Eskişehir Osmangazi University - Electrical Engineering Department Advanced Calculus Final Examination - Fall 2014

All answers must be written below or next to the questions. Anything written elsewhere will not be graded. Use the back side of the exam sheet if you need scratch paper.

1. [25 pts.] Evaluate

$$\int_C \frac{4}{(z^2+2)(z^2+4)} dz$$

where C is a positively oriented circle centered at 3i with radius 2.

1.30

2. [25 pts.] Consider $\frac{d^2y}{dx^2} + y = 0$ with its power series solution $y = c_0 + c_1x + c_2x^2 + c_3x^3 + \cdots$

- (a) Write c_2 as a function of c_0 .
- (b) Write c_{20} as a function of c_{18} .

$$\rightarrow c_2 = -\frac{1}{2}c_0, \ c_{20} = -\frac{c_{18}}{380}$$

3. [25 pts.] Find a general solution for

$$\frac{\frac{dx}{dt}}{\frac{dy}{dt}} = y \\ \frac{dy}{dt} = -2x + 3y \end{cases} \xrightarrow{\begin{subarray}{c} x = c_1 e^{2t} + c_2 e^t \\ y = 2c_1 e^{2t} + c_2 e^t \\ \frac{dy}{dt} = -2x + 3y \end{array}$$

4. [25 pts.] Find a general solution for

$$\frac{d^2y}{dt^2} + y = t^2 + 1$$
$$y = c_1 \cos t + c_2 \sin t + t^2 - 1$$

Good Luck

A. Karamancıoğlu **More Details** (1) Res at 2i $\frac{4}{(z^2+2)(z+2i)}|_{z=2i} = 0.5i$; Res at $\sqrt{2}i$: $\frac{4}{(z+\sqrt{2}i)(z^2+4)}|_{z=\sqrt{2}i} = -\frac{i}{\sqrt{2}}$ $\int_C \frac{4}{(z^2+2)(z^2+4)} = 2\pi i (0.5i - 0.707i) = 0.414\pi = 1.30$ (2) $\frac{d^2y}{dx^2} + y = 0$; $\sum_{2}^{\infty} n(n-1)c_n x^{n-2} + \sum_{0}^{\infty} c_n x^n = 0$; $\sum_{0}^{\infty} (n+2)(n+1)c_{n+2}x^n + \sum_{0}^{\infty} c_n x^n = 0$; $\sum_{0}^{\infty} [(n+2)(n+1)c_{n+2} + c_n]x^n = 0$; $c_{n+2} = -\frac{c_n}{(n+1)(n+2)}, n = 0, 1, 2, \dots$

$$\rightarrow c_2 = -\frac{1}{2}c_0, \ c_{20} = -\frac{c_{18}}{380}$$