Eskişehir Osmangazi University - Electrical Engineering Department
Advanced Calculus
Final Examination - Fall 2012

All results must be written in the blanks next to the questions. Anything written elsewhere will not be graded. Use the back side of the exam sheet if you need scratch paper.

1.

(a) Given that $C: \ |z| = 5$ described positively oriented,
evaluate $\int_C \frac{\sin z}{(z-1)^2} dz$ Ans. $2\pi i \cos 1 = 0 + i3.39$

(b) Evaluate $(1+i)^{32}$ Ans. $(\sqrt{2}e^{i\frac{\pi}{4}})^{32} = 65536e^{i8\pi} = 65536 + i0$

2. Express the differential equation $\frac{dy}{dx} = 3 + 3x^2y - xy^2$ in the form $\frac{dw}{dx} = P(x)w + Q(x)$ by using the transformation $y = 3x + \frac{1}{w}$. Ans. $\frac{dy}{dx} = 3 - \frac{1}{w^2}\frac{dw}{dx} \rightarrow$

$$3 - \frac{1}{w^2}\frac{dw}{dx} = 3 + 3x^2(3x + \frac{1}{w}) - x(3x + \frac{1}{w})^2 \rightarrow \frac{dw}{dx} = 3x^2w + x$$

$P(x) = 3x^2$
$Q(x) = x$

3. Find a general solution for

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{vmatrix} 3-\lambda & 4 \\ 1 & 3-\lambda \end{vmatrix} = \lambda^2 - 6\lambda + 5 = 0 \rightarrow \lambda_1 = 1, \ \lambda_2 = 5$$

$$\begin{bmatrix} 3-\lambda & 4 \\ 1 & 3-\lambda \end{bmatrix}_{\lambda=1} v = 0 \rightarrow v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} 3-\lambda & 4 \\ 1 & 3-\lambda \end{bmatrix}_{\lambda=5} v = 0 \rightarrow v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = c_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{5t}$$

4. Solve

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 1, \ \ x(0) = 3, \dot{x}(0) = 1$$

$$m^2 + 2m + 1 = 0 \rightarrow m = -1, -1 \rightarrow x_c = (c_1 + c_2 t)e^{-t}$$

$$\text{by UC method: } x_p = A \rightarrow x_p = 1$$

$$x = x_c + x_p = (c_1 + c_2 t)e^{-t} + 1$$

Apply the initial conditions to obtain
$x(t) = (2 + 3t)e^{-t} + 1$
Red colored texts are sufficient for the full credit!
Good Luck
A. Karamancıoğlu