

Eskişehir Osmangazi University - Electrical Engineering Department
Advanced Calculus
1st Midterm Examination - Summer 2013

All answers must be written below or next to the questions. Anything written elsewhere will not be graded. Use the back side of the exam sheet if you need scratch paper.

1. [40 pts.] Evaluate the following (express the results in cartesian form; answers should be free of trigonometric, logarithmic, hyperbolic and exponential terms):

a) $\sinh(1 + 4i)$, $\sinh x \cos y + i \cosh x \sin y = -0.7682 - i1.1678$

b) $\text{Log}(1 + 4i)$, $1.4166 + i1.3258$

c) $\frac{e^{1+i}}{1+i}$, $\frac{1.4687+i2.2874}{1+i} = 1.8780 + i0.4093$

d) 3^{2i} , $e^{2i \log 3} = e^{2i(\ln 3 + i2\pi n)} = e^{-4\pi n} e^{i2.1972} = e^{-4\pi n} (-0.586 + i0.810)$

2. [20 pts.] Evaluate

$$\int_C \frac{4 + 3z}{\cos z} dz$$

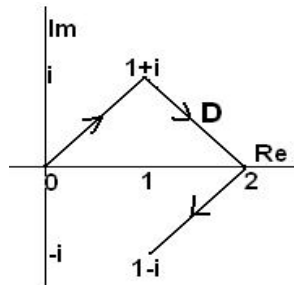
where $C : |z - 1| = 2$ is positively oriented.

$$2\pi i \left[\frac{p(z)}{q'(z)} \right]_{z=\frac{\pi}{2}} = \left[2\pi i \frac{4+3z}{-\sin z} \right]_{z=\frac{\pi}{2}} = -2\pi i \left(4 + \frac{3\pi}{2} \right) = -i(8\pi + 3\pi^2) = -i54.7416$$

3. [20 pts.] Evaluate

$$\int_D (z^2 + \cos z) dz$$

where D is concatenation of three oriented line segments shown below.



$$\begin{aligned} & \left[\frac{z^3}{3} + \sin z \right]_0^{1-i} \\ &= -0.667 - i0.667 + (1.2985 - i0.6350) \\ &= 0.6318 - i1.3016 \end{aligned}$$

4. [20 pts.] Evaluate

$$\int_E \frac{z^3}{(z-1)^2} dz$$

where $E : |z| = 2$ is positively oriented. $\frac{2\pi i}{1!} [3z^2]_{z=1} = 6\pi i$

Good Luck

A. Karamancıoğlu