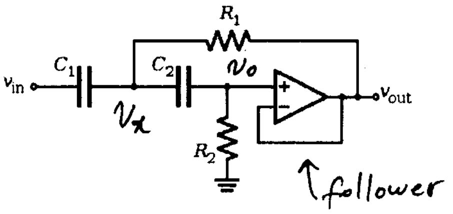
Electronics II * Exam II * April 29, 2010 * 70 minutes

ID number: ________________ Name: ____________________ Signature:____________

(40 pts)

1) Find the transfer function of the circuit given below. Indicate what kind of a filter it is (i.e., high-pass, low-pass, band-pass). Put the transfer function in the form as follows: $(as^2 + bs + c)/(s^2 + s\omega_o/Q + \omega_o^2)$.



$$\frac{V_x - V_o}{\frac{1}{sC_2}} = \frac{V_{out}}{R_2} \quad (1)$$

$$V_x - V_{out} = \frac{V_{out}}{sC_2R_2} \quad (2)$$

$$V_x = \left(1 + \frac{1}{sC_2R_2}\right)V_{out} \quad (3)$$

$$\frac{V_{in} - V_x}{\frac{1}{sC_1}} = \frac{V_x - V_{out}}{R_1} + \frac{V_x - V_{out}}{\frac{1}{sC_2}} \quad (4)$$

a = 1

b = 0

c = 0

ω_o = $\frac{1}{\sqrt{R_1C_1R_2C_2}}$

ω_o/Q = $\frac{1}{C_1R_2} + \frac{1}{C_2R_2}$

type = High pass

Substituting (2) and (3) into (4):

$$sC_1\left[V_{in} - \left(1 + \frac{1}{sC_2R_2}\right)V_{out}\right] = \frac{V_{out}}{sR_1R_2C_2} + \frac{sC_2V_{out}}{sC_2R_2}$$

$$V_{in} - \left(1 + \frac{1}{sC_2R_2}\right)V_{out} = \frac{V_{out}}{s^2C_1R_1C_2R_2} + \frac{V_{out}}{sC_1R_2}$$

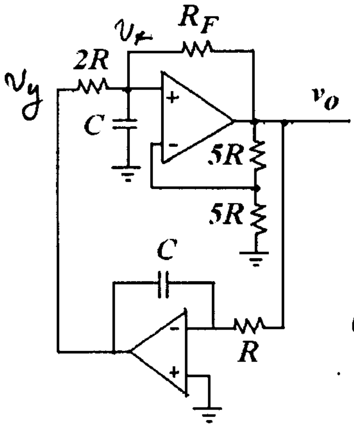
$$V_{in} = V_{out}\left[1 + \frac{1}{sC_2R_2} + \frac{1}{s^2C_1R_1C_2R_2} + \frac{1}{sC_1R_2}\right]$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + \frac{1}{s}\left(\frac{1}{C_2R_2} + \frac{1}{C_1R_2}\right) + \frac{1}{s^2C_1R_1C_2R_2}}$$

$$\frac{V_{out}}{V_{in}} = \frac{s^2}{s^2 + s\left(\frac{1}{C_1R_2} + \frac{1}{C_2R_2}\right) + \frac{1}{C_1R_1C_2R_2}} = \frac{as^2 + bs + c}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2}$$

(40 pts)

2) Find the oscillation frequency and oscillation condition for the circuit given below:



$\frac{v_o}{v_x} = 1 + \frac{5R}{5R} = 2 \rightarrow v_x = \frac{v_o}{2}$ ①

$\frac{v_y}{v_o} = -\frac{\frac{1}{sC}}{R} = -\frac{1}{sCR} \rightarrow v_y = -\frac{v_o}{sCR}$ ②

$\omega_o = \underline{1/CR}$

condition : $\underline{R_F = 2R}$

KCL at v_x : $\frac{v_y - v_x}{2R} = \frac{v_x}{\frac{1}{sC}} + \frac{v_x - v_o}{R_F}$ ③

Substituting ① and ② in ③ : $\frac{-\frac{v_o}{sCR} - \frac{v_o}{2}}{2R} = sC\frac{v_o}{2} + \frac{\frac{v_o}{2} - v_o}{R_F}$

dividing by v_o,

$$-\frac{1}{2sCR^2} - \frac{1}{4R} = \frac{sC}{2} - \frac{1}{2R_F}$$

Substitute $s = j\omega$

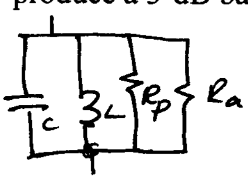
$$-\frac{1}{2j\omega CR^2} - \frac{1}{4R} = \frac{j\omega C}{2} - \frac{1}{2R_F}$$

Imaginary parts: $-\frac{1}{2j\omega CR^2} = \frac{j\omega C}{2} \rightarrow \frac{1}{\omega^2C^2R^2} = 1 \rightarrow \boxed{\omega = \frac{1}{CR}}$

Real parts : $-\frac{1}{4R} = -\frac{1}{2R_F} \longrightarrow \boxed{R_F = 2R}$

(20 pts)

3) A coil having an inductance of 10 μH is intended to be used around 1-MHz frequency. Its Q is specified to be 200. Find its equivalent parallel resistance R_p. What is the value of the capacitor required to produce resonance at 1 MHz? What additional parallel resistance R_a is required to produce a 3-dB bandwidth of 10 kHz?



$Q = \frac{R_p}{\omega_o L} \rightarrow R_p = Q\omega_o L = 200 \times 2\pi\, 10^6 \times 10 \times 10^{-6}$

$R_p = 12566\ \Omega$

$\omega_o^2 = \frac{1}{LC} \rightarrow C = \frac{1}{\omega_o^2 L} = \frac{1}{(2\pi \times 10^6)^2 \times 10 \times 10^{-6}}$

$C = 2.53 \times 10^{-9}$ F

$R_p = \underline{12566\ \Omega}$

$C = \underline{2.53 \times 10^{-9} F}$

$R_a = \underline{12566\ \Omega}$

$B = 2\pi\, 10 \times 10^3$ rad/s

$B = \frac{1}{CR} \longrightarrow R = \frac{1}{BC} = \frac{1}{2\pi \times 10000 \times 2.53 \times 10^{-9}} = 6283\ \Omega$

$\frac{1}{R} = \frac{1}{R_a} + \frac{1}{R_p} \rightarrow \frac{1}{R_a} = \frac{1}{R} - \frac{1}{R_p} = \frac{1}{6283} - \frac{1}{12566} = \frac{1}{12566}$

$R_a = 12566\ \Omega$