PROBABILITY FIRST MIDTERM EXAM

Dr. Salih Fadıl **October 22, 2008**

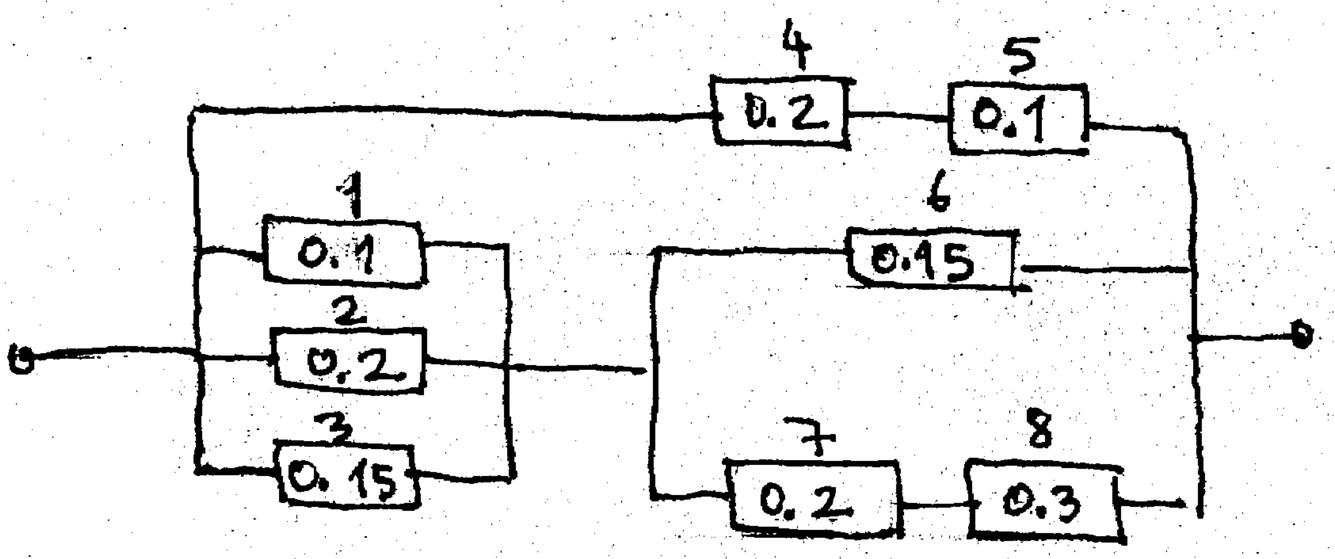
#1) A box contains five balls. Numbers 1, 2, 3, 4 and 5 are attached to those balls. A person selects a number (ball) randomly from the box and put it aside. Later on, he selects the second number randomly and put it aside. If the second number he selected is bigger than the first one, he selects the third number randomly and put it aside. If the third number he selected is bigger than the second one, he selects the fourth number randomly and put it aside and so on. Let random variable X be the number of selections that is made by the person. Determine the probability function of X.

#2) Barbara and Dianne go target shooting. Suppose that each of Barbara's shots hits the wooden duck target with the probability p_1, while each shot of Dianne's hits it with probability p_2. Suppose that they shoot simultaneously at the same target. If the wooden duck is knocked over (indicating that it was hit) what is he probability that

a) both shots hit the duck?

b) Barbara's shot hit the duck?

#3) Calculate probability that the following system works. Assume that devices work independently.



The numbers inside the rectangles show the failure probabilities of the corresponding devices

PROBABILITY FIRST MIDTERM EXAM SOLUTION MANUAL

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#1) X = # of selections that is made

$$A = \{2, 3, 4, 5\}$$

↑ increase
↓ decrease

For $X = 2$

1↓	2	3	4	5	← selection number
5	1	x	x	x	
5	2	x	x	x	
5	3	x	x	x	
5	4	x	x	x	
4	3	x	x	x	
4	2	x	x	x	
4	1	x	x	x	
3	2	x	x	x	
3	1	x	x	x	
2	1	x	x	x	

$10 \times 3! = 60$

For $X = 3$

1	↑ 2	↓ 3	4	5
1	3	2	x	x
1	4	2	x	x
1	4	3	x	x
1	5	2	x	x
1	5	3	x	x
1	5	4	x	x
2	3	1	x	x
2	4	3	x	x
2	4	1	x	x
2	5	1	x	x
2	5	3	x	x
2	5	4	x	x
3	4	2	x	x
3	4	1	x	x
3	5	1	x	x
3	5	2	x	x
3	5	4	x	x
4	5	1	x	x
4	5	2	x	x
4	5	3	x	x

$20 \times 2 = 40$

For $X = 4$

1	↑ 2	↑ 3	↓ 4	5
1	2	4	3	x
1	2	5	3	x
1	2	5	4	x
1	3	4	2	x
1	3	5	4	x
1	3	5	2	x
1	4	5	2	x
1	4	5	3	x
2	3	4	1	x
2	3	5	1	x
2	3	5	4	x
2	4	5	1	x
2	4	5	3	x

1	↑ 2	↑ 3	↓ 4	5
3	4	5	2	x
3	4	5	1	x

$15 \times 1! = 15$

For $X=5$ ← does not matter, since there are only 5 balls in the box.

1 ↑	2 ↑	3 ↑	4	5
1	2	3	5	4
1	2	4	5	3
1	3	4	5	2
2	3	4	5	1
1	2	3	4	5

} 5 cases.

X	2	3	4	5
$p(X)$	$\frac{60}{120}$	$\frac{40}{120}$	$\frac{15}{120}$	$\frac{5}{120}$
	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{8}$	$\frac{1}{24}$

#2) $B \triangleq$ Barbara hits the target $\rightarrow P(B)=P_1$, $P(\bar{B})=1-P_1$

a) $D \triangleq$ Dianne hits the target $\rightarrow P(D)=P_2$, $P(\bar{D})=1-P_2$

$H \triangleq$ The target (wooden duck) is hit

Barbara and Dianne shoot the target simultaneously.
Events B and D are independent.

$$H = B \cup D \leftarrow \{\text{The target is hit}\} = \{\text{At least one person hit the target}\}$$

$$P(H) = P(B) + P(D) - P(B)\,P(D) = P_1 + P_2 - P_1 P_2 \quad \text{or}$$

$$P(H) = 1 - P(\overline{B \cup D}) = 1 - P(\bar{B})\,P(\bar{D}) = 1 - q_1 q_2$$

$q_1 \triangleq 1-P_1$

$q_2 \triangleq 1-P_2$

$$P(B \cap D / H) = \frac{P\{(B \cap D) \cap H\}}{P(H)}$$

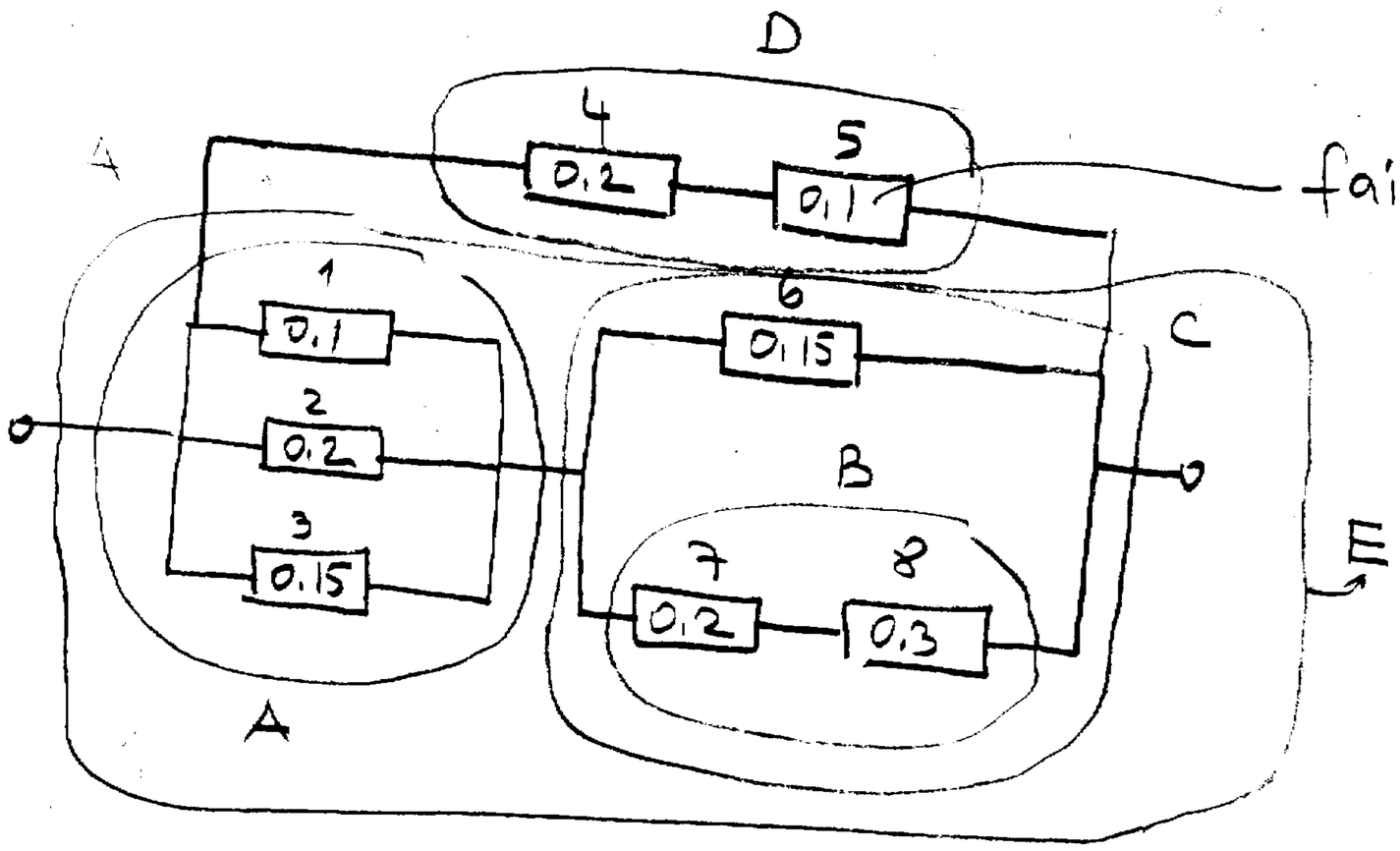
$$= \frac{P\{(B \cap D)\}}{P(H)} = \frac{P(B)\,P(D)}{P(H)} = \frac{P_1 P_2}{P_1 + P_2 - P_1 P_2} \quad \text{or}$$

$$= \frac{P_1 P_2}{1 - q_1 q_2}$$

1b) $P(B/H) = \dfrac{P\{B \cap H\}}{P(H)} = \dfrac{P(B)}{P(H)} = \dfrac{P_1}{1 - q_1 q_2}$

$= \dfrac{P_1}{P_1 + P_2 - P_1 P_2}$

#3)



Devices function independently!

$P(\bar{S}) = P(\bar{D} \cap \bar{E}) = P(\bar{D}) \cdot P(\bar{E})$

$P(\bar{D}) = P(\bar{X}_4 \cup \bar{X}_5) = 0,2 + 0,1 - 0,2 \times 0,1 = 0,28$

$P(\bar{E}) = P(\bar{A} \cup \bar{C}) = P(\bar{A}) + P(\bar{C}) - P(\bar{A})\,P(\bar{C})$

$P(\bar{A}) = P(\bar{X}_1 \cap \bar{X}_2 \cap \bar{X}_3) = P(\bar{X}_1) \cdot P(\bar{X}_2) \cdot P(\bar{X}_3) = 0,1 \times 0,2 \times 0,15$

$P(\bar{C}) = P(\bar{B} \cap \bar{X}_6) = P(\bar{B}) \cdot P(\bar{X}_6)$ $= 3 \times 10^{-3}$

$P(\bar{B}) = P(\bar{X}_7 \cup \bar{X}_8) = 0,2 + 0,3 - 0,2 \times 0,3 = 0,44$

$P(\bar{C}) = 0,44 \times 0,15 = 0,066.$

$P(\bar{E}) = 3 \times 10^{-3} + 0,066 - 3\,10^{-3} \times 0,066 = 0,068802$

$P(\bar{S}) = 0.28 \times 0.068802 = 0,01926456$

$P(s) = 1 - P(\bar{S}) = 1 - 0,01926456$

$\boxed{P(s) = 0,98073544.}$

Note that $P(s)$ can be calculated directly by using devices's working probabilities.