may be replaced by an impedance.)

Consider the following:





Convert Thevenin to Norton →

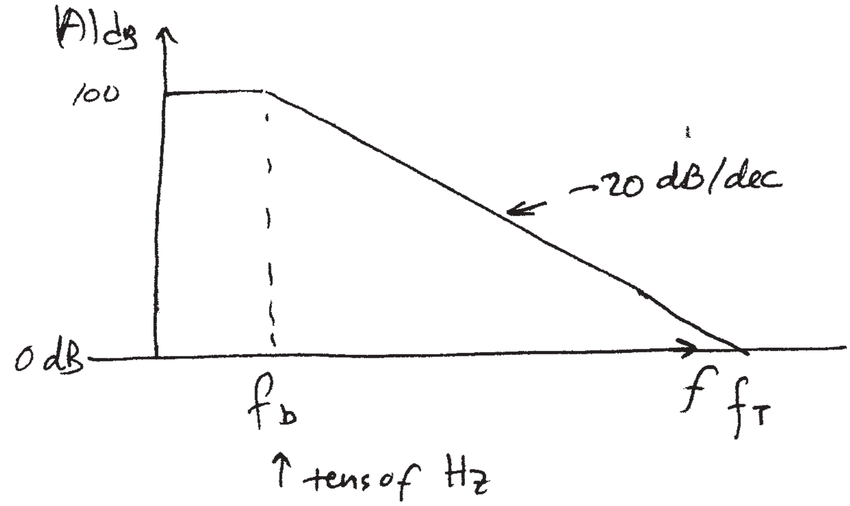


Read section 2.6 (pp. 67–76)

2.7 Nonideal Performance of OP AMPs

it is important to know the limitations so that one designs the ckt better.

Finite Open-Loop Gain and Bandwidth



internally frequency compensated opamp

(including a capacitor)

$$A(s) = \frac{A_0}{1+s/\omega_b}$$

$s = j\omega$,

$$A(j\omega) = \frac{A_0}{1+j\omega/\omega_b}$$

A_0: dc gain

ω_b: "break" frequency

high cut-off frequency

3 dB cut-off freq.

$\omega \gg \omega_b \Rightarrow A(j\omega) \simeq \frac{A_0\omega_b}{j\omega}$

$\omega_t = A_0\omega_b$ = unity-gain-bandwidth

= gain-bandwidth product.

$A(j\omega) \cong \frac{\omega_t}{j\omega}$ $\qquad A(s) \cong \frac{\omega_t}{s}$ ← integrator

Time constant $\tau = \frac{1}{\omega_t}$

Frequency Response of Closed-loop Amplifiers:

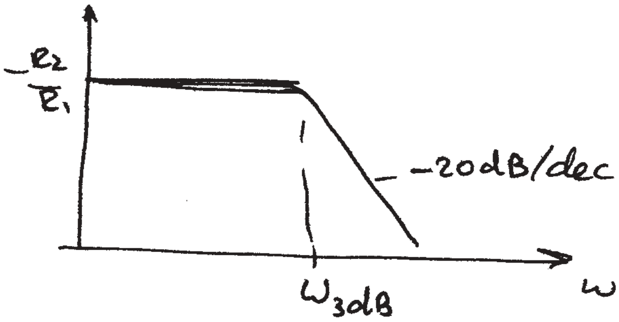
The inverting amplifier gain was found to be

$$\frac{V_o}{V_i} = \frac{-\frac{R_2}{R_1}}{1+\left(1+\frac{R_2}{R_1}\right)/A}$$

$$A(s) = \frac{A_o}{1+s/\omega_b}$$

This makes
$$\frac{V_o}{V_i} = \frac{-\frac{R_2}{R_1}}{1+\frac{1}{A_o}\left(1+\frac{R_2}{R_1}\right)+\frac{s}{\omega_t/\left(1+\frac{R_2}{R_1}\right)}}$$

$$A_o \gg 1+\frac{R_2}{R_1} \Rightarrow \quad \frac{V_o}{V_i} \cong \frac{-R_2/R_1}{1+\frac{s}{\omega_t/\left(1+\frac{R_2}{R_1}\right)}}$$



$$\omega_{3dB} = \frac{\omega_t}{1+\frac{R_2}{R_1}}$$

For the non-inverting amplifier

$$\frac{V_o}{V_i} = \frac{1+\frac{R_2}{R_1}}{1+\left(1+\frac{R_2}{R_1}\right)/A}, \quad A_o \gg 1+\frac{R_2}{R_1} \Rightarrow$$

$$\frac{V_o}{V_i} \cong \frac{1+R_2/R_1}{1+\frac{s}{\omega_t/\left(1+\frac{R_2}{R_1}\right)}} \qquad \omega_{3dB} = \frac{\omega_t}{1+\frac{R_2}{R_1}}$$

The feedback loop of the inverting and noninverting configurations are the same:

Forward path gain $\rightarrow$ $-A$

Feedback path voltage divider $\frac{R_1}{R_1+R_2} = \beta$

(feedback factor, β)

The loop gain $= -A\beta$

amount of feedback (return difference) $\equiv$ 1 − loop gain

$= 1 + A\beta$

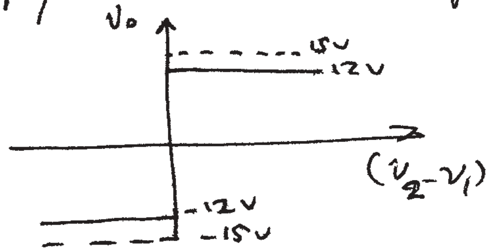
3 dB bandwidth $f_{3dB} = \beta f_t$

Large Signal Operation of OPAMPs

Output Saturation

The output voltage of an opamp is limited by the power supply. However, not all of the voltage of the suply is available for the output. It is 1-3 V less than the supply voltages in magnitude.

∓ 15 V supply $\rightarrow$ ∓ 12 V output (rated output voltage)

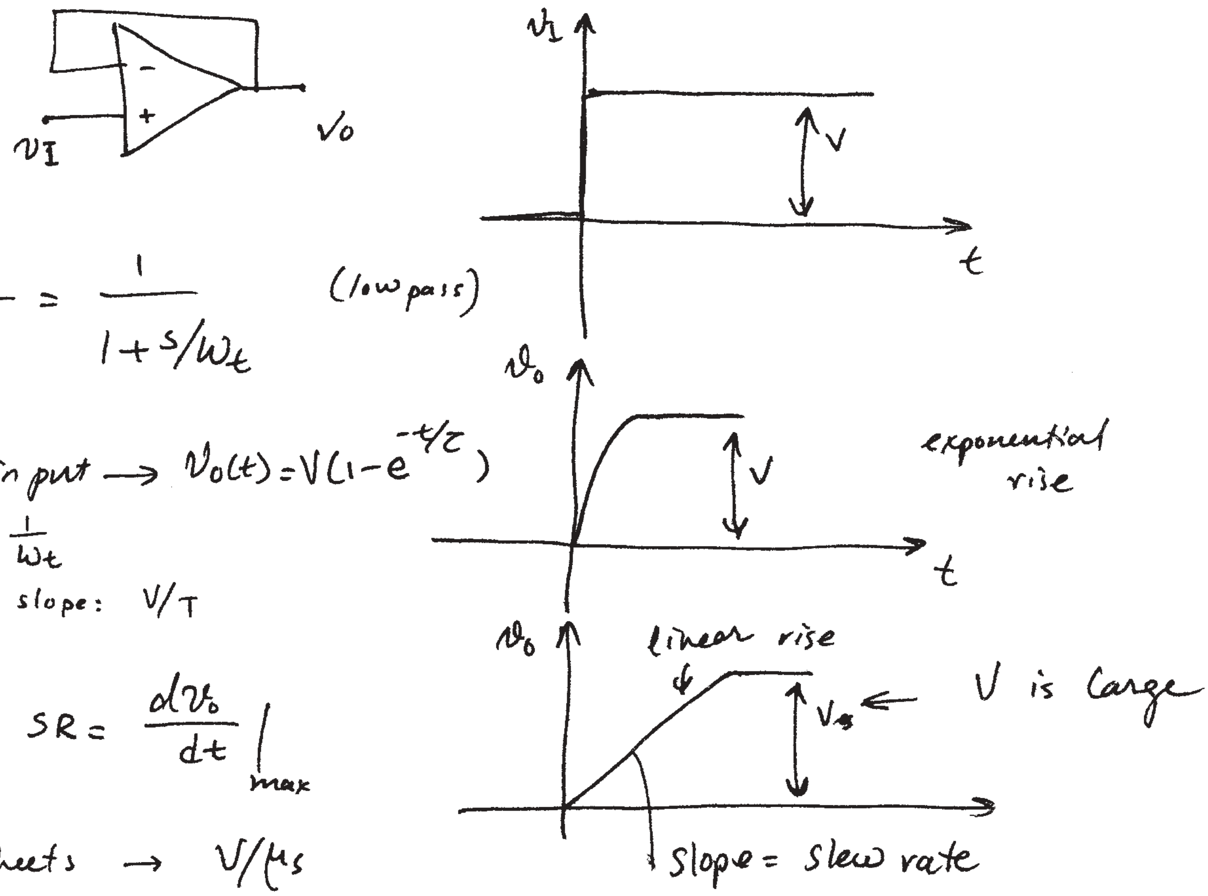


Slew Rate

The output voltage has a limit in increasing or decreasing. This is due to the internal capacitors of the operational amplifier and the limited current to charge them.

Sine/square wave $\Rightarrow$ triangular wave.

Consider the following unity gain ampl.



$$\frac{V_o}{V_i} = \frac{1}{1 + s/\omega_t} \quad \text{(low pass)}$$

step input $\rightarrow$ $v_o(t) = V(1 - e^{-t/\tau})$

$\tau = \frac{1}{\omega_t}$

initial slope: V/τ

ew Rate $\quad SR = \left.\frac{dv_o}{dt}\right|_{max}$

data sheets $\rightarrow$ $V/\mu s$

Full Power Bandwidth

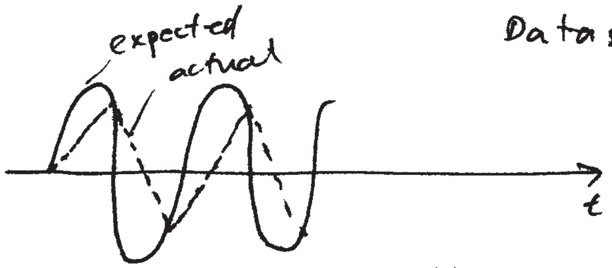
consider the unity gain amplifier.

input signal $\quad v_I = \hat{V}_i \sin\omega t$

$$\frac{dv_I}{dt} = \omega \hat{V}_i \cos\omega t \qquad \text{max slope} = \omega\hat{V}_i$$

at the zero crossing

if $\omega\hat{V}_i$ exceeds the slew rate, the output waveform will be distorted.



Datasheets f_M = full power bandwidth

(max operating freq.)

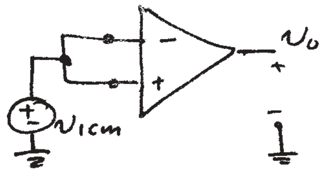
Rated - output voltage: V_{omax}

$\omega_M V_{omax} = SR \Rightarrow f_M = \frac{SR}{2\pi V_{omax}}$

$\Rightarrow$ if $\omega > \omega_m$, $\quad V_o = V_{omax}\left(\frac{\omega_m}{\omega}\right)$

max amplitude of the output w/o distortion.

Common Mode Rejection



$$\left.\begin{aligned} v_{Idm} &= v_2 - v_1 \\ v_{Icm} &= \frac{v_1+v_2}{2} \end{aligned}\right\} \qquad v_o = A_{dm} v_{Idm} + A_{cm} v_{Icm}$$

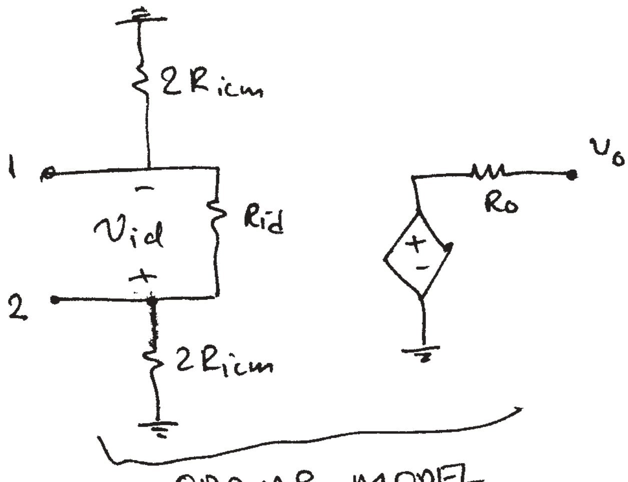
$A_{dm} \longrightarrow A \qquad\qquad A_{cm} \equiv \frac{v_o}{v_{Icm}}$

Common mode rejection ratio

$$CMRR = \frac{|A|}{|A_{cm}|} \quad \text{or} \quad 20\log\left|\frac{A}{A_{cm}}\right|$$

80 – 100 dB @ low freq.

Input and Output Resistances



OPAMP MODEL

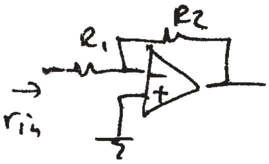
Typical values (bipolar general purpose)

$R_{id} = 1\,M\Omega$

$R_{icm} = 100\,M\Omega$

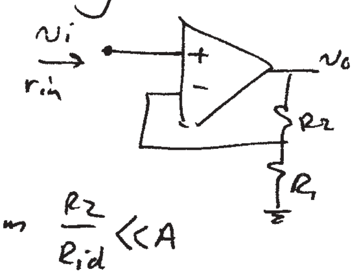
The input resistance of the closed-loop amplifier is important

inverting amp:



$r_{in} = R_1$

non inverting

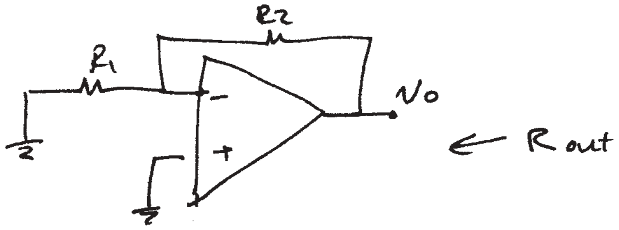


Assuming

$R_o \simeq 0 \quad R_1 \ll R_{icm} \quad \frac{R_2}{R_{id}} \ll A$

$$R_{in} \simeq \{2R_{icm}\} \,\|\, \{(1+A\beta)R_{id}\} \quad , \quad \beta = \frac{R_1}{R_1+R_2}$$

<u>Output resistance</u> $R_o \rightarrow 75 - 100\ \Omega$



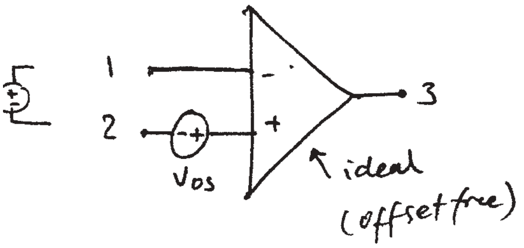
ignoring the input resistances:

$$R_{out} = [R_1 + R_2] \,\|\, \left[\frac{R_o}{(1+A\beta)}\right] \simeq \frac{R_o}{A\beta}$$

↑ smaller than R_o.

DC Problems:

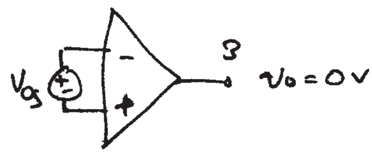
OFFSET VOLTAGE



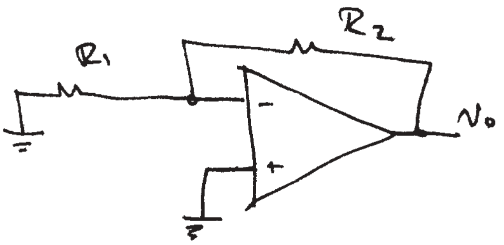
The inputs of the actual circuit is not exactly symmetrical → when both inputs are shorted, the output may not be zero.

Bias currents are needed. They may be different.

y definition



V_{os} - input offset voltage



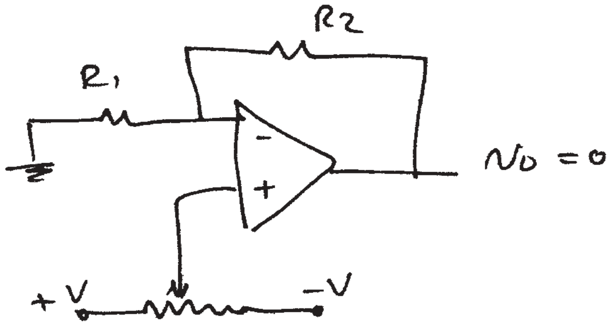
$$V_o = V_{os}\left(1 + \frac{R_2}{R_1}\right)$$

$$\frac{V_o}{1+\frac{R_2}{R_1}} = V_{os}$$

V_o is measured

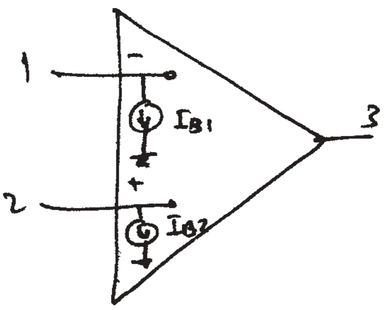
V_{os} calculated.

To adjust for the input offset voltage



Special terminals of the opamp package may be used to cancel the effects of V_{OS}.

input offset currents:

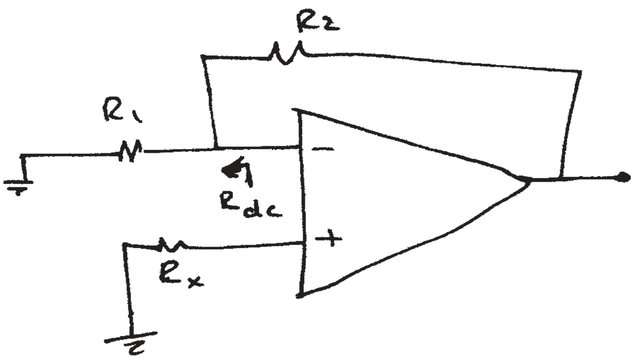


$I_B = \frac{I_{B1} + I_{B2}}{2}$ input bias current

$I_{OS} = |I_{B1} - I_{B2}|$

To reduce the effects of input offset currents,

Place a resistor in the noninverting lead:



R_{dc} the DC resistance seen by the inverting input.

Read about capacitive coupling and the rest of the chapter.

H.W.

2.16	2.22	2.32	2.40
2.17	2.23	2.33	2.41