Eskişehir Osmangazi University - Electrical Engineering Department
Fundamentals of Control Systems
First Midterm Examination - Spring 2012

For each question, put the answer just below it. Correct answers are sufficient for full credits.

1. [30 pts.] Consider the signal flow graph of a linear time invariant system shown in Fig. 1. Obtain the transfer function $\frac{Y(s)}{U(s)}$.

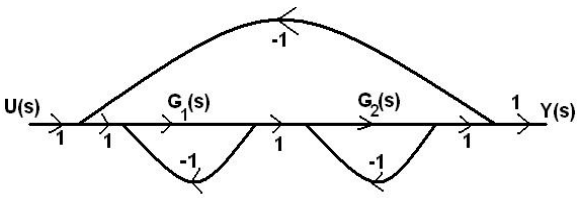
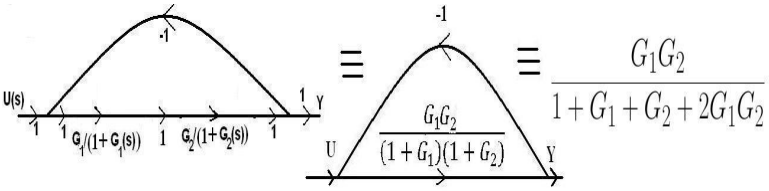


Figure 1: Signal flow graph referenced by Problem 1

Solution. By Mason's Gain Formula:
$P_1 = G_1G_2,\ \Delta_1 = 1,\ \Delta = 1 + G_1 + G_2 + 2G_1G_2 \rightarrow\ \frac{G_1G_2}{1+G_1+G_2+2G_1G_2}$
Solution. By the following block reduction:



2. Let the state space system $\dot{x}(t) = Ax(t) + Bu(t)$ be given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

with $x(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $u(t)$ is a unit step function.

(a) [10 pts.] Find e^{At}.

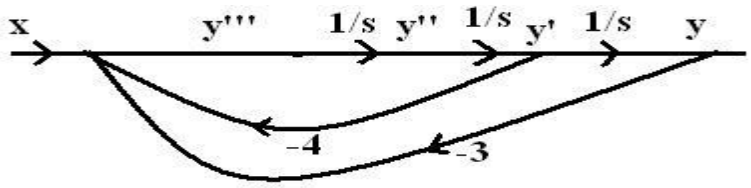
$$(sI - A)^{-1} = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix} \leftrightarrow e^{At} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

(b) [30 pts.] Find $x(t)$.

$$x(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \int_0^t \begin{bmatrix} 1 & t-\tau \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} 1 d\tau = \begin{bmatrix} \frac{t^2}{2} + 2t + 1 \\ t + 2 \end{bmatrix}$$

3. [30 pts.] Sketch the canonical realization of the transfer function

$$H(s) = \frac{1}{s^3 + 4s + 3}$$



Good Luck,
A. Karamancıoğlu