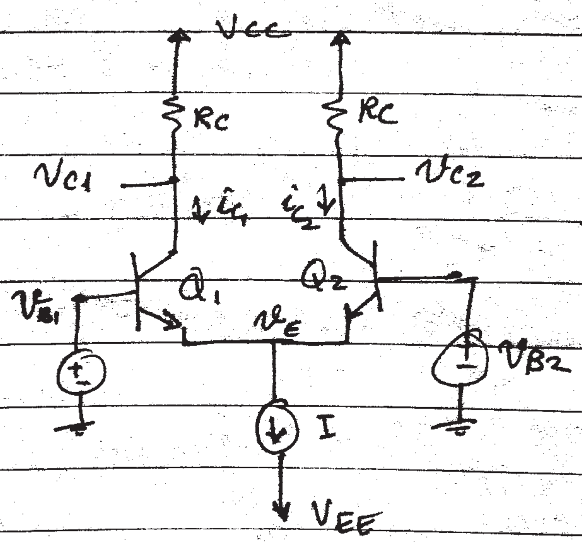
6. Differential and Multistage Amplifiers

"The most widely used circuit building block"
OPAMP input stage, ECL input stage
BJT, MOSFET, JFET, MESFET
"Biasing" is important — to be discussed here

6.1 The BJT Differential Amplifier

Topology:



Keep in mind:

1) Q_1 and Q_2 may not go into saturation mode
2) Q_1 and Q_2 are "identical."

The current I is shared between the two emitters: $I = i_{E_1} + i_{E_2}$

if $v_{B_1} = v_{B_2}$, then $i_{E_1} = i_{E_2} = \frac{I}{2}$

This is called common-mode.

Then, $i_{C_1} = i_{C_2} = \alpha i_{E_1} = \alpha \frac{I}{2} \sim \frac{I}{2}$ since $\alpha \cong 1$ for practical BJT's.

common mode → $v_{B_1} = v_{B_2} = V_{cm}$, $v_E = V_{cm} - V_{BE}$

$v_{C_1} = v_{C_2}$

$= V_{cc} - \frac{1}{2}\alpha I R_c$ ← independant of v_{B_1} or v_{B_2} (in common mode)

The amplifier rejects the common-mode signals.

Now make $v_{B_1} \neq v_{B_2}$ for example: $v_{B_1} = +1V$ $v_{B_2} = 0V$

Q_1 will have more current → $v_E = v_{B_1} - v_{BE} = 1 - 0.7 = 0.3V$

$v_{BE_2} = v_{B2} - v_E = 0 - 0.3\,V = -0.3 < 0.7 \rightarrow Q_2$ is OFF

$i_{C_1} = \alpha I$, $v_{C1} = V_{CC} - \alpha I R_C$, $v_{C2} = V_{CC}$

If you make $v_{B1} = -1$, $v_{B2} = 0$ then

Q_1 : OFF , Q_2 : ON $v_{C_1} = V_{CC}$ $v_{C2} = V_{CC} - \alpha I R_C$

Small differences between v_{B_1} and v_{B2} causes large changes in i_{C_1} and i_{C2}. $i_c = \frac{I}{2} \mp \Delta I$

$\Delta v_i \rightarrow \Delta I \rightarrow \Delta V_c = \alpha \Delta I R_C$

differential output $\rightarrow \Delta V_o = 2\alpha \Delta I R_C$?

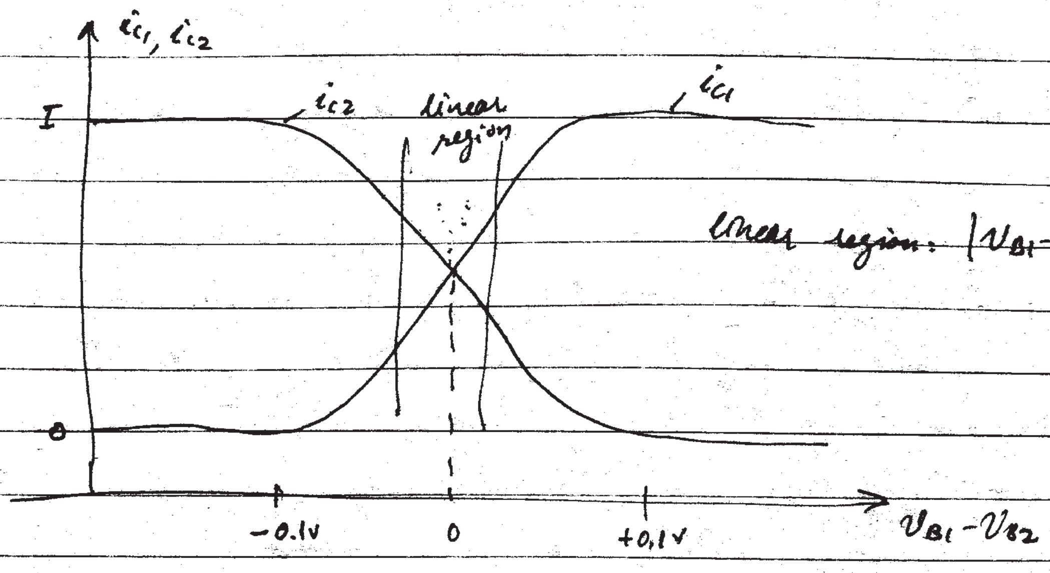
Large Signal Model

$$i_{E1} = \frac{I_S}{\alpha} e^{(v_{B1} - v_E)/V_T} , \quad i_{E_2} = \frac{I_S}{\alpha} e^{(v_{B2} - v_E)/V_T}$$

$$\frac{i_{E1}}{i_{E2}} = e^{(v_{B1} - v_{B2})/V_T} , \quad i_{E_1} + i_{E_2} = I$$

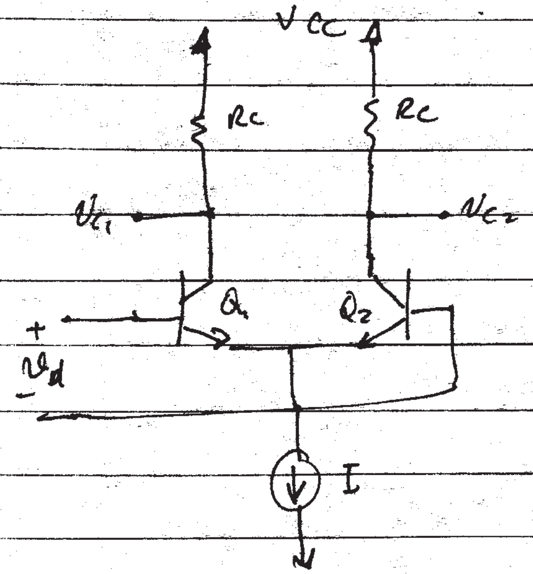
$$i_{E_1} = \frac{I}{1 + e^{(v_{B2} - v_{B1})/V_T}}$$

$$i_{E_2} = \frac{I}{1 + e^{(v_{B1} - v_{B2})/V_T}}$$



linear region: $|v_{B1} - v_{B2}| < \frac{V_T}{2}$

6.2 Small Signal Operation of BJT Diff. Amp.



differential mode

$v_{b_1} = -v_{b_2}$

$v_{b_1} - v_{b_2} = 2v_{b_1} = v_d$

$v_d \ll V_T$

$$i_{C_1} = \frac{\alpha I}{1+e^{-v_d/V_T}}, \quad i_{C_2} = \frac{\alpha I}{1+e^{v_d/V_T}}$$

multiplying the numerator and the denominator by $e^{v_d/2V_T}$

$$i_{C_1} = \frac{\alpha I e^{(v_d/2V_T)}}{e^{(v_d/2V_T)} + e^{(-v_d/2V_T)}}$$

$v_d \ll V_T \Rightarrow e^{\mp v_d/2V_T} \cong 1 \mp \frac{v_d}{2V_T}$

$$i_{C_1} \cong \frac{\alpha I \left(1 + \frac{v_d}{2V_T}\right)}{1 + \frac{v_d}{2V_T} + 1 - \frac{v_d}{2V_T}} = \frac{\alpha I}{2} + \frac{\alpha I}{2V_T}\frac{v_d}{2}$$

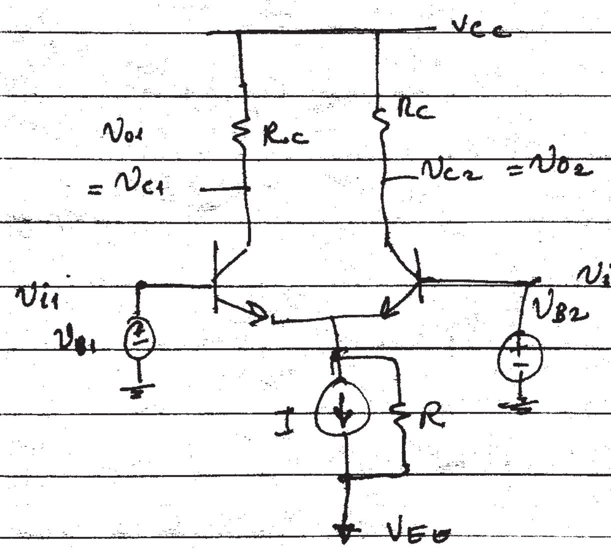
likewise,
$$i_{C_2} \cong \underbrace{\frac{\alpha I}{2}}_{\text{bias}} - \underbrace{\frac{\alpha I}{2V_T}\frac{v_d}{2}}_{\text{signal}}$$

$$i_c = -\frac{\alpha I}{2V_T}\frac{v_d}{2}$$

The transconductance $g_m = \frac{I_C}{V_T} = \frac{\alpha I/2}{V_T} = \frac{\alpha I}{2V_T}$

That means $i_{c1} = g_m \frac{v_d}{2}$, $i_{c2} = -g_m \frac{v_d}{2}$

SMALL SIGNAL MODEL FOR A BJT DIFF. AMP



Purely differential mode $\rightarrow$ $v_{i1} = -v_{i2}$

Purely common mode $\rightarrow$ $v_{i1} = v_{i2}$

for any combination : $v_{idm} = v_{i1} - v_{i2}$ diff. mode of inputs

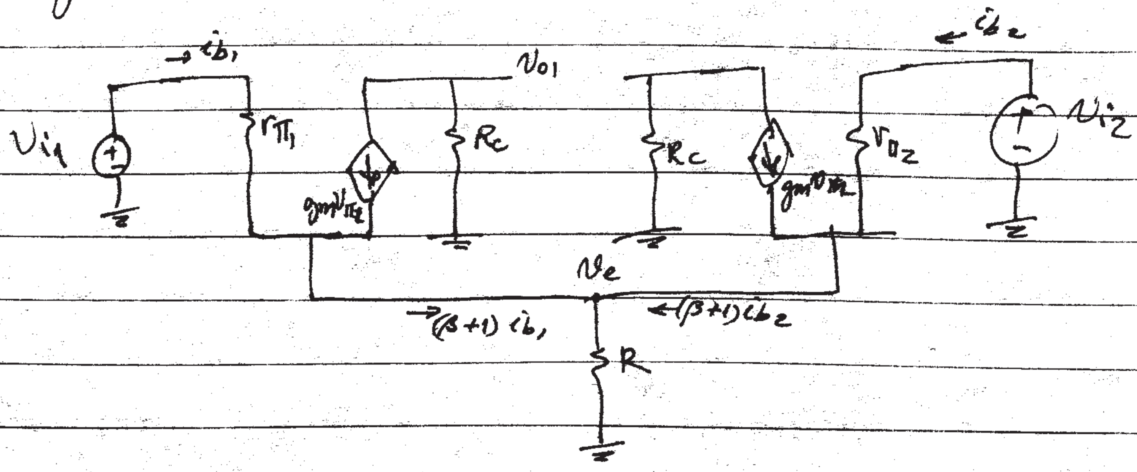
$v_{icm} = \dfrac{v_{i1} + v_{i2}}{2}$ common mode of input.

v_{i1} and v_{i2} can be reconstructed:

$$v_{i1} = v_{icm} + \frac{v_{idm}}{2}$$

$$v_{i2} = v_{icm} - \frac{v_{idm}}{2}$$

Small signal model.



$v_{o1} = -g_m v_{\pi_1} R_c$ $\qquad$ $v_{o2} = -g_m v_{\pi_2} R_c$

Purely differential mode: $v_{i_1} = -v_{i_2}$, $i_{b_1} = -i_{b_2}$, $v_{\pi_1} = -v_{\pi_2}$

since the current in R equals zero, $v_e = 0$
virtual ground.

$v_{\pi_1} = v_{i_1}$ $\qquad v_{o_1} = -g_m R_c v_{i_1} = -g_m R_c \frac{v_{idm}}{2}$

Single-ended, diff mode gain: $A_{dm\,se_1} = \frac{v_{o_1}}{v_{idm}} = -\frac{g_m R_c}{2}$

$A_{dm\text{-}se_2} = \frac{v_{o_2}}{v_{idm}} = +\frac{g_m R_c}{2}$

The diff mode input resistance: $r_{in\text{-}dm} \equiv \frac{v_{idm}}{i_{b_1}}$

$r_{in\text{-}dm} = \frac{2 v_{i_1}}{i_{b_1}} = 2 r_{\pi}$ $\qquad r_{out\text{-}~~se~~} = R_c$

Differential mode of the output is possible:

$A_{dm\text{-}diff} \equiv \frac{v_{o_1} - v_{o_2}}{v_{idm}} = -g_m R_c$

$r_{out\text{-}diff} = 2R_c$

—o—

Common mode analysis: $v_{i_1} = v_{i_2} = v_{icm}$

in this case, $v_e \neq 0$

$v_e = R\,(i_{e_1} + i_{e_2})$

$v_e = R\,(\beta+1)\,(i_{b_1} + i_{b_2})$

$v_e = 2R\,(\beta+1)\,i_{b_1}$

$v_{i_1} = v_e + i_{b_1} r_{\pi}$

$v_{i_1} = [r_{\pi} + 2R(\beta+1)]\, i_{b_1}$ $\qquad v_{o_1} = -g_m v_{\pi_1} R_c$

$= -\beta i_{b_1} R_c$

$A_{cm\text{-}se_1} = \frac{v_{o_1}}{v_{icm}} = \frac{-\beta R_c}{r_{\pi} + 2(\beta+1) R}$ $\qquad A_{cm\text{-}se_2} = \frac{-\beta R_c}{r_{\pi} + 2(\beta+1) R}$

ideal current source $\rightarrow R = \infty \Rightarrow A_{cm-se} = 0$

Common mode

input resistance: $r_{icm} \equiv \frac{v_{icm}}{i_{b_1}} = \frac{v_{i_1}}{i_{b_1}} = r_\pi + 2(\beta+1)R$

output resistance $r_{out-se} = R_C$

Common mode rejection ratio: $CMRR = \left| \frac{A_{dm}}{A_{cm}} \right|$

Sometimes in dB $= 20 \log \left| \frac{A_{dm}}{A_{cm}} \right|$

For our ckt: $CMRR = \dfrac{g_m R_C/2}{\dfrac{\beta R_C}{r_\pi + 2(\beta+1)R}}$

$g_m = \frac{\beta}{r_\pi} \Rightarrow CMRR = \frac{r_\pi + 2(\beta+1)R}{2r_\pi} \approx \frac{(\beta+1)R}{r_\pi}$

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Read section 6.2 for details.

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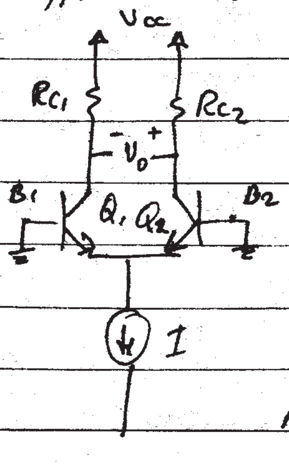
6.3 Non-ideal Characteristics of The Diff-Amp.

input offset voltage



Since no two things are absolutely identical

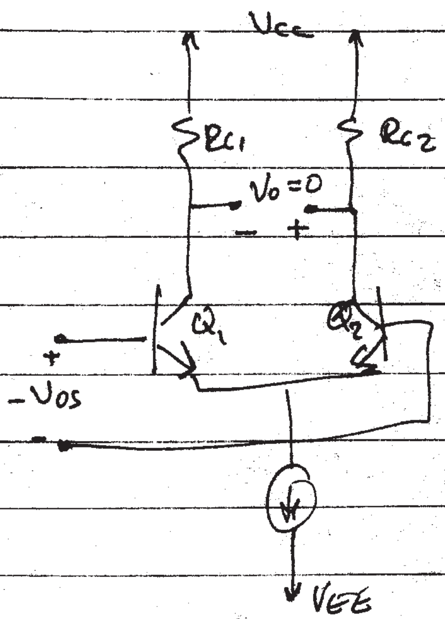
$\rightarrow i_{c_1} \neq i_{c_2} \quad R_{c_1} \neq R_{c_2}$

$\rightarrow V_o$ is nonzero when both inputs are zero.

input offset voltage: $V_{os} = V_o / A_{dm}$

V_o: output offset voltage

if one applies $-V_{OS}$ between the input terminals, the output voltage $= 0$.



Read Section 6.3 for a detailed analysis

V_{OS} can be due to mismatch between R_{C_1} and R_{C_2} or between I_{S_1} and I_{S_2} of Q_1 and Q_2.

$$V_{OS} = V_T \sqrt{\left(\frac{\Delta R_C}{R_C}\right)^2 + \left(\frac{\Delta I_S}{I_S}\right)^2}$$

R_C, I_S: average values

Input bias and Offset currents

$$I_{OS} = |I_{B1} - I_{B2}| = I_B \left(\frac{\Delta\beta}{\beta}\right)$$

average

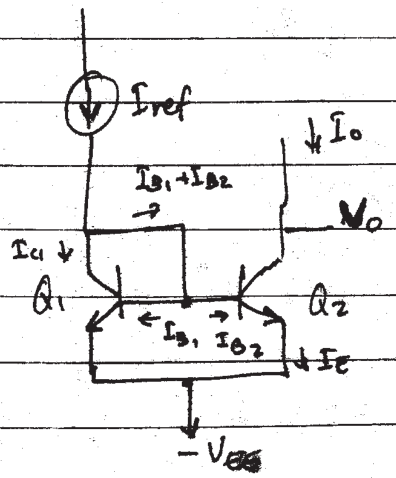
Input common mode range

The range is determined from the condition that Q_1 and Q_2 must stay in active mode. An additional condition comes from the current source.

BIASING THE BJT INTEGRATED CKTS

Resistors and capacitors take too much space on an integrated ckt → they are avoided in IC design.

The current mirror



Q_1 and Q_2 are identical

$V_{BE_1} = V_{BE2}$

$\Rightarrow \quad I_{C_1} = I_{C_2} = I_o$

$I_{ref} = I_{C1} + I_{B_1} + I_{B_2} = I_o + 2 I_{B_1}$

if $\beta \gg 1$ then the base currents can be neglected

$\rightarrow \quad I_o \simeq I_{ref}$

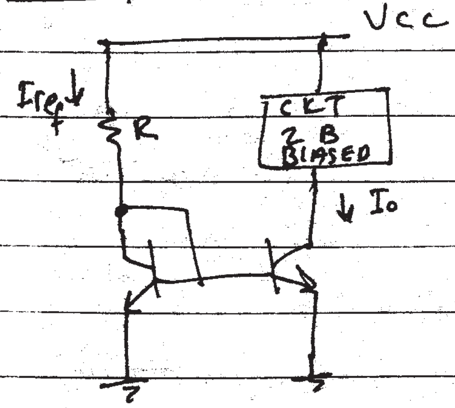
To be more precise:

$$I_o = \frac{\beta}{\beta+1} I_E \,, \quad I_{ref} = \frac{\beta+2}{\beta+1} I_E$$

$$\frac{I_o}{I_{ref}} = \frac{\beta}{\beta+2} = \frac{1}{1+\frac{2}{\beta}}$$

(this ignores the Early effect)

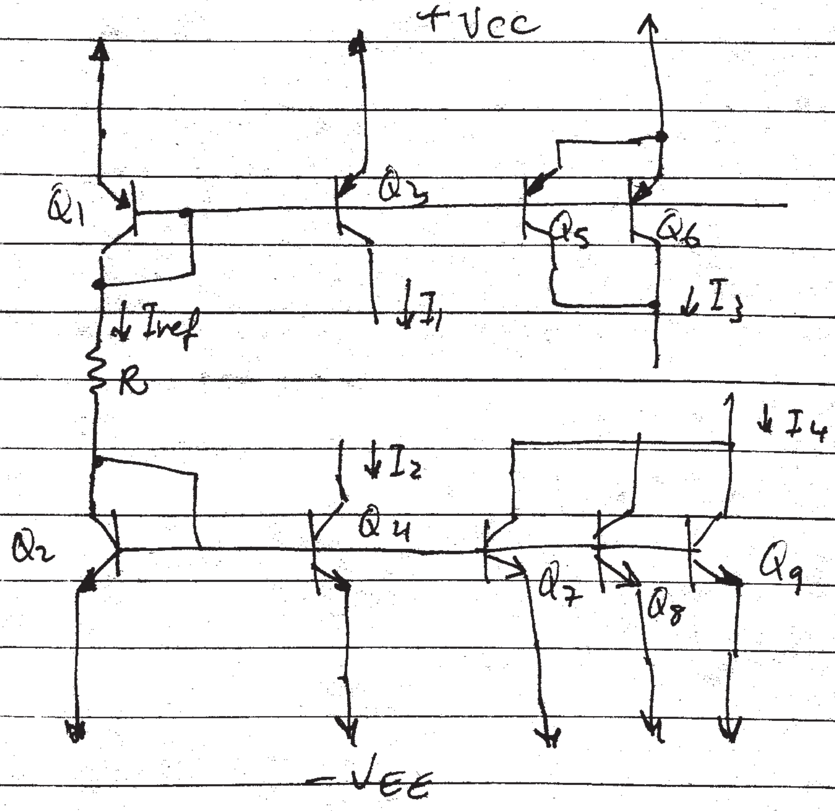
Example



$$I_{ref} = \frac{V_{CC} - V_{BE}}{R}$$

$$\left.\begin{array}{l} I_o = 1\,mA \\ V_{CC} = 5V \end{array}\right\} \Rightarrow \quad R = \frac{5-0.7}{1} = 4.3\,k\Omega$$

Current Steering Circuits



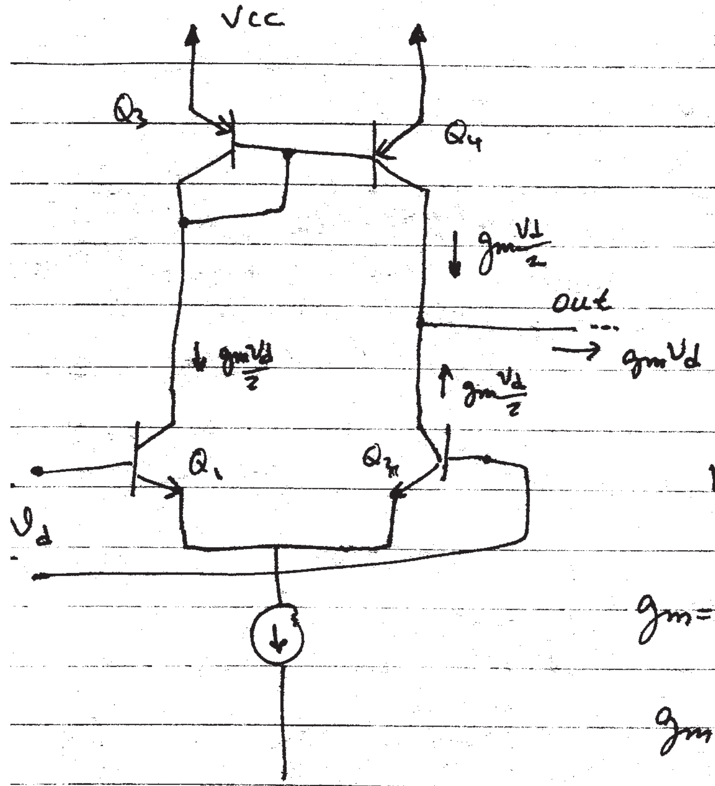
I_{REF} provides the base currents for all transistors

Read details in section 6.4 including Wilson + Widlar sources.

Study Example 6.2 on p. 438

6.5 The BJT Differential Amplifier w/ Active Load

A transistor operated as a current source has a large small signal resistance ⟶ it may be utilized as a high-value resistor.



$V_o = g_m V_d R_o$

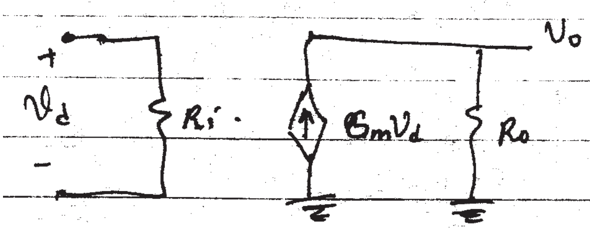
$R_o = r_{o2} \| r_{o4}$

$r_{o2} = r_{o4} = r_o \Rightarrow \quad V_o = \frac{g_m V_d r_o}{2}$

$g_m = \frac{I_C}{V_T}$, $r_o = \frac{V_A}{I_C}$, $I_C = \frac{I}{2}$

$g_m r_o = \frac{V_A}{V_T}$

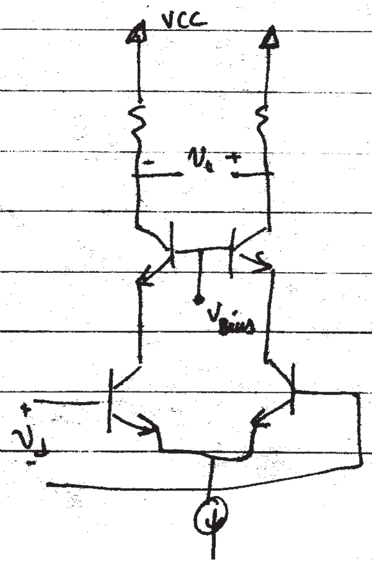
$V_A = 100V \Rightarrow g_m r_o = 4000$, gain $\simeq 2000$



$R_i = 2r_\pi$

$G_m = g_m = \frac{I/2}{V_T}$

The Cascode configuration

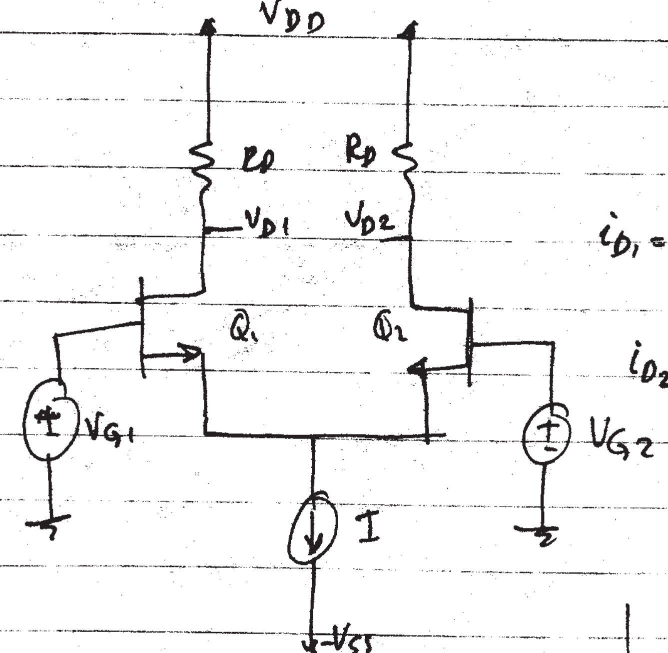


~~current~~ Voltage gain does not change much. However freq. response is improved.

JFET and MOSFET DIFF. PAIRS

— high input resistance —

$v_{G1} - v_{G2} = v_{id}$



identical JFETs

$$i_{D1} = \frac{I}{2} + v_{id}\frac{1}{-2V_p}\sqrt{2\frac{I_{DSS}}{I} - \left(\frac{v_{id}}{V_p}\right)^2\left(\frac{I_{DSS}}{I}\right)^2}$$

$$i_{D2} = \frac{I}{2} - v_{id}\frac{1}{-2V_p}\sqrt{\frac{2I_{DSS}}{I} - \left(\frac{v_{id}}{V_p}\right)^2\left(\frac{I_{SS}}{I}\right)^2}$$

$$i_D = I_{DSS}\left(1 - \frac{v_{GS}}{V_p}\right)^2$$

Small signal operation.

$$v_{d1} = -g_m \frac{v_{id}}{2} R_D$$

$$v_{d2} = +g_m \frac{v_{id}}{2} R_D$$

MOSFET $i_D = k(v_{GS} - V_t)^2$

$$i_{D1} = \frac{I}{2} + \left(\frac{I}{V_{GS} - V_t}\right)\left(\frac{v_{id}}{2}\right)\sqrt{1 - \left(\frac{v_{id}/2}{V_{GS} - V_t}\right)^2}$$

$$i_{D2} = \frac{I_{\ }}{2} - \left(\qquad\right)\left(\qquad\right)\sqrt{\qquad}$$

$$g_m = \frac{2I_D}{V_{GS} - V_t} = \frac{I}{V_{GS} - V_t}$$

Read the rest of the section