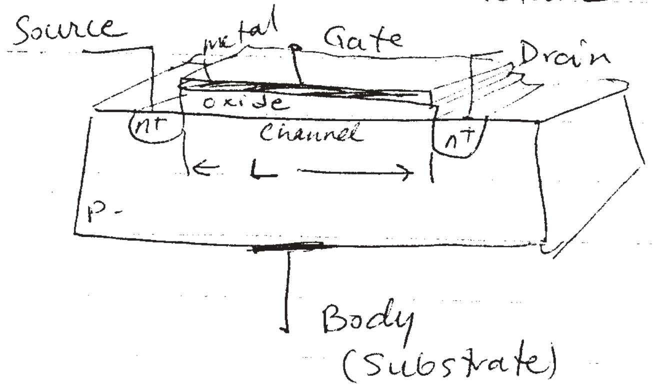
5. Field Effect Transistors

- Unipolar device - only electrons or only holes carry current.
- Smaller than BJTs in size
- most digital ckts are made of FETs
- JFET, ~~M~~MOSFET (S), MESFET

MOSFET device Structure:



$1\ \mu m < L \leq 10\ \mu m$

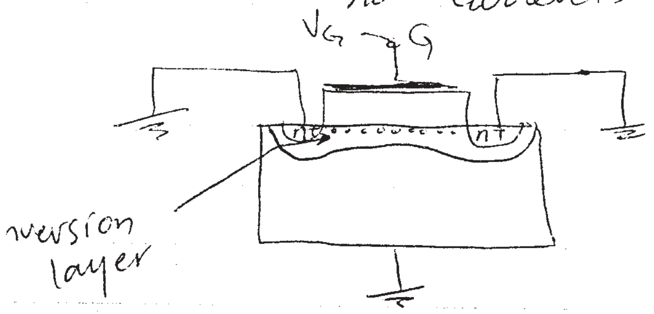
Oxide Thickness $0.1\ \mu m$

$1000\ \mathring{A}$

IGFET Insulated gate Field Effect transistor.

Operation w/ Gate open ckt

→ Back to back two diode
no currents flow.



When V_G = Threshold voltage, a channel is formed between the two n^+ regions

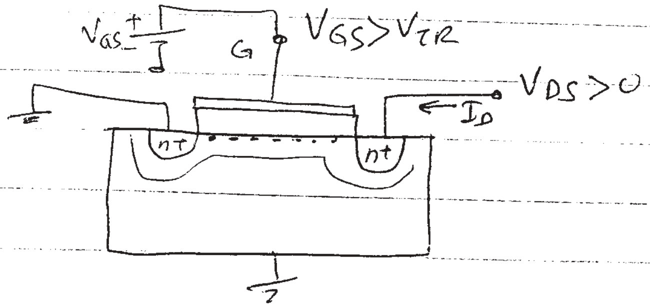
V_t

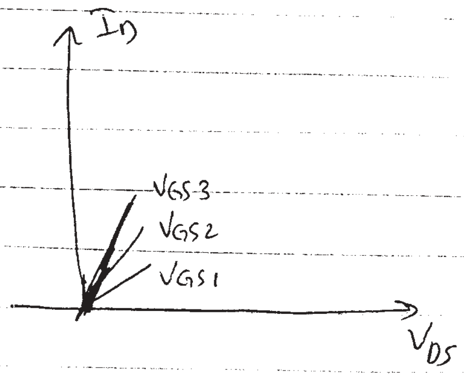
why is it called "field effect" transistor?

Threshold voltage $= V_t,\ V_T,\ V_{TR}$

Thermal voltage $= V_T = \frac{kT}{q}$

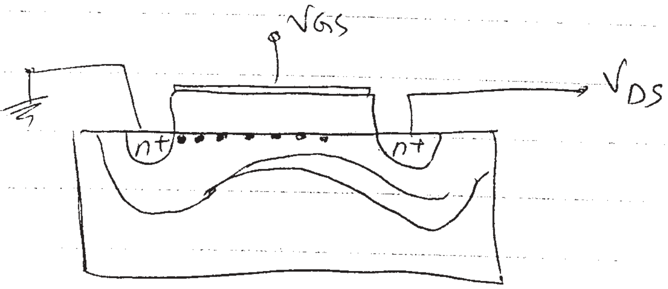
Now apply a voltage to the drain terminal

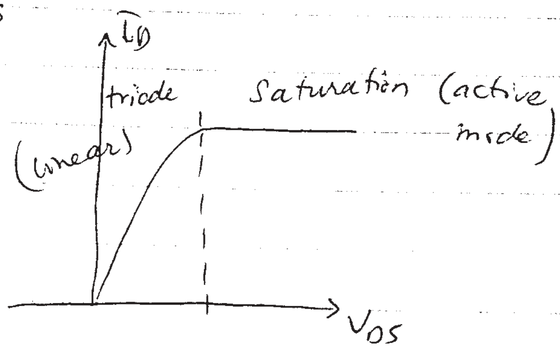




for small V_{DS}, I_D is proportional to the gate excess voltage $(V_{GS}-V_t)\,V_D$

For larger V_{DS}, the voltage along the channel varies, and electron density varies too.





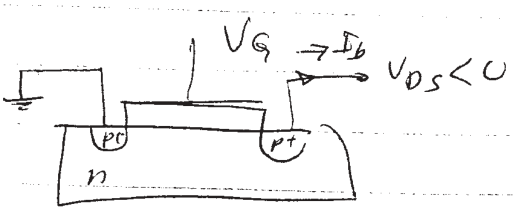
$V_{GS} > V_t$

$V_{DS} < V_{GS} - V_t$: triode

$V_{DS} > V_{GS} - V_t$: saturation

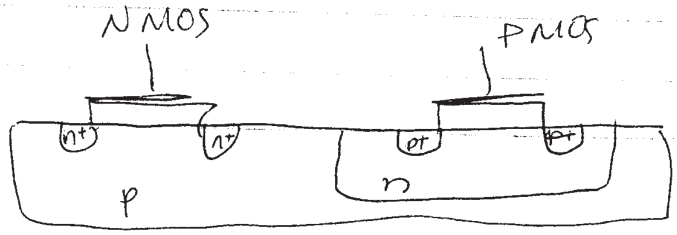
channel is pinched off at saturation → constant current.

PMOS (P- channel MOS)



$V_t < 0$

CMOS

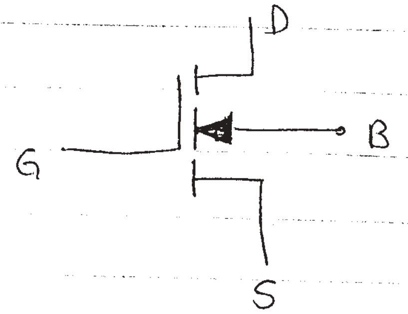


Current Voltage Characteristics of MOSFET

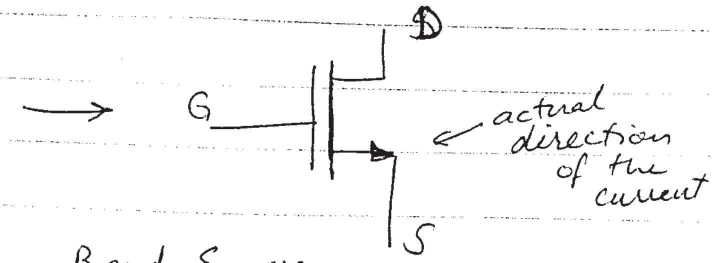
n-channel $\quad V_t > 0 \Rightarrow$ enhancement mode

$\quad\quad\quad\quad V_t < 0 \Rightarrow$ depletion mode

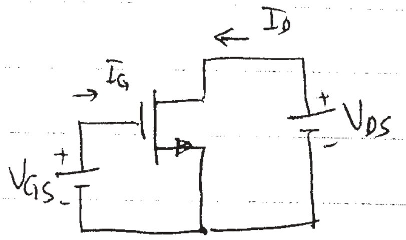
enhancement mode



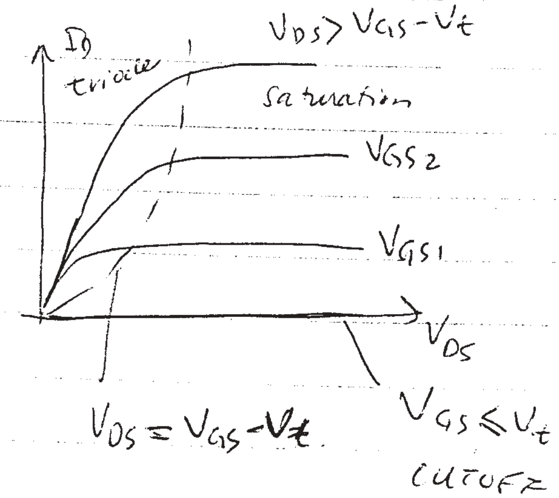
$\longrightarrow$



B and S are tied together



$I_G = 0$



Three modes of Operation:

CUTOFF $\longrightarrow \quad V_{GS} \le V_t \quad \longrightarrow \quad I_D = 0$

Triode Mode (linear mode): $\quad V_{GS} > V_t$

$\quad\quad\quad\quad V_{DS} < V_{GS} - V_t$

$$i_D = K\left[2(V_{GS} - V_t)\, V_{DS} - V_{DS}^2\right]$$

for small V_{DS}

$$i_D \simeq 2K(V_{GS} - V_t)\, V_{DS}$$

SATURATION MODE $\quad V_{GS} > V_t \,, \quad V_{DS} > V_{GS} - V_t$

$$i_D = K\left(V_{GS} - V_t\right)^2$$

K: Conductance parameter – related to the physical properties of the device

$$K = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)$$

W: width

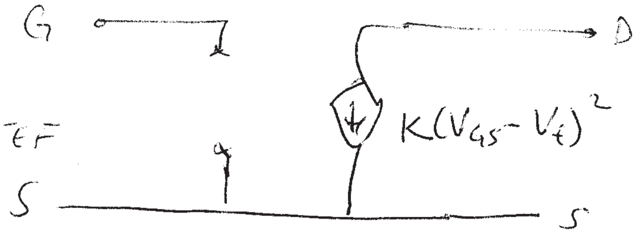
L: length of channel

C_{ox}: oxide capacitance

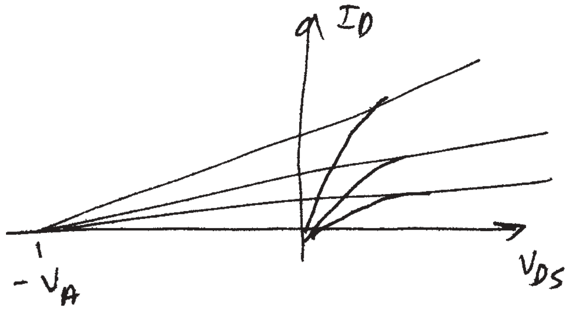
μ_n: Electron mobility

$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$ $\frac{W}{L}$: aspect ratio

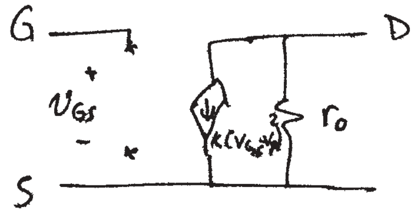
Large signal eq. of a MOSFET for SATURATION MODE



Finite output resistance – channel length modulation



$$i_D = K(V_{GS}-V_t)^2(1+\lambda v_{DS})$$



$r_o \cong \frac{1}{\lambda I_D}$

$\frac{1}{\lambda} = V_A$

$30 < V_A < 200$

ideal $\rightarrow V_A \rightarrow \infty$

P-channel MOSFET

$V_{GS} > V_t \rightarrow$ cutoff

$V_{GS} \le V_t$, $v_{DS} \geq V_{GS}-V_t$ triode

$$i_D = K[2(V_{GS}-V_t)v_{DS} - v_{DS}^2]$$

$V_{GS} \le V_t$, $v_{DS} \le V_{GS}-V_t \rightarrow$ saturation

$$i_D = K(V_{GS}-V_t)^2(1+\lambda v_{DS})$$

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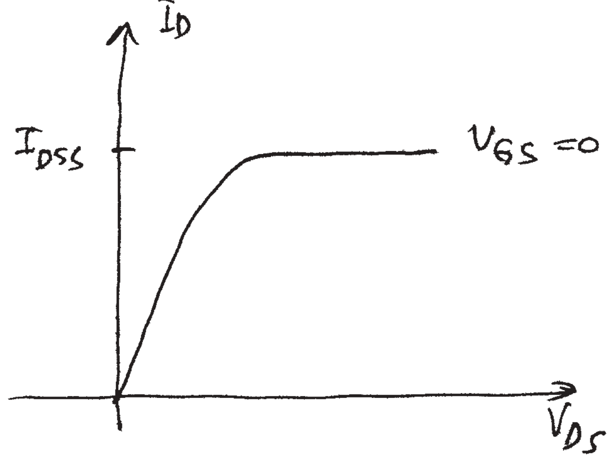
Both K and V_t are temperature dependent
$|V_t|$ decreases 2mV/°C with temperature
K decreases w/ Temp.
Overall effect I_D decreases as $T\uparrow$.

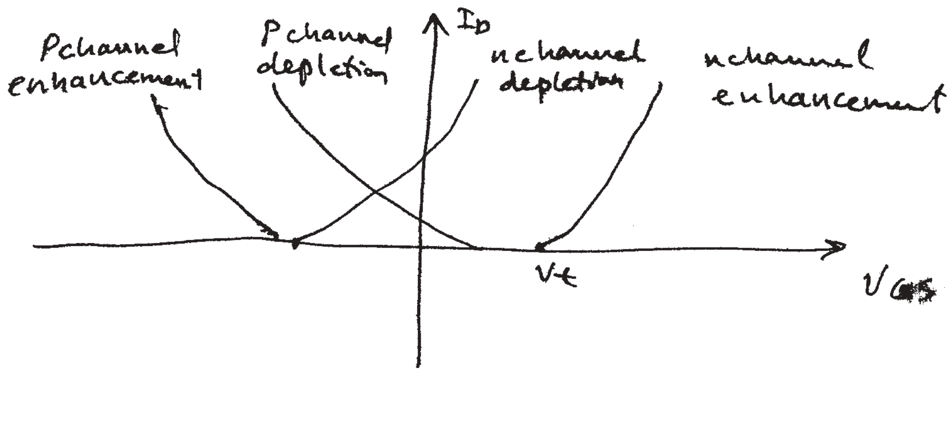
Depletion mode device (n-channel)
$V_t > 0 \Rightarrow$ no channel when $V_{GS}=0$
Enhancement mode
$V_t < 0 \Rightarrow$ there is a channel when $V_{GS}=0$
$\Rightarrow$ depletion mode.



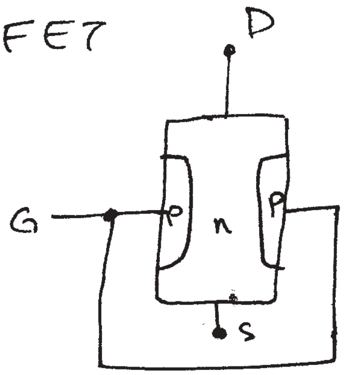
$$I_{DSS} = I_D\Big|_{V_{GS}=0} = K(V_{GS}-V_t)^2\Big|_{V_{GS}=0} = KV_t^2$$

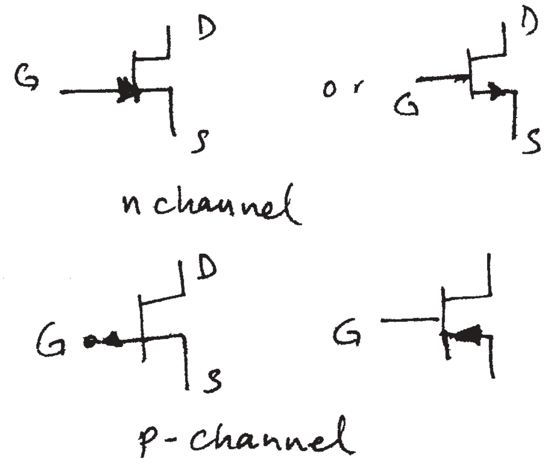
P-channel: $V_t < 0$ enhancement
$V_t > 0$ depletion mode

Transconductance curves (saturation)



JFET

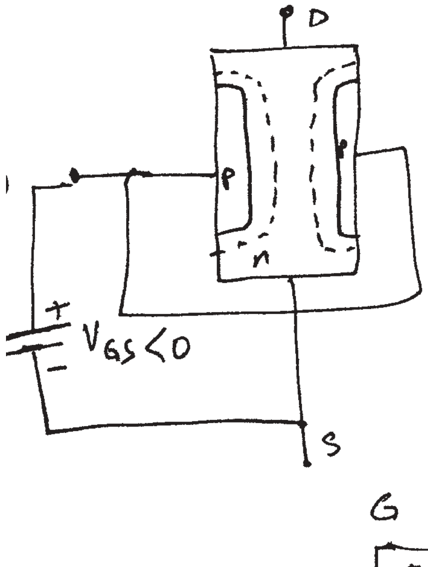




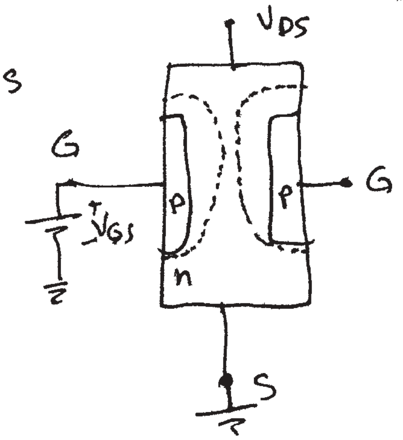
Operation:

a channel exists between Drain and Source. if a voltage is applied to the drain, current flows.

A reverse voltage between G and S would increase the depletion region — channel resistance increases → less current



Larger V_{DS} → voltage gradient along the channel nonuniform (gradual) depletion width ⟶ nonuniform channel cross section



when $V_{DS}=0$, $V_{GS}=V_p$ would deplete the entire channel →

when $V_{DS} \geq V_{GS}-V_p$

~~entire channel wa~~

the channel is pinched off at the drain end.

Modes of operation:

$V_{GS} < V_p \rightarrow$ cut off $I_D = 0$

$V_{GS} \geq V_p$ + $V_{DS} < V_{GS} - V_p$ Triode mode

$$i_D = K[2(V_{GS} - V_p) v_{DS} - v_{DS}^2]$$

$V_{GS} > V_p$ + $V_{DS} > V_{GS} - V_p$ constant current mode

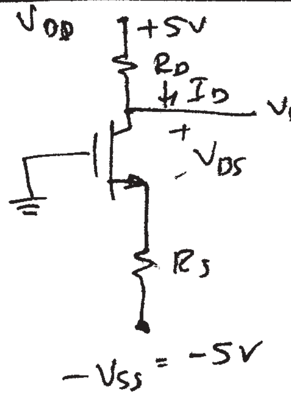
$$i_D = K(V_{GS} - V_p)^2$$

$V_p \rightarrow$ ~~the~~ same as V_t for a MOSFET

$K = ?$ $V_{GS} = 0 \Rightarrow$ $i_D = K V_p^2 = I_{DSS}$

$\Rightarrow \boxed{K = \frac{I_{DSS}}{V_p^2}}$ $\boxed{V_t = V_p}$

EXAMPLES



Determine R_D and R_S so that $I_D = 0.4$ mA
$V_D = 1.0$ V

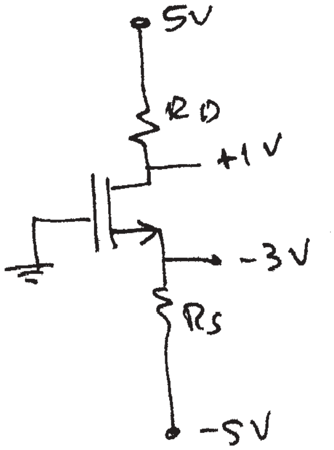
$V_t = 2V$
$\mu_n C_{ox} = \frac{20 \mu A}{V^2}$, $L = 10 \mu m$, $W = 400 \mu m$
$\lambda = 0$

$$K = \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L}\right) = \frac{20}{2} \times \frac{400}{10} = \frac{800}{2} \mu A/V = 0.4 mA/V$$

Constant current mode $\rightarrow$ $I_D = K(V_{GS} - V_t)^2$

$0.4\,mA = 0.4\,(V_{GS}-V_t)^2 \Rightarrow V_{GS}-V_t = 1$

$V_{GS} = V_t + 1 = 2+1 = 3\,V.$



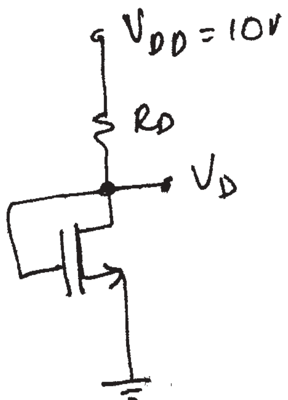
$V_G = 0\,,\quad V_S = V_G - V_{GS} = 0-3 = -3\,V$

$R_S I_D = -3-(-5) = 2\,V$

$R_S = \dfrac{2\,V}{0.4\,mA} = 5\,k\Omega$

$I_D R_D = 5-1 = 4\,V \qquad R_D = \dfrac{4\,V}{0.4\,mA} = 10\,k\Omega$

— o —



Find R_D so that $I_D = 0.4\,mA$

$\mu_n C_{ox} = 20\,\mu A/V^2$
$W = 100\,\mu m$
$L = 10\,\mu m$

$K = \frac{1}{2}\, 20 \times \frac{100}{10} = 0.1\, mA/V^2$

$V_t = 2\,V$

$V_{DS} = V_{GS} \qquad V_t > 0 \Rightarrow V_{DS} > V_{GS} - V_t$

$\Rightarrow$ Constant current mode for $V_{DS} > V_{TR}$

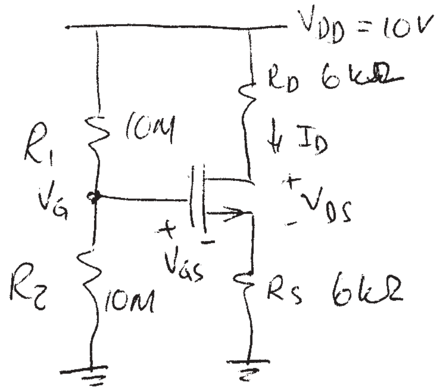
$0.4 = 0.1\,(V_{GS}-V_t)^2 \Rightarrow (V_{GS}-V_t)^2 = 4$

$V_{GS} - V_t = 2$

$V_{GS} = V_t + 2 = 4\,V.$

$V_D = 4\,V\,, \quad R_D = \dfrac{10-4}{0.4} = \dfrac{6}{0.4} = 15\,k\Omega$

Example



$\lambda = 0$

$K = 0.5\ mA/V^2$

$V_t = 1V$

find I_D and V_{DS}

Assume Constant current mode

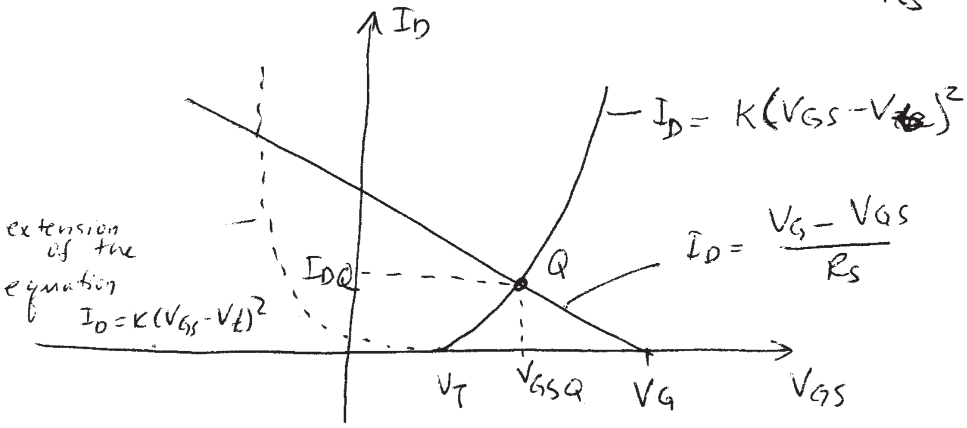
$I_D = K(V_{GS} - V_t)^2$

$V_{DS} \geq V_{GS} - V_{TR}$

$V_G = 5V$

$V_G = I_D R_s + V_{GS} \Rightarrow V_{GS} = V_G - I_D R_s$

$\Rightarrow I_D = \dfrac{V_G - V_{GS}}{R_s}$



Analytical solution of these equations would yield two V_{GS} values — only one is valid.

$$\left.\begin{aligned} I_D &= \frac{V_G - V_{GS}}{R_S} \\ I_D &= K(V_{GS} - V_t)^2 \end{aligned}\right\} \quad V_G - V_{GS} = KR_s(V_{GS}^2 - 2V_{GS}V_{TR} + V_{TR}^2)$$

$$0 = KR_sV_{GS}^2 + (1 - 2V_{TR}KR_s)V_{GS} - V_G + V_{TR}^2KR_s$$

$$0 = V_{GS}^2 + \left(\frac{1}{KR_s} - 2V_{TR}\right)V_{GS} + V_{TR}^2 - \frac{V_G}{KR_s}$$

Solve for V_{GS1} and V_{GS2}

$V_{GS} > V_t \rightarrow$ is the solution

$V_{GS} = 5 - 6 I_D$

$I_D = k(V_{GS} - V_t)^2$

$= 0.5(5 - 6 I_D - 1)^2 \Rightarrow 18 I_D^2 - 25 I_D + 8 = 0$

$I_{D_1} = 0.89 mA$

$I_{D_2} = 0.5 mA$

I_{D_1} is not acceptable :

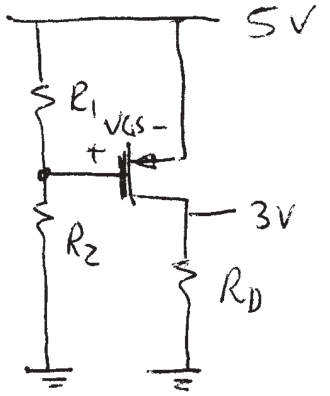
because $6 \times 0.89 = 5.34V$

$5.34 > V_G$ not possible

$I_D = 0.5 mA \rightarrow$

$\left.\begin{array}{l} V_S = 0.5 \times 6 = 3V \\ V_D = 10 - 6 \times 0.5 = 7V \end{array}\right\} V_{DS} = 7 - 3 = 4V.$

Example



Design for $I_D = 0.5mA$

$V_D = +3V$

$V_t = -1V$

$K = 0.5 mA/V^2$

$\lambda = 0$

$$I_D = K(V_{GS} - V_t)^2$$

$$0.5 = 0.5(V_{GS} + 1)^2$$

$$(V_{GS} + 1)^2 = 1$$

$V_{GS} + 1 = 1$ or $V_{GS} + 1 = -1$

$V_{GS} = 0$ ← not acceptable $\underline{V_{GS} = -2}$

$V_{DS} = -2V$

$V_{GS} = -2V$

$V_G = 5 - 2 = 3V.$

Let $R_2 = 3M\Omega$

$R_1 = 2M\Omega$

$$R_D = \frac{3}{0.5} = 6k\Omega$$

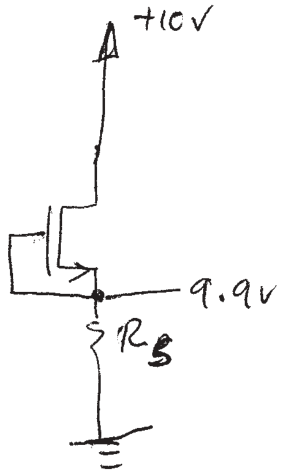
What is the largest value of R_D that would still keep the MOSFET in saturation mode?

$$V_{DS} \leqslant V_{GS} - V_t = -1$$

That means V_D can be as large as 4V.

$$R_{Dmax} = \frac{4}{0.5} = 8k\Omega$$

Example



Design a ckt so that $V_S = 9.9V$
What would be the effective resistance between the drain and the source?

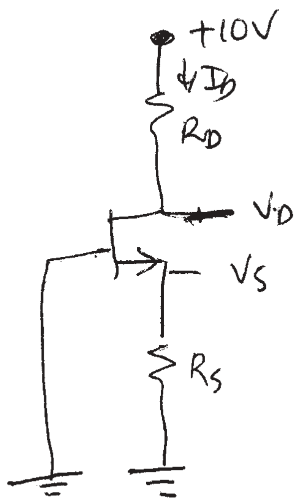
$V_t = -1V$
$K = 0.5 mA/V^2$

$V_{DS} = 0.1V < V_G - V_t = 0+1$
⇒ triode mode

$$I_D = K\left[2(V_{GS} - V_t)V_{DS} - V_{DS}^2\right]$$
$$= 0.5\left[2(0+1)0.1 - 0.01\right] \simeq 0.1mA$$

$$R_S = \frac{9.9}{0.1} = 99\,k\Omega \simeq 100\,k\Omega$$

$$r_{ds} = \frac{V_{DS}}{I_D} = \frac{0.1\,V}{0.1\,mA} = 1\,k\Omega$$



$I_D = 4mA$
$V_D = 6V$
$V_p = -4V$
$I_{DSS} = 16mA$
$\lambda = 0$
$K = \frac{I_{DSS}}{V_p^2} = \frac{16}{16} = 1 mA/V^2$

Determine R_S and R_D

1) Find V_{GS} to have $I_D = 4mA$.

$$I_D = K(V_{GS} - V_t)^2$$

$$V_{GS_1} = \sqrt{\frac{I_D}{K}} + V_t = 2 - 4 = -2V$$

$$V_{GS_2} = -\sqrt{\frac{I_D}{K}} + V_t = -(2+4) = -6V$$

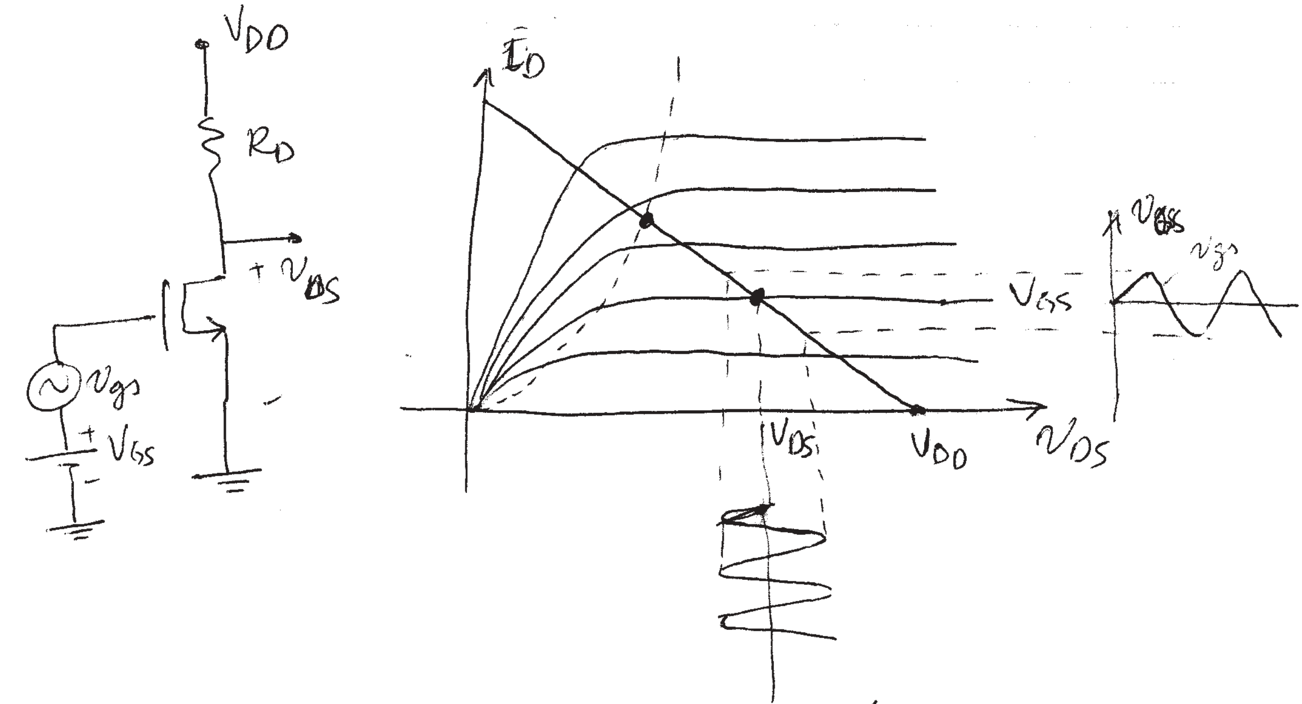
↑ not acceptable

$V_{GS} = -2V$, $V_G = 0$, $V_S = 2V$, $V_{DS} = 6 - 2 = 4V > V_{GS} - V_t = -2 + 4 = 2V$

$$R_S = \frac{2}{4} = 0.5\,k\Omega$$

$$R_D = \frac{10-6}{4} = 1\,k\Omega$$

THE FET AS AN AMPLIFIER



Suppose that V_{GS} and V_{DS} are large enough to keep the MOSFET in saturation mode

$$I_D = K(V_{GS} - V_t)^2 \qquad \text{when } v_{gs} = 0$$
$$V_{DS} = V_{DD} - I_D R_D$$

With signal,

$$v_{GS} = V_{GS} + v_{gs}$$

$$i_D = K(v_{GS} - V_t)^2 = K(V_{GS} + v_{gs} - V_t)^2$$

$$= \underbrace{K(V_{GS} - V_t)^2}_{I_D} + \underbrace{2K(V_{GS} - V_t)v_{gs} + Kv_{gs}^2}_{i_d}$$

$$i_d = \underbrace{2K(V_{GS} - V_t)\, v_{gs}}_{\text{linear}} + \underbrace{K v_{gs}^2}_{\text{quadratic}}$$

undesirable

$v_{gs} \ll 2(V_{GS} - V_t)$ then nonlinear term is negligible

$$i_d \cong 2K(V_{GS} - V_t)\, v_{gs} \quad = g_m v_{gs}$$

define transconductance : $g_m \equiv \dfrac{i_d}{v_{gs}} = 2K(V_{GS} - V_t)$

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{v_{GS} = V_{GS}}$$

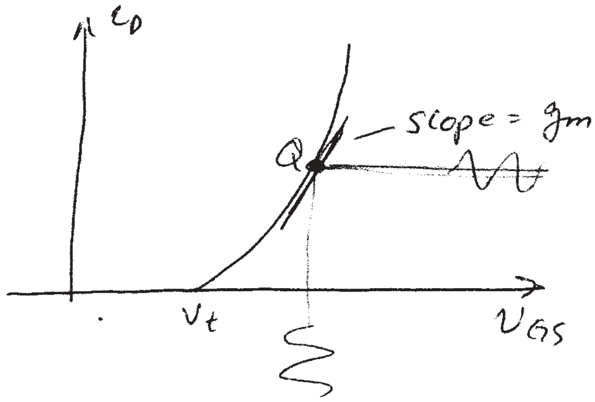
Remember $I_D = K(V_{GS} - V_t)^2$

$$\sqrt{I_D} = \sqrt{K}\,(V_{GS} - V_t)$$

$$\sqrt{\frac{I_D}{K}} = (V_{GS} - V_t)$$

$$g_m = 2K(V_{GS} - V_t) = 2K\sqrt{\frac{I_D}{K}} = 2\sqrt{\frac{K^2 I_D}{K}} = 2\sqrt{K I_D}$$

$$g_m = \sqrt{2\mu_n C_{ox}}\,\sqrt{W/L}\,\sqrt{I_D}$$



in the ckt, $v_D = V_{DD} - R_D i_D$

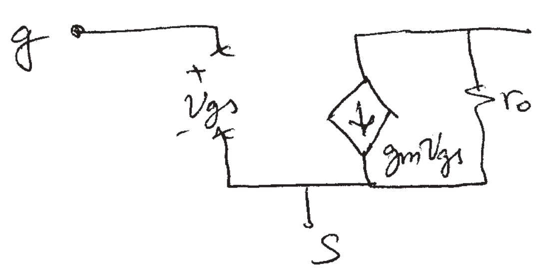
$$v_D = \underbrace{V_{DD} - R_D I_D}_{\text{bias} = V_D} - \underbrace{R_D i_d}_{\text{signal} = v_d}$$

$$v_d = -i_d R_D$$
$$= -g_m v_{gs} R_D$$

Voltage gain $\dfrac{v_d}{v_{gs}} = -g_m R_D$

These expressions are valid as long as the device remains in constant current mode and the input signal small.

Small signal model for a FET (saturation mode)
(Low and medium frequencies)



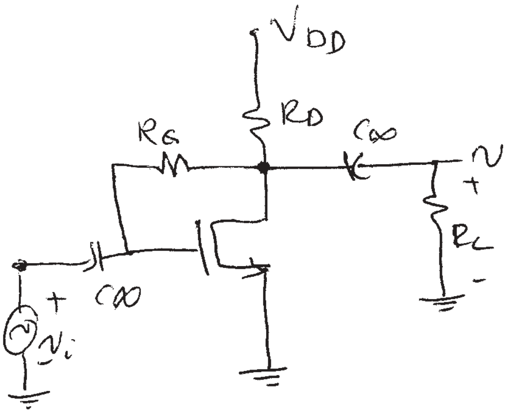
r_o is ignored most of the time.

$$r_o = \frac{|V_A|}{I_D}$$

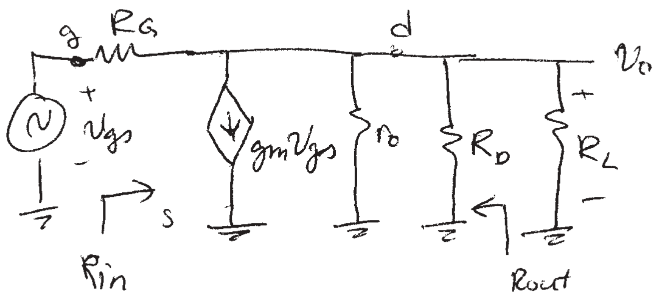
$$g_m = 2\sqrt{K I_D}$$

For a JFET $\quad g_m = \dfrac{2 I_{DSS}}{|V_P|}\sqrt{\dfrac{I_D}{I_{DSS}}}$

Example



small signal eq. ckt:



$V_t = 1.5\text{V}$
$K = 0.175 \text{mA/V}^2$
$V_A = 50\text{ V}$
$R_G = 10 \text{ M}\Omega$
$R_D = 10\text{ k}\Omega$
$R_L = 10\text{k}\Omega$
$V_{DD} = 15\text{V}$
$\lambda = 0$

$V_{GS} = 4.4\text{V} = V_D$
$I_D = 1.06 \text{ mV}$

$g_m = 2\sqrt{K I_D} = 0.725 \text{ mA/V}$

$r_o = \dfrac{50}{1.06} = 47\text{k}\Omega$

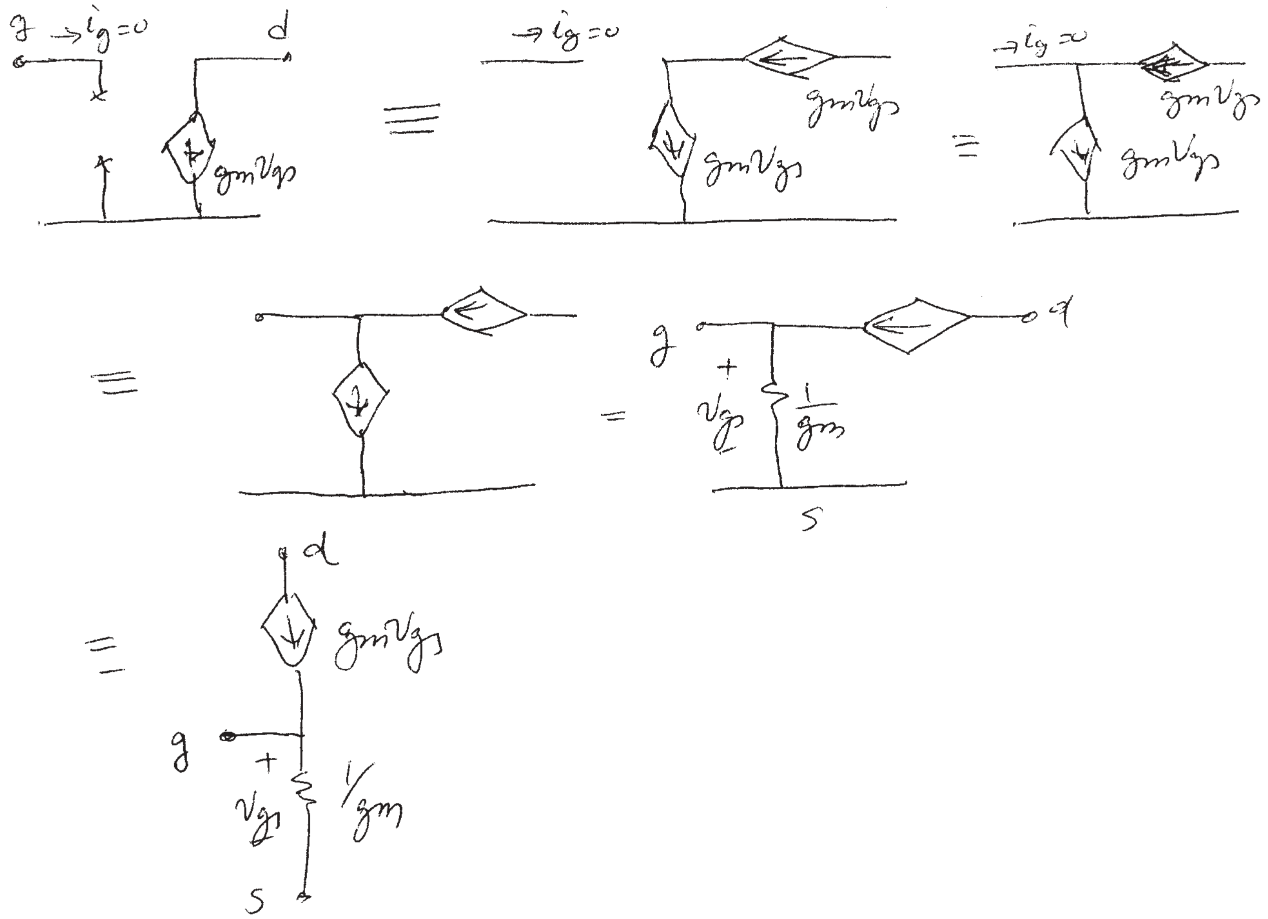
$\dfrac{v_o}{v_i} \cong -g_m(R_D \| r_o \| R_L)$

$= -3.3$

Current in R_G can be ignored since R_G is large (Compared to the current $g_m v_{gs}$)

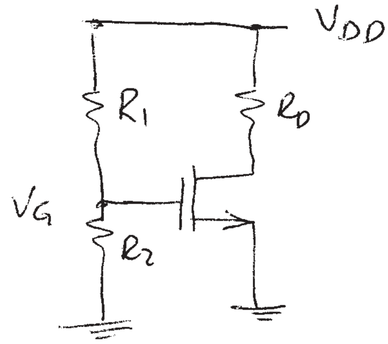
$$R_{in} = \frac{v_{gs}}{i_{in}} = \frac{v_{gs}}{(v_{gs} - v_o)/R_G} = R_G \frac{1}{1 - \frac{v_o}{v_{gs}}} = \frac{R_G}{4.3} = 2.33 \text{ M}\Omega$$

T equivalent circuit



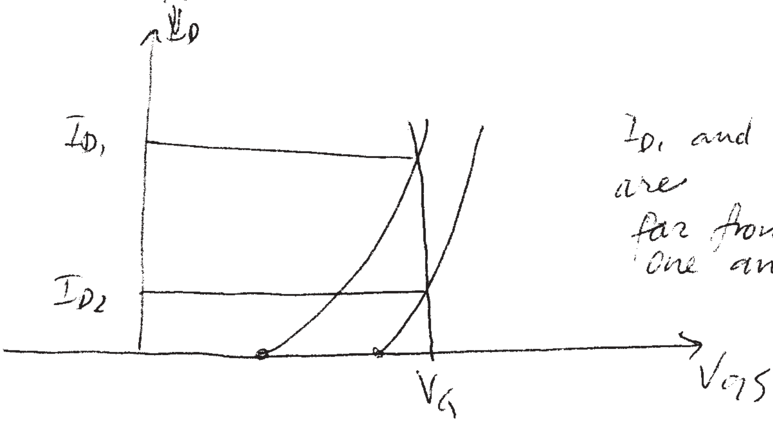
FET BIAS CIRCUITS

direct bias



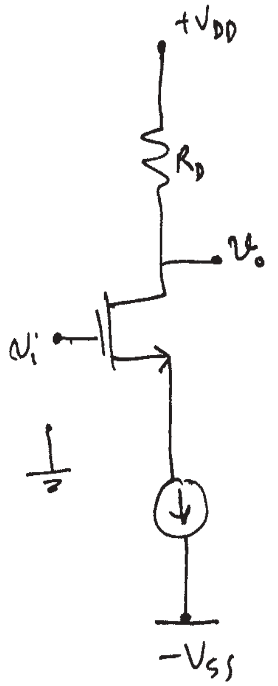
$$V_G = V_{DD} \frac{R_2}{R_1 + R_2} \quad \text{(constant)}$$

$$V_{GS} = V_G = \text{constant}$$



I_{D1} and I_{D2} are far from one another.

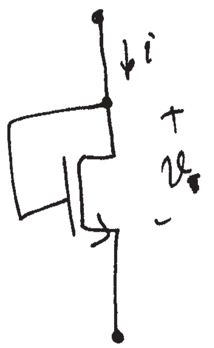
Integrated ckt MOS Amplifiers



R_D takes more space than a MOSFET

is it possible to replace R_D w/ a MOSFET?

Consider the following ckt:

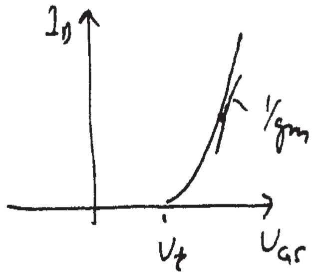


$V_T > 0$

$\Rightarrow$ always in Saturation mode.

$V = V_{GS}$; $i = K(V - V_t)^2$

its small signal resistance is $\frac{1}{g_m}$

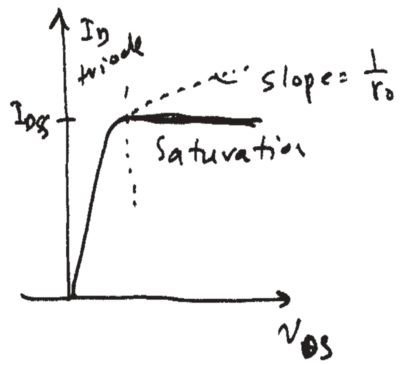


Another one

$V_{GS} = 0$

$v = V_{DS}$

$V_t = V_{tD} < 0$



$\leftarrow$ In triode mode the device would act like a low-valued resistor r_{ds}

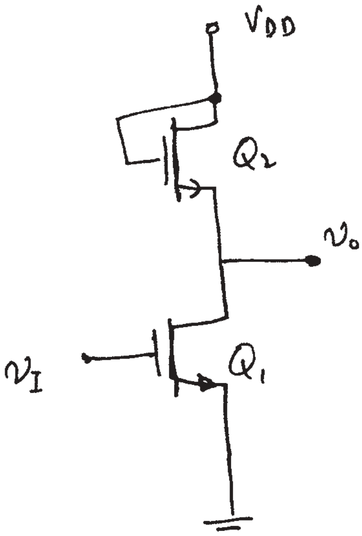
in the sat mode $\rightarrow$ large valued resistor $\rightarrow r_o$.

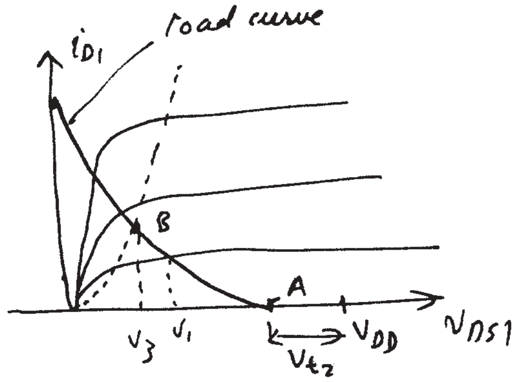
triode region:

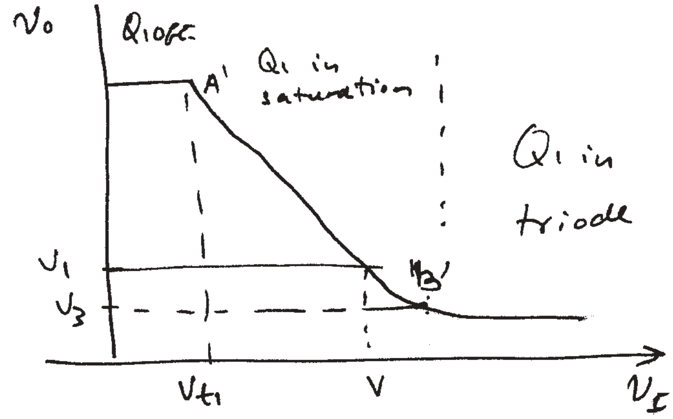
$$i = K(-2V_{tD}v - v^2)$$

onset of saturation $\quad i = KV_{tD}^2 = I_{DSS}$

Channel length mod. $\quad i \simeq KV_{tD}^2\left(1 + \frac{v}{V_A}\right)$







$$i_{D1} = K_1(v_{GS1} - V_t)^2$$

$$v_{GS1} = v_I$$

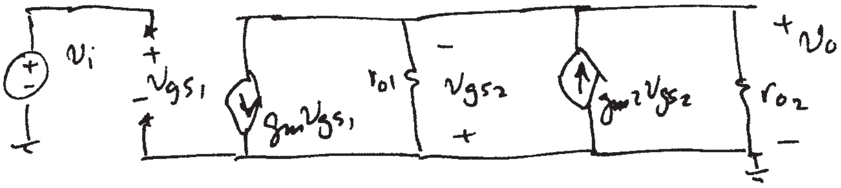
$$i_{D1} = i_{D2} = K_2(v_{GS2} - V_t)^2 = i_D$$

$$v_{GS2} = V_{DD} - v_o \Rightarrow i_D = K_2(V_{DD} - v_o - V_t)^2$$

$$\boxed{v_o = \left(V_{DD} - V_t + \sqrt{\frac{K_1}{K_2}}\, V_t\right) - \sqrt{\frac{K_1}{K_2}}\, v_I}$$

voltage gain $\dfrac{\Delta v_o}{\Delta v_I} = -\sqrt{\dfrac{K_1}{K_2}}$

using the small signal models

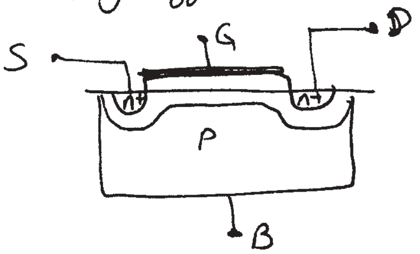


$$g_m = 2\sqrt{K I_D}$$

$$\frac{v_o}{v_i} = \frac{-g_{m1}}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}} \approx \frac{-g_{m1}}{g_{m2}}$$

$$-\frac{g_{m1}}{g_{m2}} = \frac{-\sqrt{K_1}}{\sqrt{K_2}}$$

Body effect:



if B is not tied to S, there will be "body effect"

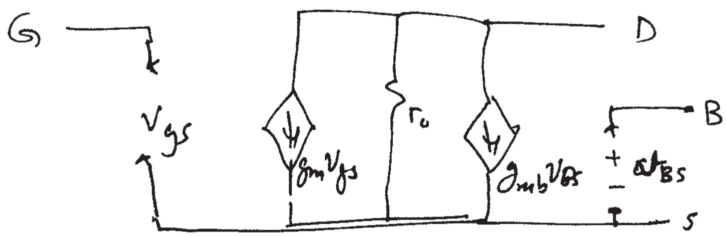
The "body" voltage affects V_t hence it affects I_D.

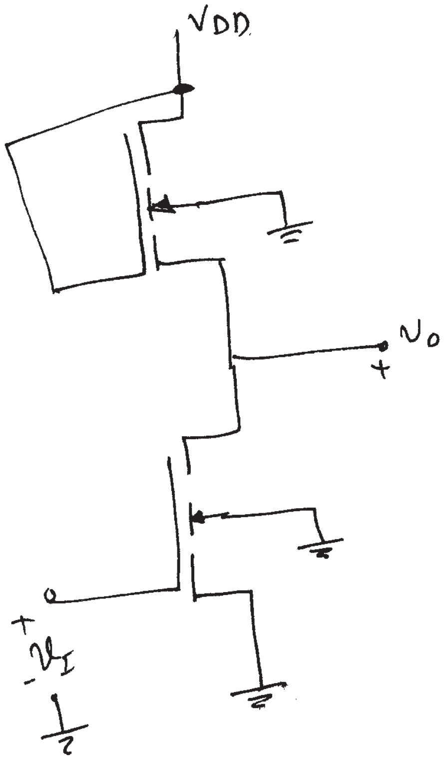
body transconductance $\rightarrow$ $g_{mb} = \left.\dfrac{\partial i_D}{\partial v_{BS}}\right|_Q$

$$g_{mb} = \chi g_m$$

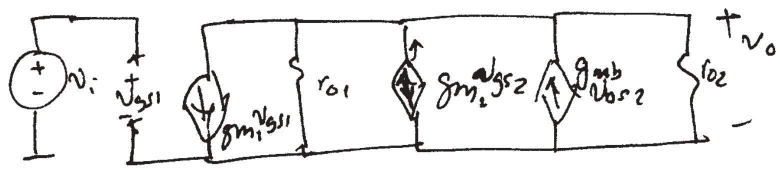
$$\chi = \frac{\partial V_t}{\partial V_{SB}} = \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}} \qquad 0.1 < \chi < 0.3$$

Modified Mosfet model:





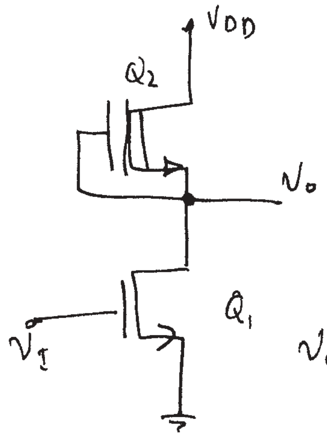
small signal model

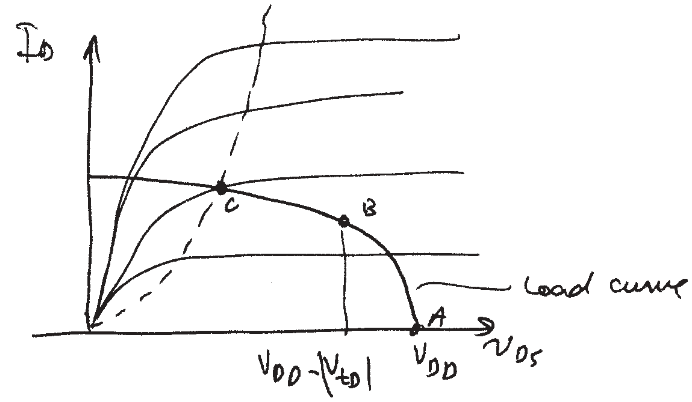


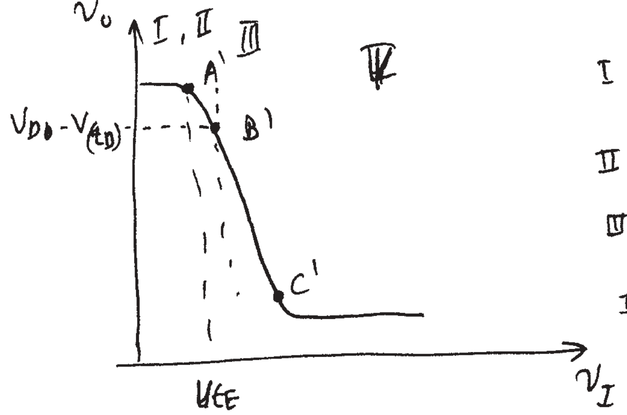
$$\frac{v_o}{v_i} = -\frac{g_{m1}}{g_{m2} + g_{mb2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}} \simeq \frac{-g_{m1}}{g_{m2} + g_{mb2}}$$

$$= \frac{-g_{m1}}{g_{m2}} \frac{1}{1+\chi}$$

NMOS Amplifier w/ depletion Load







I : Q_1 OFF, Q_2 TRIODE

II : Q_1 SAT, Q_2 TRI

III Q_1 : SAT, Q_2 : SAT

IV Q_1 : TRI, Q_2 : SAT

if the operation is in region III, $A_v = \dfrac{v_o}{v_i} = -g_{m1}(r_{o1} \| r_{o2})$

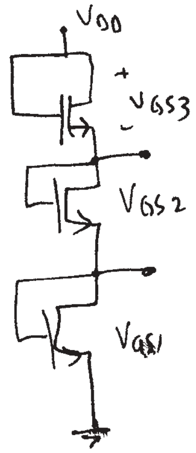
including the body effect:

$$A_v = -g_{m1}\left[\frac{1}{g_{mb2}} \| r_{o1} \| r_{o2}\right] \simeq -\frac{g_{m1}}{g_{mb2}} = -\frac{g_{m1}}{g_{m2}}\frac{1}{\chi}$$

Bias current $\simeq I_{DSS}$

DON'T WE NEED RESISTORS TO BIAS THE MOSFET?

One can use MOSFETS as voltage dividers



$$V_{GS1} + V_{GS2} + V_{GS3} = V_{DD}$$

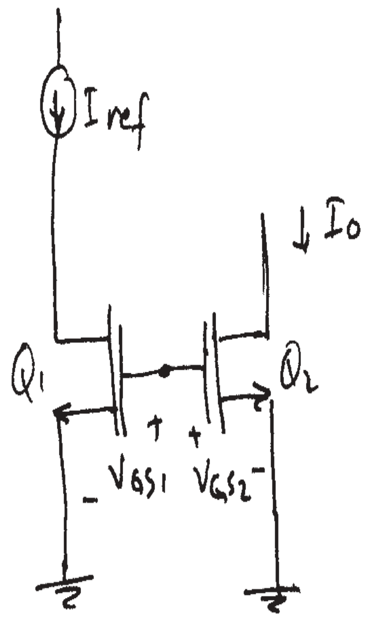
if $K_1 = K_2 = K_3$

$V_t \to$ same + I_D = equal

$$V_{GS1} = V_{GS2} = V_{GS3} = \frac{V_{DD}}{3}$$

By making K different one can adjust the voltages.

Current mirror:

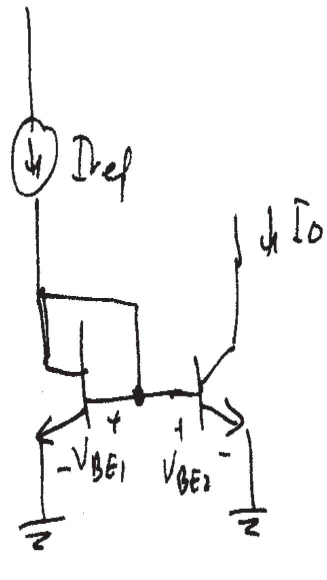


if Q_1 and Q_2 are identical and they both in saturation modes then $I_o = I_{ref}$.

if the two transistors are not identical but both in saturation mode → I_o and I_{ref} are related because $V_{GS1} = V_{GS2}$

$$\left.\begin{aligned} I_{ref} &= K_1(V_{GS1} - V_t)^2 \\ I_o &= K_2(V_{GS2} - V_t)^2 \end{aligned}\right\} \quad \frac{I_o}{I_{ref}} = \frac{K_2}{K_1}$$

W/ BJT



Read the sections
- FET switches
- GaAs Devices