Eskişehir Osmangazi University - Electrical Engineering Department
Fundamentals of Control Systems
Final Examination - Spring 2009

1. [15 pts.] For the configuration in Figure 1, find the transfer function $\frac{Y(s)}{R(s)}$. [15 pts.] Is the closed loop system BIBO stable?

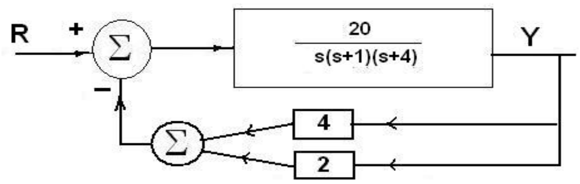

Figure 1: Configuration referenced by Problem 1

2. [20 pts.] Consider the configuration in Figure 2. Plot the pole locations of the closed loop system as K varies from 0 to ∞. [10 pts.] For which values of K, is the closed loop system BIBO stable?

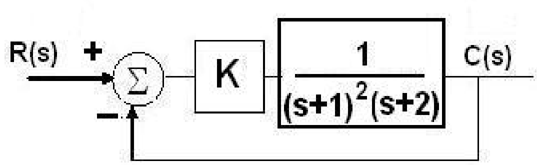

Figure 2: Configuration referenced by Problem 2

3. [20 pts.] Find the image of the rectangular path in Figure 3 under the mapping $H(s) = (s-1)^2 + 1$. [20 pts.] Use mapping theorem to figure out how many roots of $H(s)$ are in the rectangular path in Figure 3.

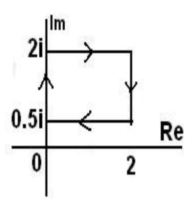

Figure 3: Rectangular path referenced by Problem 3

Good Luck,
A. Karamancıoğlu

Solutions

1.

$$\frac{Y(s)}{R(s)} = \frac{20}{s(s+1)(s+4)+120} = \frac{20}{s^3+5s^2+4s+120}$$

```
[   1,   4]
[   5, 120]
[ -20,   0]
[ 120,   0]
```

Due to sign changes in the Routh table, the closed loop system is unstable.

2.

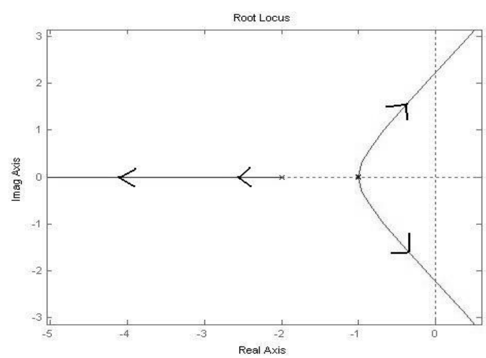

Figure 4: Root locus of the system in Problem 2

For those who like to use MATLAB, the relevant codes are:

```
syms s s=tf('s') G=1/((s+1)^2*(s+2)); rlocus(G)
```

The closed loop system's transfer function:

$$\frac{K}{(s+1)^2(s+2)+K} = \frac{K}{s^3+4s^2+5s+2+K}$$

The Routh table:

```
[ 1,      5]
[ 4,   2+K]
[ 18-K    ]
[ 2+K     ]
```

The closed loop system is stable for $K < 18$.

3.

For those who like to use MATLAB, the relevant codes are:

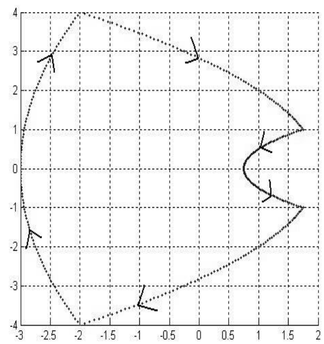

Figure 5: Image of the rectangular path

```
clc
hold on
for k=25:100
s=0.02*k*i;
G=(s-1)^2+1;
rho=abs(G);
theta=angle(G);
polar(theta,rho)
end

hold on
for k=0:100
s=0.02*k +2*i;
G=(s-1)^2+1;
rho=abs(G);
theta=angle(G);
polar(theta,rho)
end

hold on
for k=25:100
s=2+ 0.02*k*i;
G=(s-1)^2+1;
rho=abs(G);
theta=angle(G);
polar(theta,rho)
end

hold on
for k=0:100
```

```
s=0.02*k+0.5*i;
G=(s-1)^2+1;
rho=abs(G);
theta=angle(G);
polar(theta,rho)
end
```

$H(s)$ has no pole in the rectangular path. Thus $P = 0$. Number of encirclement of the origin is one in the clockwise direction. Thus $N = 1$. Using the mapping theorem formula $Z - P = N$ implies $Z - 0 = 1 \rightarrow Z = 1$, in words H(s) has one root in the rectangular path.