Osmangazi University - Electrical Engineering Department
Fundamentals of Control Systems
Second Midterm Examination - Spring 2005

1. Let $C(s) = K$ be a constant gain and $H(s) = \frac{1}{(s^2+2s+1)(s+3)}$ as depicted in Figure 1.
a) (10 pts.) Write the closed loop transfer function $T(s)$
b) (30 pts.) Sketch the root locus of the system. At which value of K does the root locus cross the jw axis?
2. (30 pts.) Let $C(s) = K_1 + K_2 s$ and $H(s) = \frac{s}{(s^2+3s)}$ in Figure 1. Find K_1 and K_2 such that the closed loop system has poles at $s = 0$ and $s = -1$.

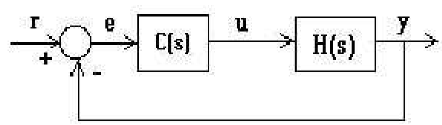

Figure 1: System referenced by Problems 1 and 2

3. (30 pts.) Consider a transfer function $G(s) = \frac{1}{s+3}$. Obtain its steady state response to the input signal $4\cos(5t)$.
Good Luck,
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Solutions

1.a

$$T(s) = \frac{K}{(s^2+2s+1)(s+3)+K} = \frac{K}{s^3+5s^2+7s+3+K}$$

1.b

Initially all the roots are in the LHP. As K increases the two roots move towards the RHP. Instability begins at the K value where the root locus crosses the iw axis. Let us form the Routh table to find out where the instability starts:

$$\begin{array}{cc} 1 & 7 \\ 5 & 3+K \\ \frac{32-K}{5} & \\ 3+K & \end{array}$$

Positivity of the leftmost column requires $K < 32$. Hence $K = 32$ is the iw axis crossing gain.

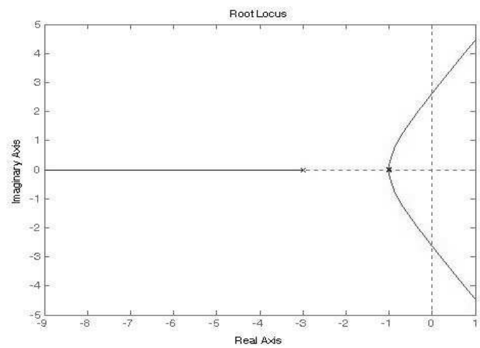

Figure 2: Root Locus of $H(s)$ of Problems 1

2.

$$T(s) = \frac{s(K_1 + k_2 s)}{(1 + K_2)s^2 + (3 + K_1)s}$$

One root is already at $s = 0$. The other root is at $s = -\frac{3+K_1}{1+K_2}$. For a root at $s = -1$ we may select $K_1 = 1$ and $K_2 = 3$.

3.

$$G(s) = \frac{1}{i5 + 3} \longrightarrow |G(i5)| = \frac{1}{\sqrt{34}}; \quad \angle G(i5) = -1.03$$

The output is hence $\frac{4}{\sqrt{34}} cos(5t - 1.03)$ or $0.686 cos(5t - 1.03)$