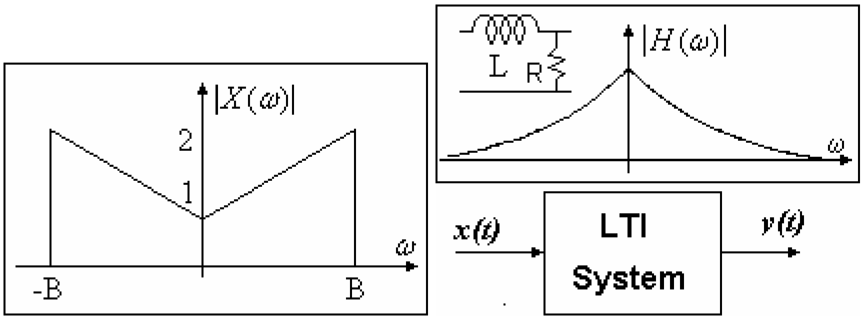
No : Solutions Name : Answers

Eskişehir Osmangazi University - Faculty of Engineering and Architecture

Department of Electrical Engineering & Electronics 02.05.2006

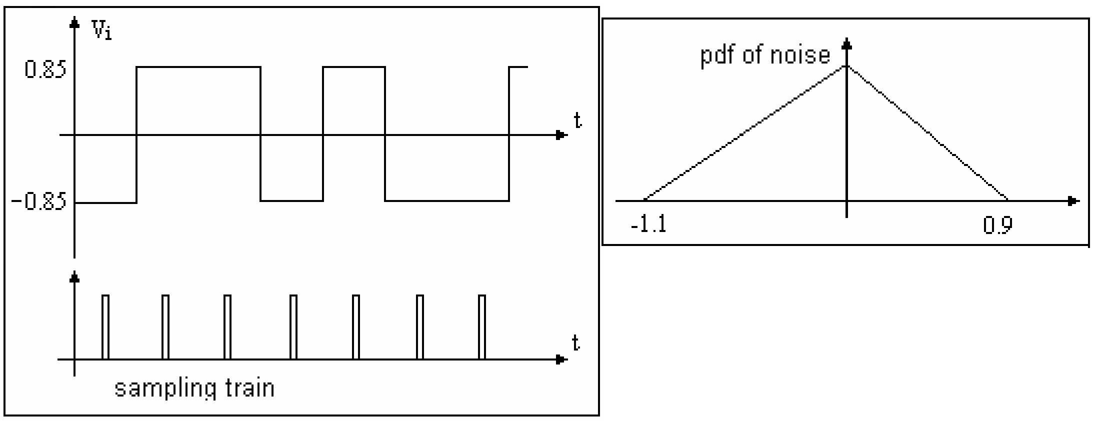
"*Communications*" 2nd Midterm Exam.

1. The input of a linear time invariant system is an ergodic random process $x(t)$. $|X(\omega)|$ and $|H(\omega)|$ are given. Find $E\{Y\}$ (expected value of the output).

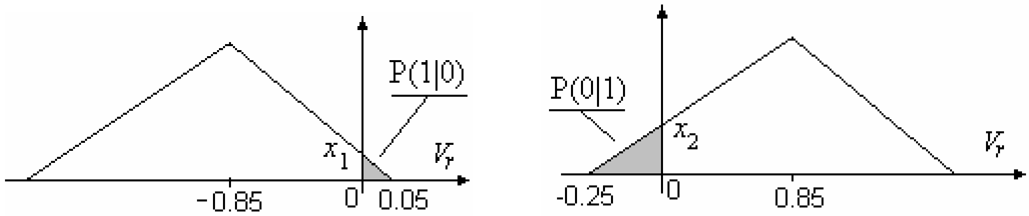


$E\{Y\} = m_y$ depends only on m_x and $H(\omega)$ at $\omega = 0$. That is $E\{Y\} = m_y = m_x H(0)$. For the given LR circuit $H(0) = 1$. m_x equals to the DC value of $x(t)$ which is given as 1 in the $|X(\omega)|$ spectrum. So, $E\{Y\} = 1 \times 1 = 1$,

2. On a differential binary, line binary zero (0) and one (1) are repesented by -0.85 and 0.85 Volts respectively. At the receiver-end the signal is sampled as shown. The decision circuitry uses 0 V as threshold. The line is under AWN whose pdf is shown. The probability of transmitting a binary 0 is 0.7. Calculate the probability of decision error at the receiver.



The probabilities of receiving 1 when 0 was sent and receiving 0 when 1 was sent are the areas shown.



Note : No books, notes allowed in exam. 1 hour 15 minutes (seventy-five minutes)

Using similarity of triangles $x_1 = \dfrac{0.05}{0.9}$ and $x_2 = \dfrac{0.25}{1.1}$. The areas are then

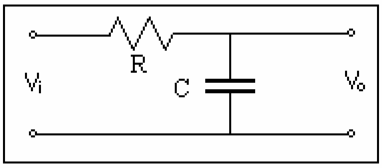
$$P(1|0) = \frac{0.05}{0.9} \times 0.05/2 = \frac{0.0025}{1.8} \cong 0.0014 \quad \text{and}$$

$$P(0|1) = \frac{0.25}{1.1} \times 0.25/2 = \frac{0.0625}{12.2} \cong 0.0284.$$

Summing the error probabilities $P_e = P(0)P(1|0) + P(1)P(0|1)$

$$P_e = 0.7 \times 0.0014 + 0.3 \times 0.0284 \cong 0.0095$$

3. An LPF with R=1kΩ is given. White Gaussian noise at the input has the power spectral density of $S_n(\omega) = \dfrac{N_0}{2} = 10^{-12}\left[W/Hz\right]$. In order for the total output noise power P_{no} to be less than $6.28 \times 10^{-3}\left[\mu W\right]$ find the minimum value of the capacitor C.



$$S_o(\omega) = S_i(\omega)\left|H(\omega)\right|^2, \quad H(j\omega) = \frac{1}{1 + j\omega RC}, \quad \left|H(\omega)\right| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$$P_o = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_o(\omega)d\omega = \frac{N_0}{2\pi}\int_0^{\infty}\frac{d\omega}{1 + \omega^2 R^2 C^2}$$

$$P_o = \frac{N_0}{2\pi R^2 C^2}\int_0^{\infty}\frac{d\omega}{\omega^2 + \dfrac{1}{R^2C^2}} = \frac{N_0}{2\pi R^2 C^2} RC\tan^{-1}(RC\omega)\Big|_0^{\infty} = \frac{N_0}{2\pi RC}\left[\tan^{-1}(\infty) - \tan^{-1}(0)\right]$$

$$P_o = \frac{10^{-12}}{1 \times 10^3 \times \pi \times C}\frac{\pi}{2}$$

For this to be equal to the maximum allowed noise power C should be

$$C = \frac{10^{-12}}{1 \times 10^3 \times 6.28 \times 10^{-9} \times 2} = 0.08 \times 10^{-6}\left[F\right] = 80\left[nF\right] \text{ (minimum capacitance.)}$$

$\sin A \cos B = \frac{1}{2}(\sin(A-B) + \sin(A+B))$ $\qquad \cos^2 A = \frac{1}{2} + \frac{1}{2}\cos(2A)$ $\qquad \sin^2 A = \frac{1}{2} - \frac{1}{2}\cos(2A)$

$\cos A \cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$ $\qquad \cos(A \pm B) = \cos A \cos B \pm \sin A \sin B$

$\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$ $\qquad e^{-at}u(t) \Leftrightarrow (a + j\omega)^{-1}$ $\qquad u(t) \Leftrightarrow \pi\delta(\omega) - j\omega^{-1}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$ $\qquad \int \frac{xdx}{x^2 + a^2} = \frac{1}{a}\ln(x^2 + a^2) + c$ $\qquad \int e^x dx = e^x + c$

$\int \frac{x^2 dx}{x^2 + a^2} = x - a\tan^{-1}(\frac{x}{a}) + c$ $\qquad \int \frac{xdx}{x^2 - a^2} = \frac{1}{2}\ln(x^2 - a^2) + c$