Eskişehir Osmangazi University - Electrical Engineering Department
Fundamentals of Control Systems
First Midterm Examination - Spring 2007

1. A linear time invariant system is described by

$$\dot{x} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

Obtain $y(t), \quad t \geq 0$ for the initial condition $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and unit step input u.

2. For the block diagram of Figure 1, find the transfer function $\frac{Y(s)}{U(s)}$ Good

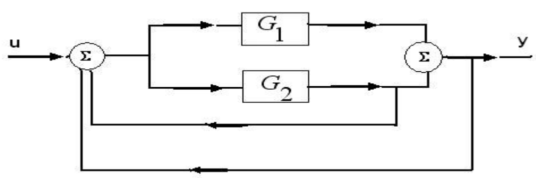

Figure 1: Block diagram referenced by Problem 2

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Solutions

1.

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$$

$$x(t) = \begin{bmatrix} e^t & 0 \\ 0 & e^{4t} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \int_0^t \begin{bmatrix} e^{t-\tau} & 0 \\ 0 & e^{4(t-\tau)} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} 1d\tau$$

$$x(t) = \begin{bmatrix} e^t \\ 0 \end{bmatrix} + \int_0^t \begin{bmatrix} e^{t-\tau} \\ e^{4(t-\tau)} \end{bmatrix} d\tau$$

$$x(t) = \begin{bmatrix} 2e^t - 1 \\ \frac{e^{4t}}{4} - \frac{1}{4} \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \rightarrow y = 2e^t - 1$$

2. Mason's formula yields:

$$\frac{Y(s)}{U(s)} = \frac{G_1 + G_2}{1 - G_1 - 2G_2}$$