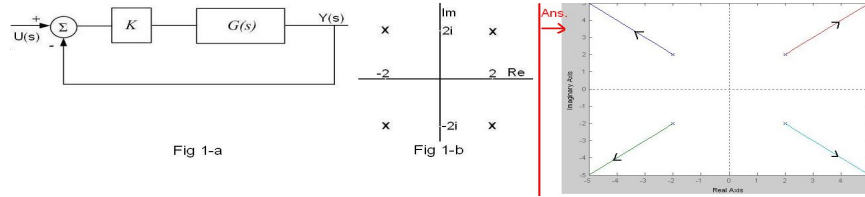


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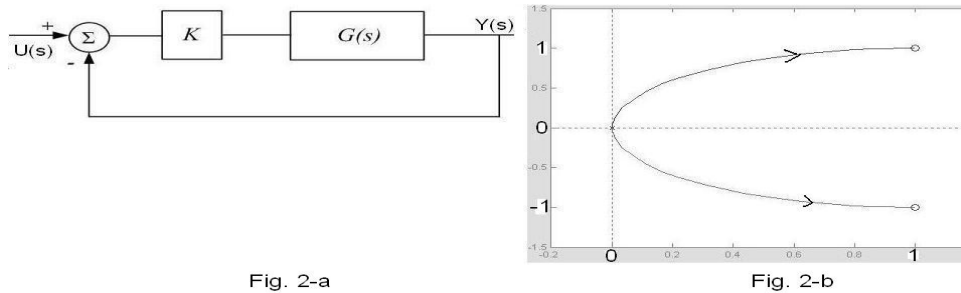
Eskişehir Osmangazi University - Electrical Engineering Department  
Fundamentals of Control Systems  
Second Midterm Examination - Spring 2013

For each question, put the answer just below it. Correct answers are sufficient for full credits.



1. [33 pts.-No partial credits] Consider the configuration in Fig. 1-a. The poles of  $G(s) = \frac{1}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$  are denoted by 'x' in Fig. 1-b. Which elements of the set  $\{0, 2, 2i, 1 + i, 4 + 4i, 5 + 5i\}$  **cannot** be poles of the closed-loop transfer function  $\frac{Y(s)}{U(s)}$  as  $K$  changes from 0 to infinity?

**Ans.** Given that four poles and no zeros, clearly there four asymptotes centred at 0 with angles  $45^\circ, 135^\circ, 225^\circ, 315^\circ$  as depicted rightmost above. Alternatively one can check the angle condition for each of them. The test point locations are conveniently given so that one can do the checking easily. The answer is therefore  $\{0, 2, 2i, 1 + i\}$



2. [33 pts.] Consider the configuration in Fig. 2-a. Determine a transfer function  $G(s)$  such that the pole locations of the closed-loop transfer function  $\frac{Y(s)}{U(s)}$  are as shown in Fig. 2-b as  $K$  changes from 0 to infinity.

**Ans.** This graphics occurs if there are zeros at  $1 + i$  and  $1 - i$ , and there are two poles at the origin. Thus  $G(s) = \frac{(s-1+i)(s-1-i)}{s^2} = \frac{s^2 - 2s + 2}{s^2}$

3. Input  $x(t) = \sin(t) + \sin(2t)$  is applied to an LTI system with transfer function  $G(s) = \frac{1}{s^2 + 2s + 1}$ .

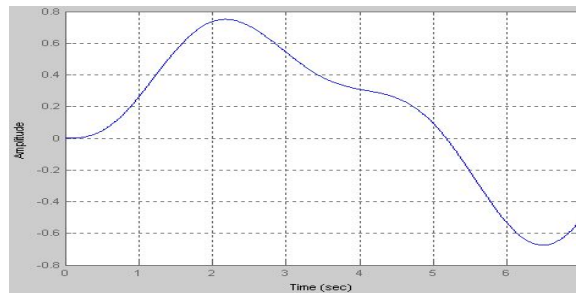
(a) [20 pts.] Find the steady state output  $y(t)$ .

(b) [14 pts.] Evaluate the steady state output  $y(t)$  at  $t = 7$ .

**Ans.** (a) At  $w = 1$ , we have  $G(w) = 0.5 \angle -1.57$ ; at  $w = 2$  we have  $G(w) = 0.2 \angle -2.214$ .

Thus  $y(t) = 0.5 \sin(t - 1.57) + 0.2 \sin(2t - 2.214)$ .

(b)  $y(7) = -0.516$ . It may be verified by the actual response graphics below.



Good Luck,  
A. Karamancioğlu