a) $P_{n_0} = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df = \frac{N_0}{2} \int_{-c}^{c} |H(f)|^2 df$

$= \cancel{2} \times \frac{N_0}{\cancel{2}} \left[ \int_{0}^{c/2} |H(f)|^2 df + \int_{c/2}^{c} |H(f)|^2 df \right]$

$= N_0 \left[ \int_{0}^{f_m} \left( \frac{f^2}{4f_m^2} - \frac{f}{f_m} + 1 \right) df + \int_{f_m}^{2f_m} 1/4 \, df \right]$

$= N_0 f_m \cdot \frac{10}{12} = \frac{5}{6} N_0 f_m$ or $\frac{5}{12\pi} N_0 \omega_m$ -($2\pi f_m = \omega_m$)-

b) $r(t) = A \{ \cos[2\pi(f_c + f_m)t] \cdot \cos[2\pi f_c t]$
$+ \sin[2\pi f_c t] \cos[2\pi f_c t] \}$

by using ; $\cos A \cos B = 1/2 [\cos(A+B) + \cos(A-B)]$
$\cos A \sin B = 1/2 [\sin(A+B) - \sin(A-B)]$

$r(t) = \frac{A}{2} \{ \cos[2\pi(2f_c + f_m)t] + \underline{\cos[2\pi f_m t]} + \sin[2\pi(2f_c)t] - \cancel{\sin(0)}^{\,0} \}$

since $f_c \gg f_m$ ($2f_c \gg 2f_m = C$)
only the $A/2 \cos[2\pi f_m t]$ part will pass the LPF!

part b)
cont.

So; $y(t) = A/2 \cos[2\pi f_m t] \times \underbrace{1/2}_{\text{amplitude of LPF at } f_m}$

$P_y(t) = (A/4)^2 \times \left(\underbrace{\text{Power of } \cos}_{P_{cos}}\right)$

$P_{cos} =$

i) $\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \cos^2(2\pi f_m t)\,dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left( \frac{1 + \cos(4\pi f_m t)}{2} \right) dt$

$= 1/2$

OR simply;

ii) $(\text{rms of } \cos)^2 = (1/\sqrt{2})^2 = 1/2$

$P_y = \left(\frac{A}{4}\right)^2 \times 1/2 = A^2/32$

$P_{n_o} = \frac{5}{6} N_0 f_m$

$\frac{P_y}{P_{n_o}} = SNR_o = \frac{A^2/32}{N_0 f_m 5/6} = \frac{3A^2}{40 N_0 f_m}$

c) $\mathcal{F}\{K\delta(t)\} = K$

Simply put $K^2$ instead of $\frac{N_0}{2}$ (PSD of white noise) in part b.

$SNR_o' = \frac{K^2 f_m \times 5/3}{N_0 f_m} = \frac{K^2}{N_0} \times \frac{5}{3}$