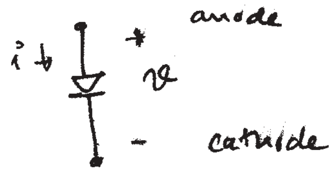
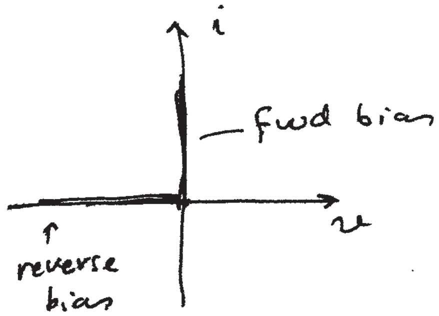
3. DIODES

3.1 IDEAL DIODE





$v < 0 \Rightarrow$

$i = 0$

"cutoff"

$i > 0 \Rightarrow v = 0$

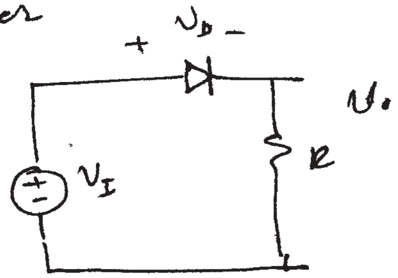
"ON"

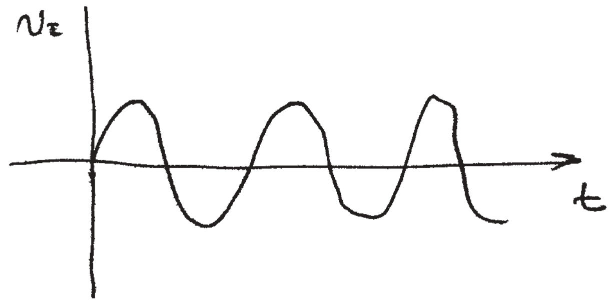
nonlinear

piecewise linear

The current is limited by the external circuit.

Rectifier





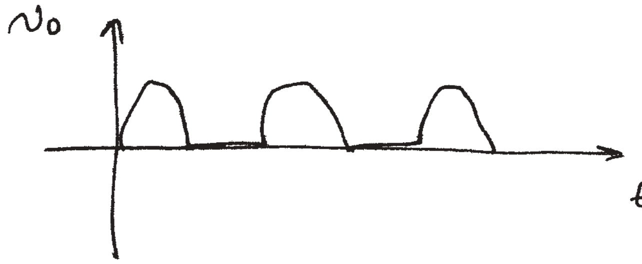
average values

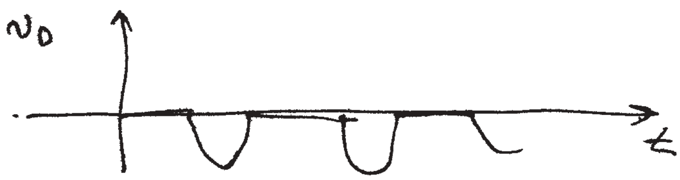
$\langle v_I \rangle = 0$

$\langle v_o \rangle \neq 0$ and positive

$\Rightarrow$ rectifier

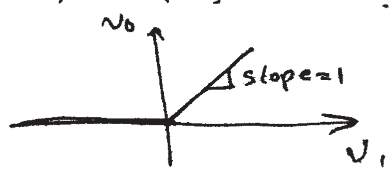
AC $\xrightarrow{\text{rectifier}}$ DC





Transfer characteristics

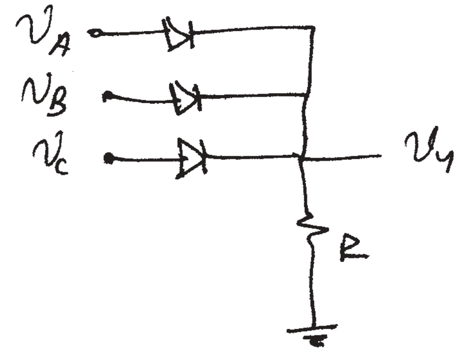
v_o vs v_I



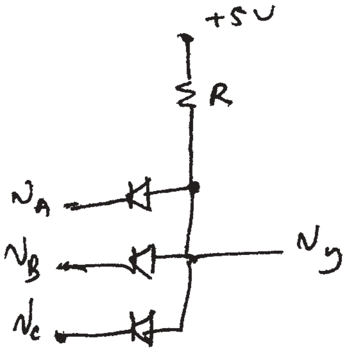
Diode logic gates

Positive logic

0V → "0"   5V → "1"

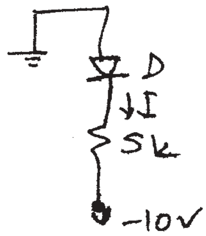


$V_y = V_A + V_B + V_C$ OR gate



$V_y = V_A \cdot V_B \cdot V_C$ AND gate

D is ON

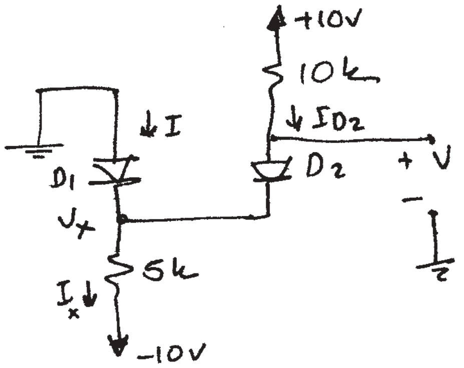


$I = \frac{0-(-10)}{5k}$

$= 2mA$

Example

Find I and V in the ckt below.



ARE THE DIODES ON OR OFF?

ASSUME D1 is OFF, D2 IS ON → ~~V~~ I = 0 ~~V~~.

$V_x = ?$

$I_{D_2} = \frac{10-(-10)}{10+5} = \frac{20}{15}$ mA

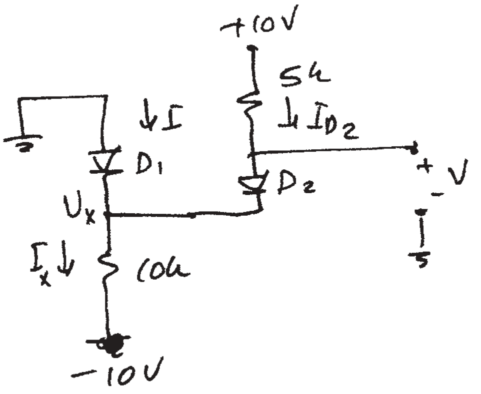
$V_x = -10 + 5 \times \frac{20}{15} = -10 + \frac{20}{3} = -3.33\ V$

This would make $V_{D1} = 3.33V$ → DIODE IS ON

$\Rightarrow V_x = 0\ V \Rightarrow$ $I_x = \frac{0-(-10)}{5k} = 2mA$, $I_{D2} = \frac{10-0}{10} = 1mA$

$I = 2-1 = 1mA$

Example



Assume D_1 : ON $D2$: ON

D_1 : ON ⟶ $V_x = 0V$ $I_x = 1mA$

$I + I_{D2} = I_x = 1mA$

$I_{D2} = \frac{10 - V}{5} = 2mA.$ ⟹ $I = I_x - I_{D2} = 1 - 2mA = -1mA$

not possible → $D1$: OFF.

$I = I_{D_1} = 0$

$I_x = I_{D2}$

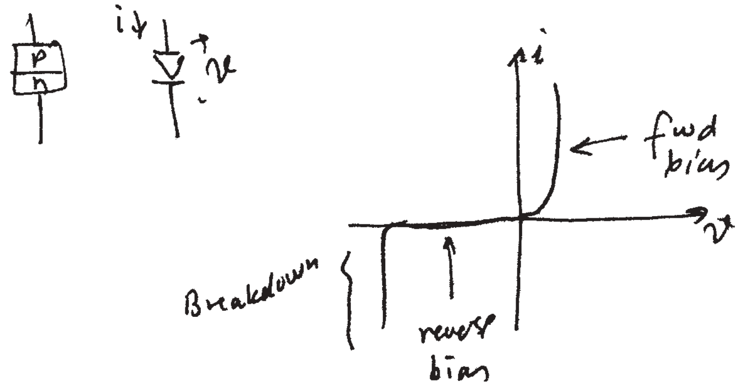
$I_{D2} = \frac{10-(-10)}{10+5} = \frac{20}{15} mA.$

$\frac{10-(-10)}{5+10} \times 10 - 10 = V$

$\frac{200}{15} - 10 = V$

$3.33 = V$

Terminal characteristics of Junction Diodes



η = ideality factor

$1 < \eta < 2$

Reverse and saturation region → $i = I_s(e^{v/\eta V_T} - 1)$

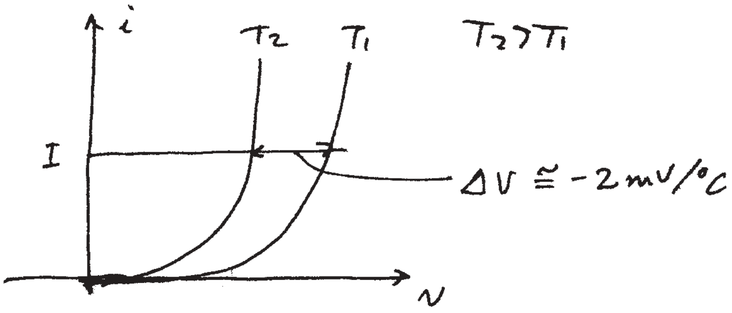
I_s : (reverse) saturation current , $V_T = \frac{kT}{q}$ thermal voltage

$= 0.0258$ V at Room Temp 300K

25mV for practical purposes.

if $v \gg \eta V_T$ then $e^{v/\eta V_T} \gg 1 \Rightarrow I \cong I_s e^{v/\eta V_T}$

$\Rightarrow \quad v = \eta V_T \ln \frac{i}{I_s}$

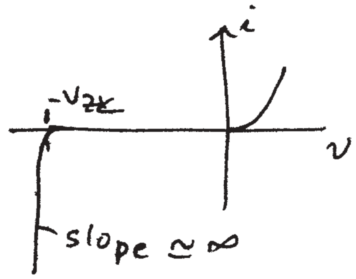


(Is doubles for every 10° increase in temperature)

suppose at

300 K	$I_s = 10^{-9}$ A
310 K	$I_s = 2\times10^{-9}$ A
320 K	$I_s = 4\times10^{-9}$ A

Breakdown

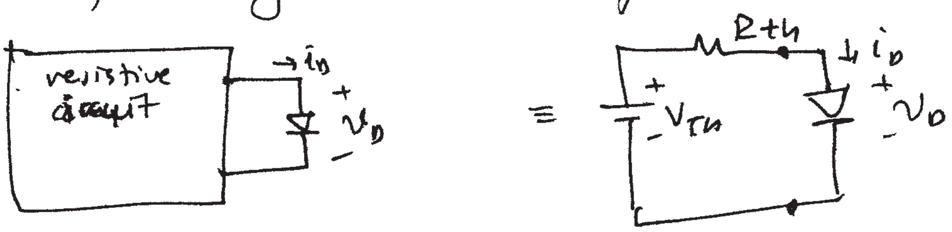


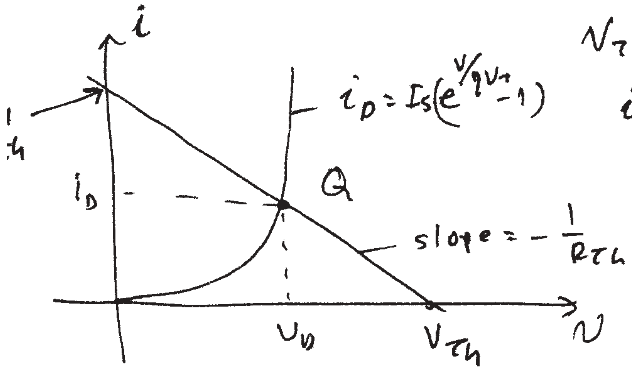
In breakdown, the voltage across the diode remains almost constant → voltage reference or voltage regulator.

Analysis of Diode Ckts

<u>graphical analysis</u>

Valid for any nonlinear component





$V_{Th} = V_D + I_D R_{th}$

$i_D = \frac{V_{th} - V_D}{R_{th}}$ → slope = $-\frac{1}{R_{th}}$

iterative analysis → numerical analysis:

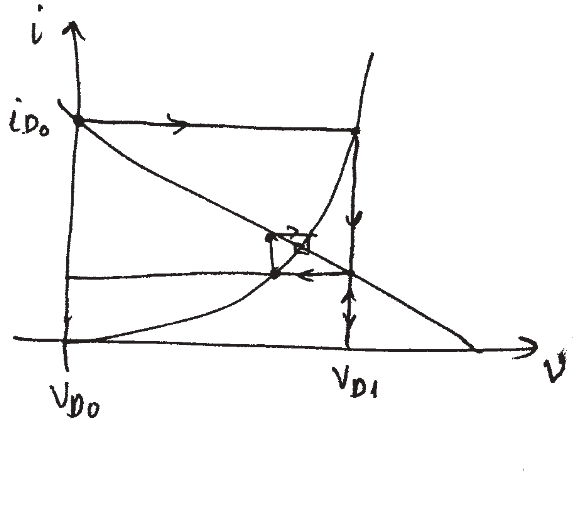
Assume $v_{D_0} = 0$ ⇒ $i_{D_0} = \frac{V_{Th} - V_{D_0}}{R_{th}}$

then find $v_D = \eta V_T \ln \frac{i_D}{I_S} = V_{D1}$

Then recalculate $i_{D_1} = \frac{V_{Th} - V_{D1}}{R_{th}}$

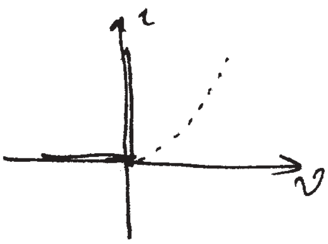
find $v_{D2} = \eta V_T \ln \frac{i_{D1}}{I_s}$ → calculate $I_{D_2} = \frac{V_{Th} - V_{D2}}{R_{th}}$

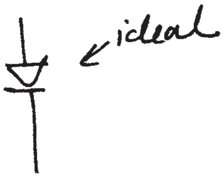
until v_D and I_D converge.

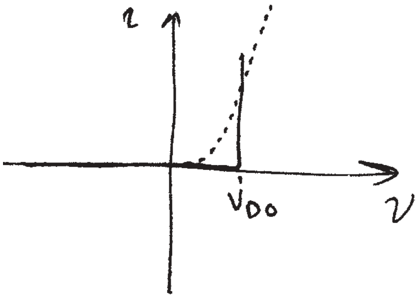


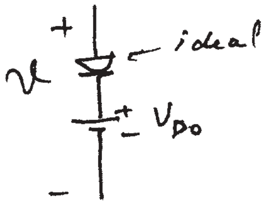
Piecewise linear Models

Battery-plus-resistance model.









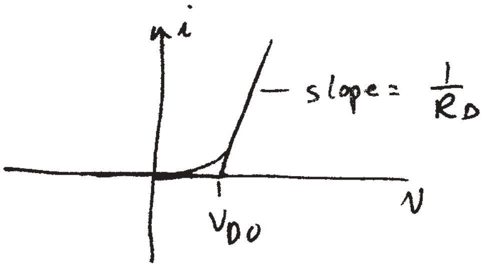
example:

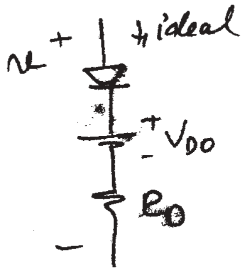
v_D	i_D
0.63V	1 mA
0.72V	10 mA

find

V_{D0}

R_D

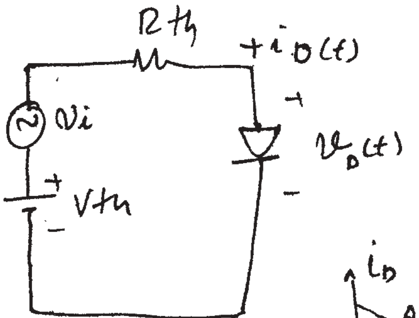


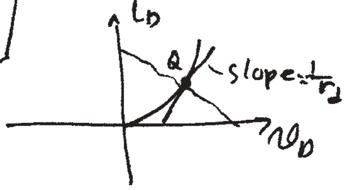


$$v_D = V_{D0} + I_D R_D$$

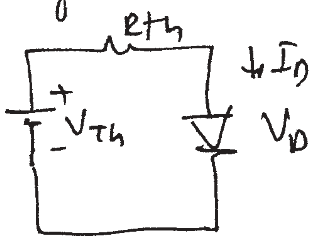
$$R_D = \frac{V_{D2} - V_{D1}}{I_{D2} - I_{D1}} = \frac{0.9\,V}{9\,mA} = 0.1\,k\Omega$$

Small signal model



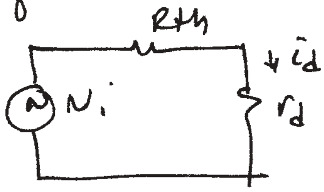


DC equivalent



r_d is found from the slope of the characteristic curve at the operating point.

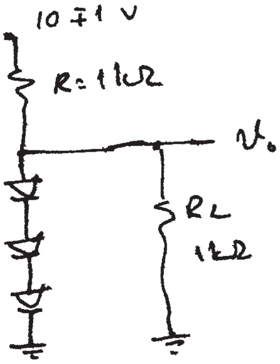
AC equivalent



$i_D \simeq I_s \, e^{v_D/\eta V_T}$, $\dfrac{di_D}{dv_D} = \dfrac{1}{\eta V_T} I_s e^{v_D/\eta V_T} = \dfrac{i_D}{\eta V_T}$

$$\frac{1}{r_d} = \frac{di_D}{dv_D}\Big|_Q = \frac{I_{DQ}}{\eta V_T} \Rightarrow r_d = \frac{\eta V_T}{I_{DQ}}$$

Example:

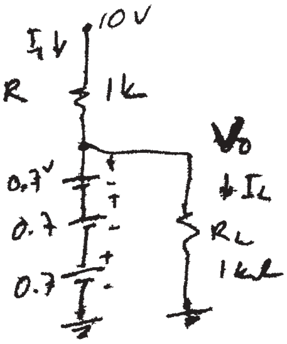


find V_o

DC model

$V_{D0} \cong 0.7$ V

$\eta = 2$



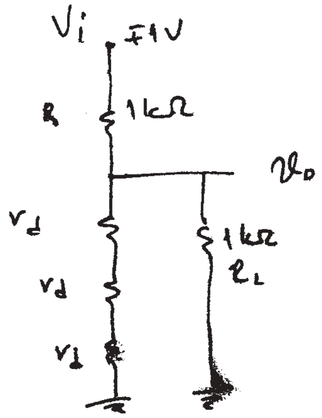
$V_o = 3 \times 0.7 = 2.1$ V

$I_1 = \dfrac{10 - 2.1}{1k} = 7.9$ mA

$I_L = \dfrac{2.1\text{ V}}{1k} = 2.1$ mA

$I_D = I_1 - I_L = 7.9 - 2.1 = 5.8$ mA.

ac model



$$r_d = \frac{\eta V_T}{I_D} = \frac{2 \times 0.025 \text{ mV}}{5.8 \text{ mA}} = \frac{50\text{mV}}{5.8\text{mA}} = 8.62\ \Omega$$

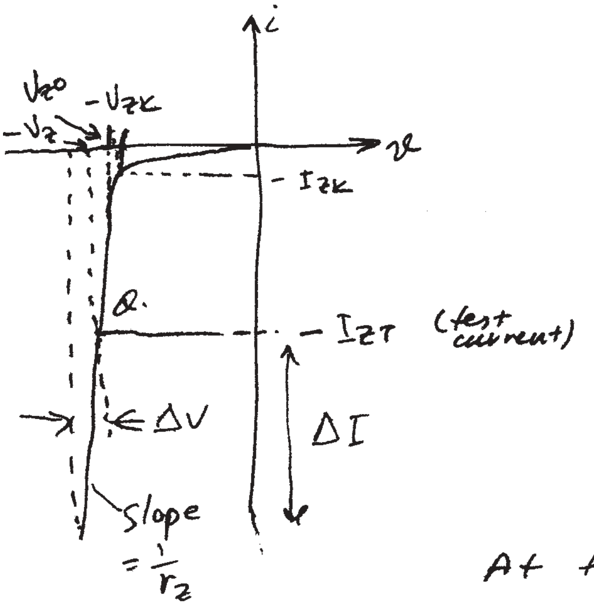
$$v_o = \frac{1000 \,\|\, 3\times 8.62}{1000 + 1000 \,\|\, 3\times 8.62}(\mp 1) = \frac{25.21}{1025.21} = 0{,}0246(\mp 1)$$

$v_o = \mp 24.6$ mV.

$$\frac{v_o}{v_i} = \frac{0.0246}{1} = 2.46\%$$

reduction: 97.54 % reduction in ripple

Zener Diodes

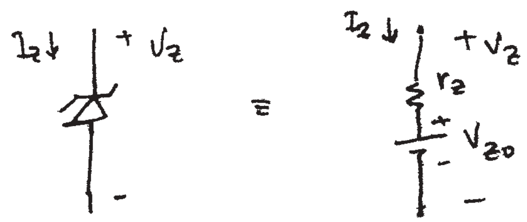


manufacturer specifies I_{zk}, V_z @ I_{zT}

Breakdown curve is almost linear below I_{zk}.

incremental resistance → $r_z = \frac{1}{slope}$
dynamic resistance

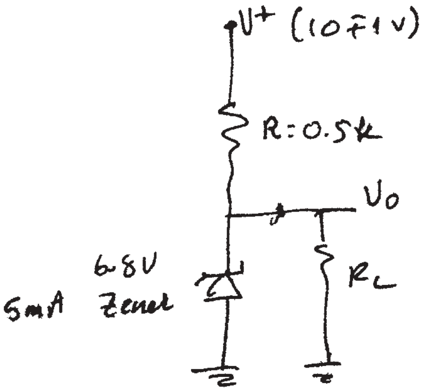
At the Q point the diode can be modeled as follows:



r_z is in Ω's (small)
⇒ voltage source

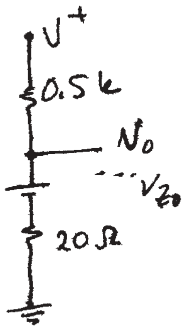
NOT: If I_z falls below I_{zk}, this model becomes invalid!

Example



$V_z = 6.8V$ @ 5ma
$r_z = 20\,\Omega$, $I_{zk} = 0.2mA$

a) find V_0 with $R_L \to \infty$



$$I_z = \frac{V^+ - V_z}{0.5 + 0.02} = \frac{10 - 6.7}{520}$$

$= 6.35$ mA.

$V_0 = 6.35 \times 0.020 + 6.7 = $ 6.83 mA

$$V_{z0} = V_z - I_{zT} r_z = 6.8 - 5 \times 20 = 6.7V$$

b) find $\frac{\Delta V_o}{\Delta V^+}$. $\frac{\Delta V_o}{\Delta V^+} = \frac{r_z}{R + r_z} = \frac{20}{520} = 0,0385$

$\Delta V^+ = \mp 1\,V \Rightarrow \Delta V_o = \mp 38.5\,mV$

c) When $R_L = 2\,k\Omega$, find ΔV_o.

$V_o \simeq 6.8\,V \Rightarrow I_z = 6.35 - \frac{V_o}{R_L} = 6.35 - \frac{6.8}{2k} = (6.35 - 3.4)\,m$

change in $I_z \rightarrow \Delta I_z = 3.4\,mA$

$\Delta V_z = 3.4 \times 20 = $ " $58\,mV$

d) Find V_o when $R_L = 0.5\,k\Omega$

expected value of V_o is $6.8\,V$

expected value of load current $6.8/0.5 = 13.6\,mA$

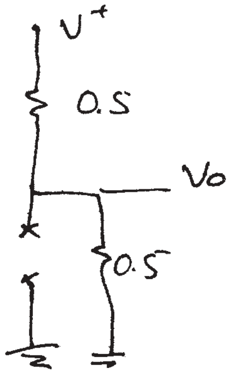
$\Rightarrow$ This is not possible

initially $I_R \simeq (10 - 6.8)/0.5\,k = 6.4\,mA$.

$I_R = I_L + I_z$ $I_z = I_R - I_L = 6.4 - 13.6 = \underline{-7.2\,mA}$ not possible.

$\Rightarrow I_z \cong 0$ "blocking mode" (reverse bias w/o breakdown)

$\rightarrow$ zener diode is <u>almost</u> an open ckt.



$V_o = V^+ \frac{0.5}{0.5 + 0.5} = \frac{V^+}{2} = 5\,V \mp 0.5\,V$

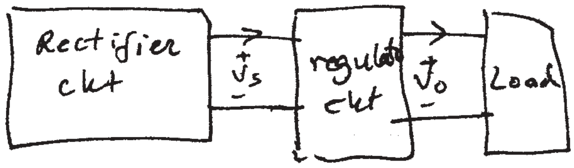
e) min value of $R_L = ?$ (To keep zener in breakdown)

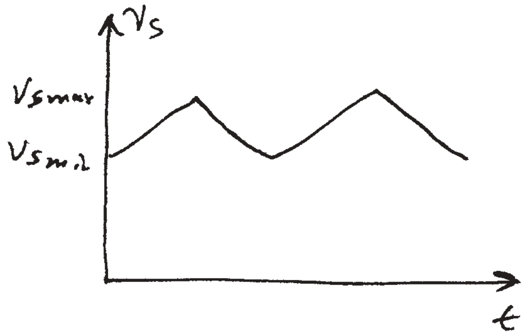
$V^+_{min} = 10 - 1 = 9\,v.$ $I_{R_{min}} = \frac{9 - 6.7}{0.5} = 4.6\,mA$

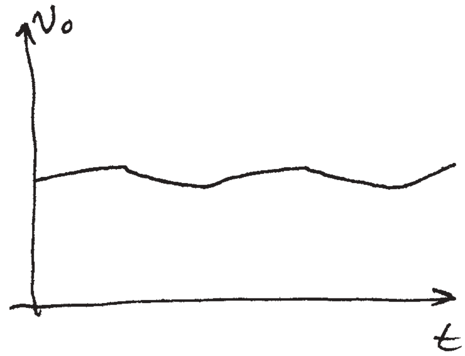
$V_z \cong V_{z0} = 6.7\,V$, $I_z = I_{zk} = 0.2\,mA$

$I_L = I_{R_{min}} - I_z = 4.6 - 0.2 = 4.4\,mA$, $R_L = V_o/I_L = \frac{6.7}{4.4} = 1.52\,k\Omega$

Design of Zener Shunt Regulator

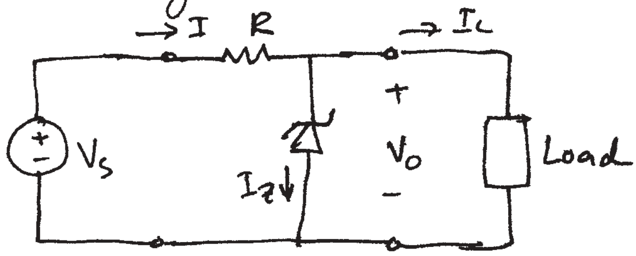






A zener diode that operates in breakdown can be used in the regulator.

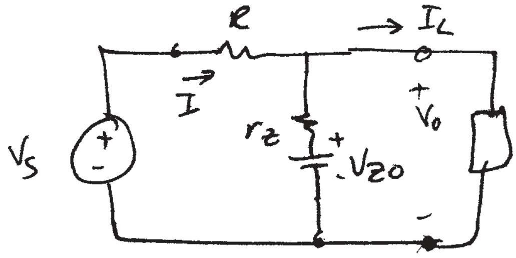
Shunt Regulator



$$\text{Line regulation} \equiv \frac{\Delta V_o}{\Delta V_s} \quad \frac{mV}{V}$$

$$\text{Load regulation} = \frac{\Delta V_o}{\Delta I_L}$$

Replace the zener diode with its equivalent



$$V_o = V_{z0}\frac{R}{R+r_z} + V_s\frac{r_z}{R+r_z} - I_L(R \| r_z)$$

$$\text{line regulation} = \frac{r_z}{R+r_z}$$

$$\text{load regulation} = -(r_z \| R_L)$$

$$R = \frac{V_{smin} - V_{z0} - r_z I_{zmin}}{I_{zmin} + I_{Lmax}}$$

In the design or analysis "the worst case scenario" technique is used.

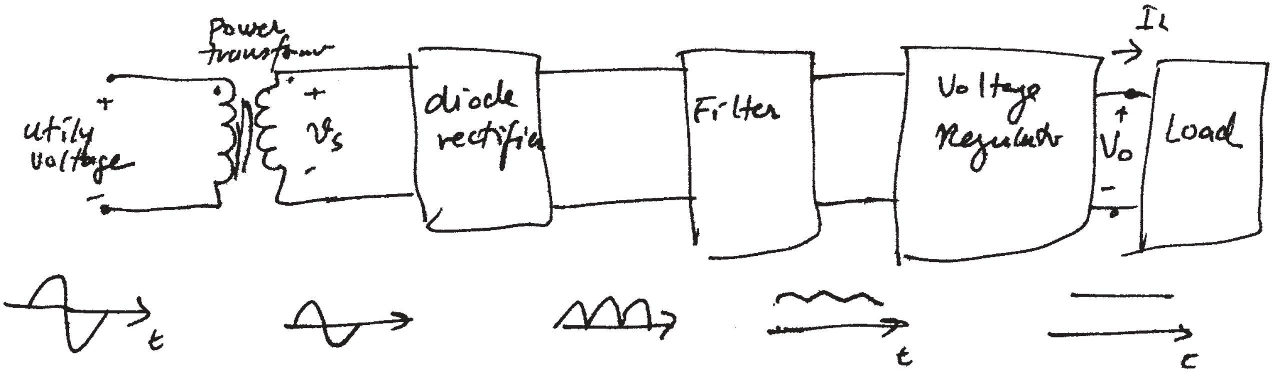
For proper regulation, $I_Z \geq I_{Zmin} = I_{ZK}$ (~ 1 mA) consider $V_{S\,min}$, $I_{L\,max}$

for safety $I_{Zmax} \leq \dfrac{P_{Z\,nominal}}{V_{ZK}}$ consider $I_{L\,min}$, $V_{s\,max}$

One has to consider the power dissipation in the series resistor R.

In the most of the practical cases $R_Z \cong 0$.

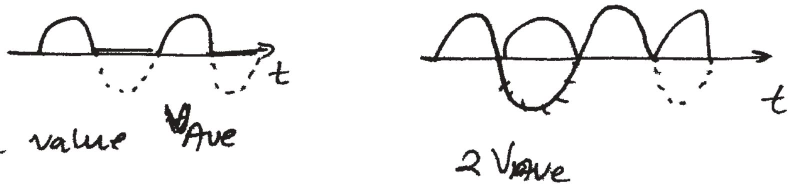
Rectifier Circuits



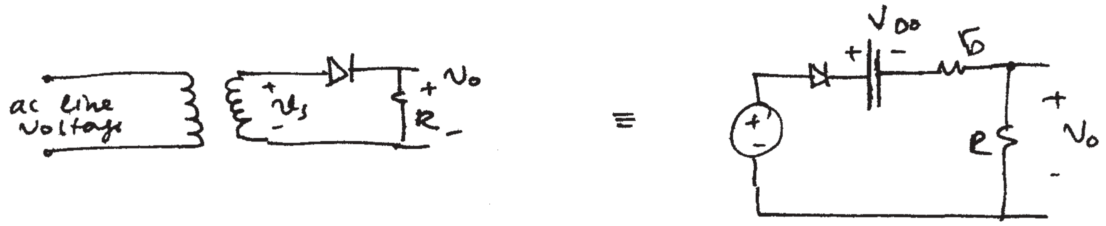
This is a blockdiagram of a dc power supply.

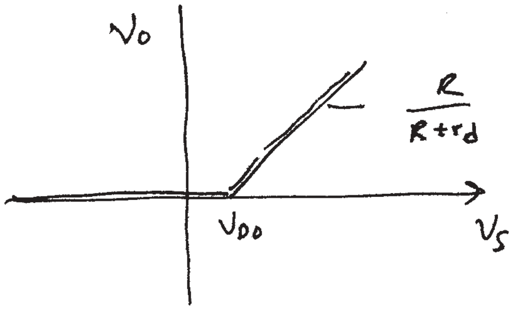
Power transformer → steps down the voltage
→ provides isolation

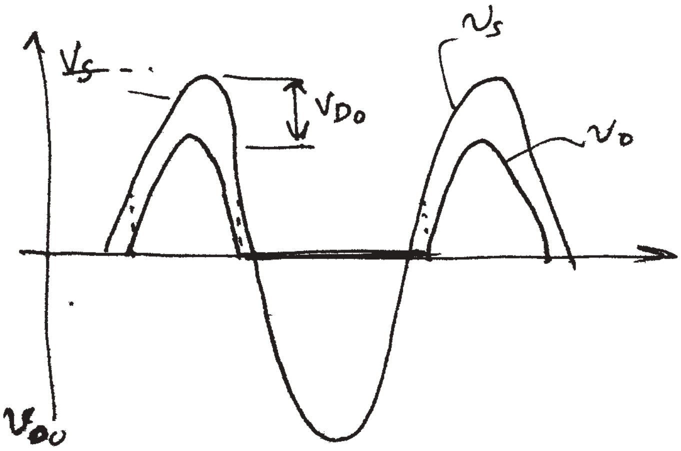
Diode rectifier Half-wave / Full wave



Half-wave rectifier







$v_o \simeq v_s - V_{D0}$ when $v_s \geq V_{D0}$

in practice silicon diodes are used → $V_{D0} \simeq 0.7\,V$ $0.8\,V$

Peak inverse voltage PIV : max reverse bias voltage on the diode ⟶ here $PIV = V_s$

Choose a diode such that $V_{ZK} \geq 1.5\,V_s$

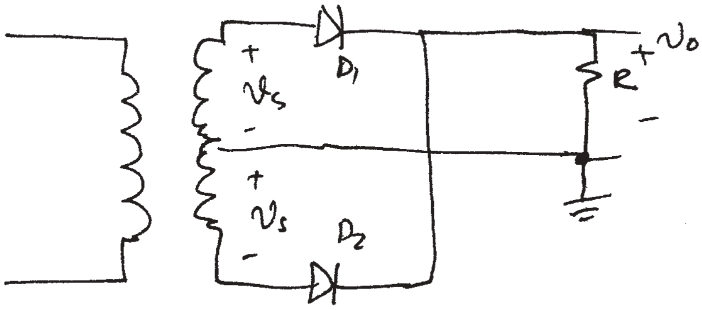
Peak of $v_o \simeq V_s - V_{D0}$

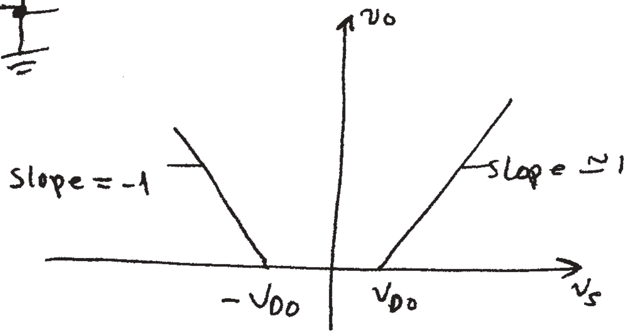
average of v_o $= \frac{1}{T}\int_0^T v_o(t)\,dt \simeq \frac{1}{T}\int_0^{T/2} V_m \sin\omega t\,dt$

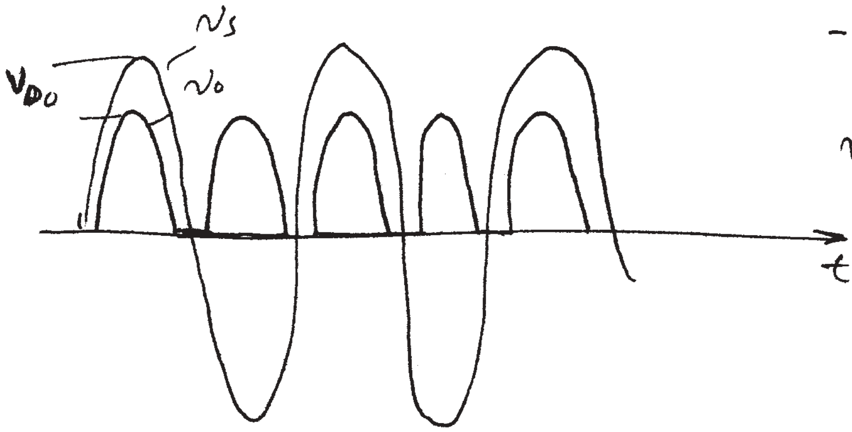
$= \frac{V_m}{\pi}$

$V_o = \frac{V_s}{\pi} - \frac{V_{D0}}{2}$ can be found from a more detailed analysis.

Full-wave rectifier w/ center tapped transformer





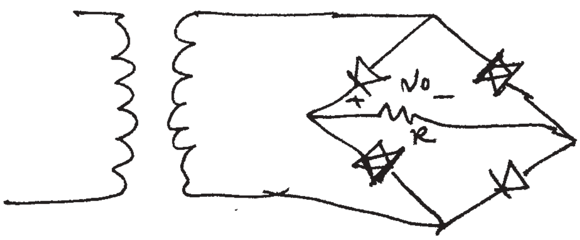


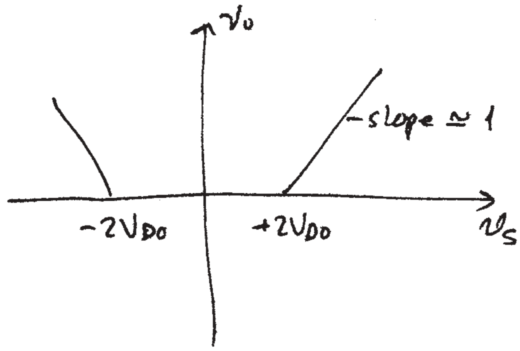
$v_s < 0 \rightarrow D_2$ conducts
$v_s > 0 \rightarrow D_1$ conducts

$PIV = 2V_s - V_{D0}$

average output voltage $= \frac{2V_s}{\pi} - V_{D0}$

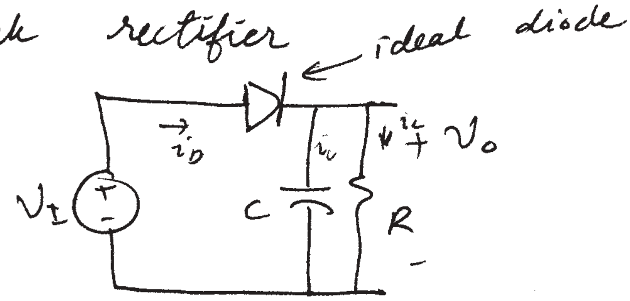
Bridge Rectifier

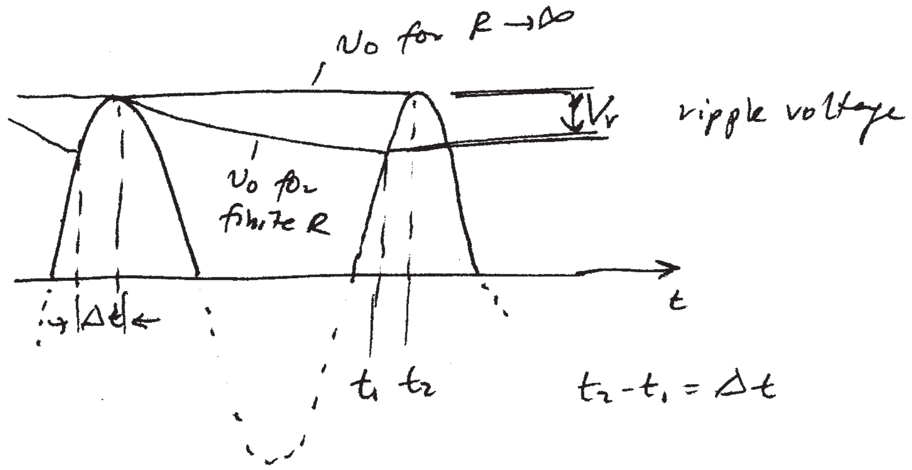




$$V_o \cong \frac{2}{\pi} V_s - 2V_{D0}$$

Peak rectifier





Diode conducts from t_1 to t_2. → Capacitor is charged.
after t_2, the capacitor discharges.

$$i_D = i_C + i_L$$

$$= C\frac{dv_I}{dt} + i_L$$

during the discharge $i_D = 0$

$$i_L = -C\frac{dv_I}{dt} \quad \cong \text{constant}$$

$$\Delta t \ll T \Rightarrow \quad i_L = -C\frac{\Delta V_I}{\Delta t} = -C\frac{V_r}{T} \Rightarrow |V_R| = \frac{T\, i_L}{C}$$

$$i_L \cong \frac{V_O}{R} \quad \Rightarrow \quad |V_R| \simeq \frac{T V_O}{RC} = \frac{1}{f}\frac{V_O}{RC}$$

$$\Rightarrow \quad V_O = V_P - \frac{1}{2}V_r$$

(V_P: peak voltage.)

Example: input 50 Hz, 24 V rms

$R = 10k\Omega$

find C so that $V_r = 2V$

Peak voltage $24\sqrt{2} = 33.94V$

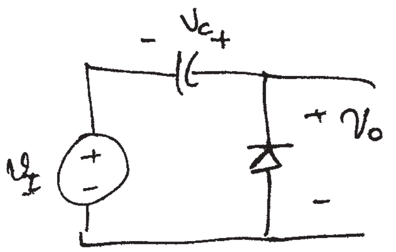
$$C = \frac{1}{f}\frac{V_p}{V_r R} = \frac{1}{50}\frac{33.94}{2\times10\times10^3} = 0.3394\times10^{-4}F = 33.94\,\mu F.$$

Fullwave rectifier → charging + discharging freq → 2f

$$V_r = \frac{V_p}{2fCR}$$ has lower ripple.

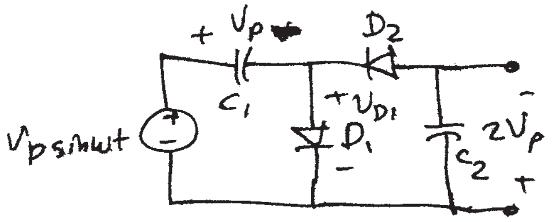
Read sections: Limiting and Clamping ckts.

DC restorer / clamped capacitor:



$V_o = V_I + \max(V_I)$

Voltage doubler:



Also read Physical Operation of Diodes — Basic Semiconductor Concepts

HW Problems:

3.32, 3.37, 3.45, 3.55