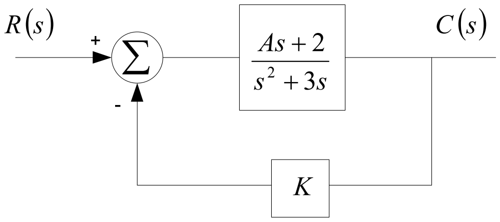
Electrical Engineering Department-Osmangazi University
Fundamentals of Control Systems-1st Midterm Examination-Spring 2005

1.



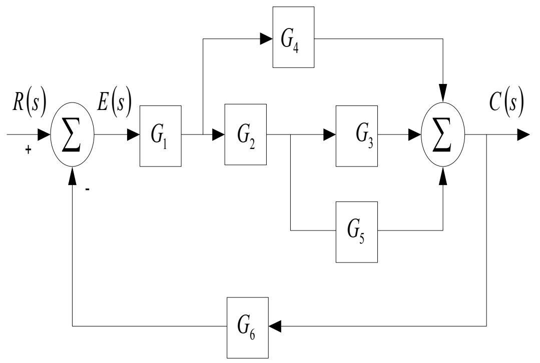
a. Write the transfer function $C(s)/R(s)$ when $A=1$, $K=4$.

b. Write the time domain expression for the impulse response of the system when $A=1$, $K=4$.

c. When $A=0$, obtain the least K value such that the system is underdamped.

d. Compute the rise time, the peak time, and the peak value of $C(t)$ when $A=0$, $K=2$.

2. Obtain the transfer function $C(s)/R(s)$ and $E(s)/R(s)$ in a simplified form for the block diagram given below.



Good Luck!. A. Karamancıoğlu

Solutions

1.

a. $$\frac{C(s)}{R(s)}=\frac{\dfrac{s+2}{s^2+3s}}{1+\dfrac{s+2}{s^2+3s}\cdot 4}=\frac{s+2}{s^2+7s+8}$$

b.

$$C(s)=\frac{s+2}{s^2+7s+8}\cdot 1=\frac{s+2}{(s+5.56)(s+1.44)}$$

$$=\frac{0.86}{s+5.56}+\frac{0.14}{s+1.44}$$

$$c(t)=0.86e^{-5.56t}+0.14e^{-1.44t}\ ,\quad t\geq 0$$

c. $$\frac{C(s)}{R(s)}=\frac{\dfrac{2}{s^2+3s}}{1+\dfrac{2}{s^2+3s}\cdot K}=\frac{2}{s^2+3s+2K}$$

When the denominator polynomial has complex roots, the response is underdamped:

The roots $-1.5\pm\sqrt{2.25-2K}$ must satisfy $2K>2.25$ or $K>1.125$

d. $$\frac{C(s)}{R(s)}=\frac{2}{s^2+3s+4}=\frac{1}{2}\cdot\frac{4}{s^2+3s+4}$$

$$\rightarrow \omega_n=2,\quad \xi=0.75$$

Rise time :

$$\frac{\pi-\beta}{\omega_d}=\frac{\pi-\cos^{-1}\xi}{\omega_n\sqrt{1-\xi^2}}=\frac{\pi-0.7227}{2\sqrt{1-0.5625}}$$

$$=\frac{2.4188}{1.3228}=1.8284\text{ sec.}$$

Peak time: $\dfrac{\pi}{\omega_d}=2.3748$ sec.

$$2\times\text{Peak value}=1+e^{-\frac{\xi\omega_n\pi}{\omega_d}}=1+e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$=1+e^{-\frac{2.3561}{0.6614}}=1.0283$$

Peak value$=0.514$

2.

Forward paths:

$P_1=G_1G_2G_3$, $P_2=G_1G_2G_5$, $P_3=G_1G_4$

Determinants:

$\Delta=1+G_1G_4G_6+G_1G_2G_3G_6+G_1G_2G_5G_6$;

$\Delta_1=\Delta_2=\Delta_3=1$

Use Mason's formula:

$$\frac{C(s)}{R(s)}=\frac{G_1G_4+G_1G_2G_3+G_1G_2G_5}{1+G_1G_4G_5+G_1G_2G_3G_6+G_1G_2G_5G_6}$$

$$\frac{E(s)}{R(s)}=\frac{1}{1+G_1G_4G_6+G_1G_2G_3G_6+G_1G_2G_5G_6}$$