

Eskişehir Osmangazi University - Electrical Engineering Department
Fundamentals of Control Systems
First Midterm Examination - Summer 2012

1. Let

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u; \quad y = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a) Obtain the transfer function $\frac{Y(s)}{U(s)}$.

b) If u is a unit step function, find the steady state $y(t)$.

ANS. a) $\frac{4s+3}{s^2-s-1}$ because $(sI-A) = \begin{bmatrix} s-1 & -1 \\ -1 & s \end{bmatrix}$; $(sI-A)^{-1} = \frac{1}{s^2-s-1} \begin{bmatrix} s & 1 \\ 1 & s-1 \end{bmatrix}$;

$$C(sI-A)^{-1}B = \begin{bmatrix} 2 & 1 \end{bmatrix} \frac{1}{s^2-s-1} \begin{bmatrix} s & 1 \\ 1 & s-1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{4s+3}{s^2-s-1}$$

b) as $t \rightarrow \infty$; $y(t) \rightarrow \infty$

2. Find transfer function $C(s)/R(s)$.

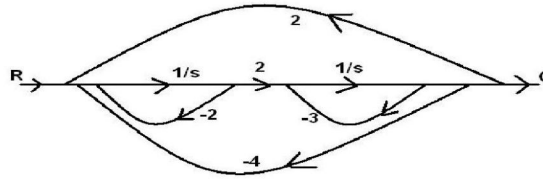


Figure 1: Signal flow graph referenced by Problem 2

ANS. $\frac{2}{s^2+5s+10}$

3. How many roots of $s^5 - 9s^4 + 25s^3 - 15s^2 - 26s + 24$ are in the OLHP?

ANS. 1, because its Routh Table is as follows:

$$\begin{array}{l} [\quad 1, \quad 25, \quad -26] \\ [\quad -9, \quad -15, \quad 24] \\ [\quad 70/3, \quad -70/3, \quad 0] \\ [\quad -24, \quad 24, \quad 0] \\ [\quad -48, \quad 0, \quad 0] \\ [\quad 24, \quad 0, \quad 0] \end{array}$$

4. Find steady state $y(t)$ if it is the output of $G(s) = \frac{1}{s^2+5s+6}$ for a unit step input.

ANS. $\frac{1}{6}$, because, in the standard form $G(s) = \frac{1}{6} \times \frac{6}{s^2+5s+6}$; steady state response of $G(s) = \frac{6}{s^2+5s+6}$ to unit step is 1 therefore $G(s) = \frac{1}{6} \times \frac{6}{s^2+5s+6}$ has steady state unit step response $\frac{1}{6}$.

Good Luck,
A. Karamancioğlu