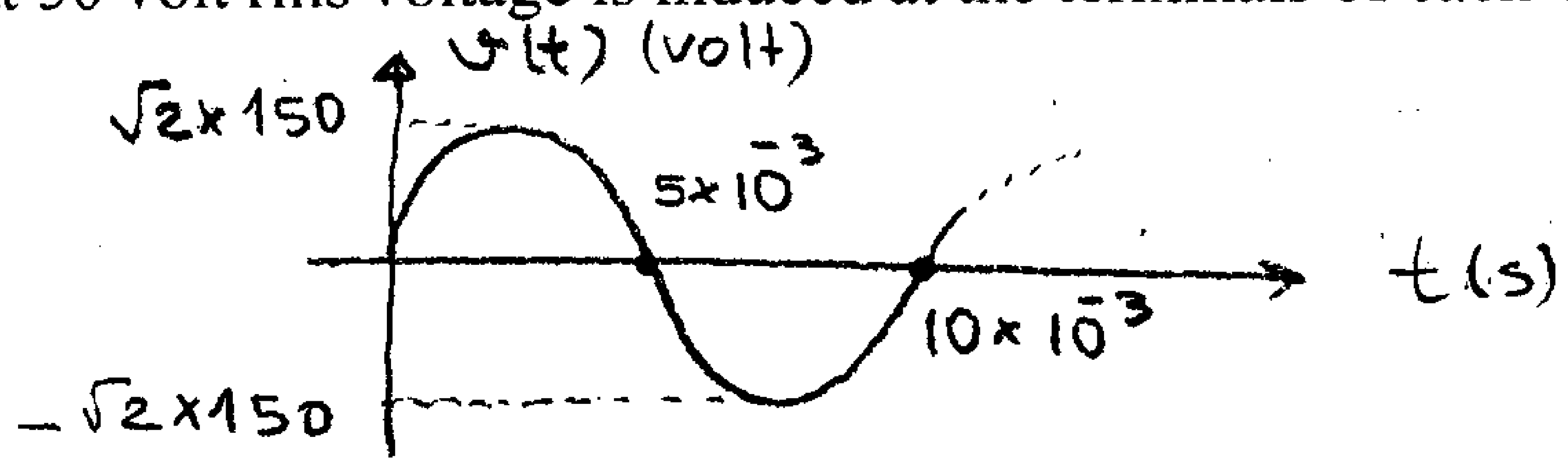
PRINCIPLES OF ENERGY CONVERSION FIRST MIDTERM EXAM

Dr. Salih FADIL **March 27, 2009**

#1) In a primitive single phase synchronous generator, when the armature windings in the rotor are connected as series, the following voltage is observed at the terminals of the brushes. Assume that 50 volt rms voltage is induced at the terminals of each winding.



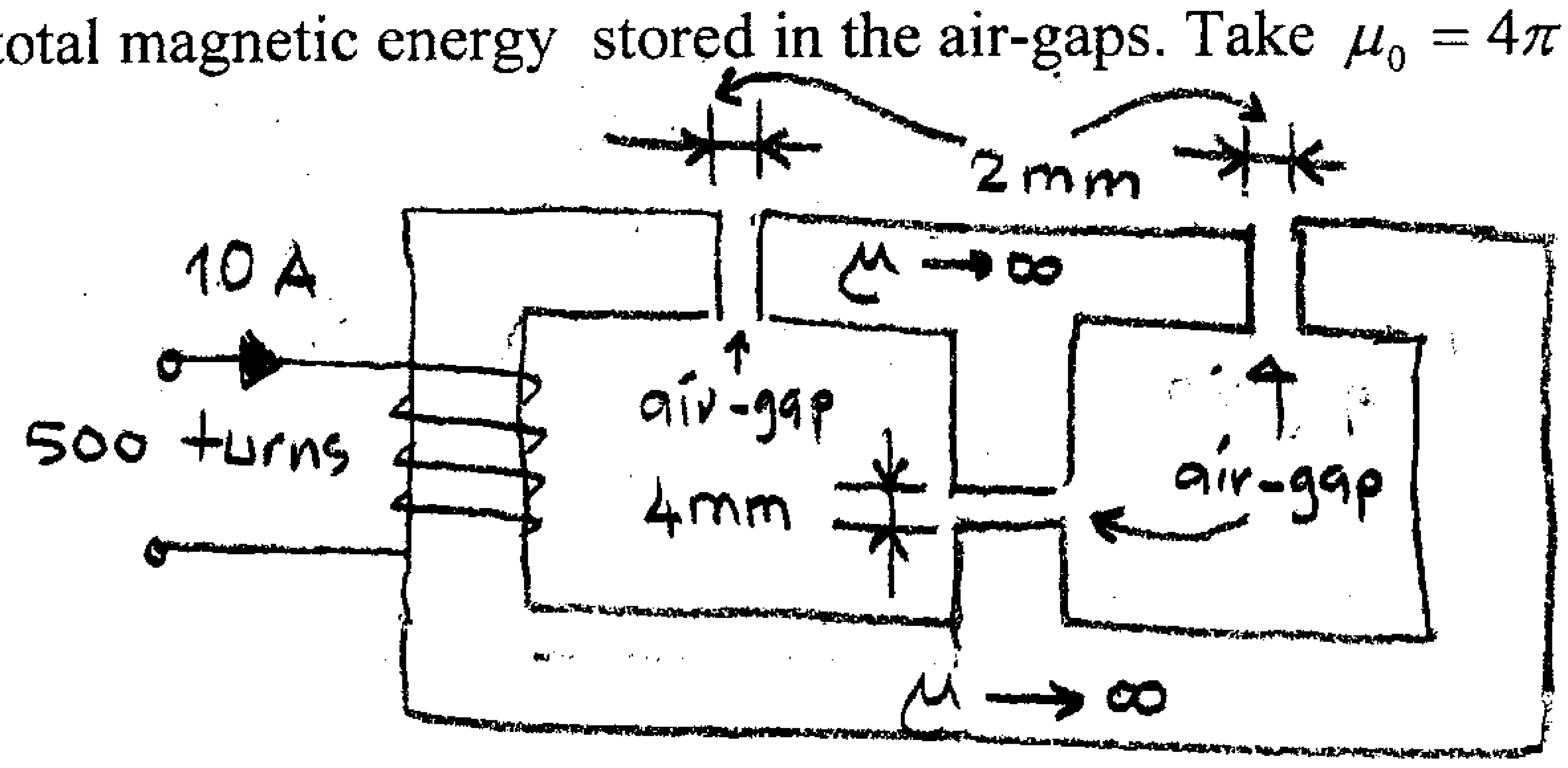
a) Draw the cross-sectional view of the generator, label each winding terminal on the rotor as $A-A'$ and $B-B'$ etc. , and show the geometric angle between the two consecutive poles in your drawing.

b) Calculate the mechanical angular velocity of the rotor as r/s. Calculate the speed of the rotor as rpm.

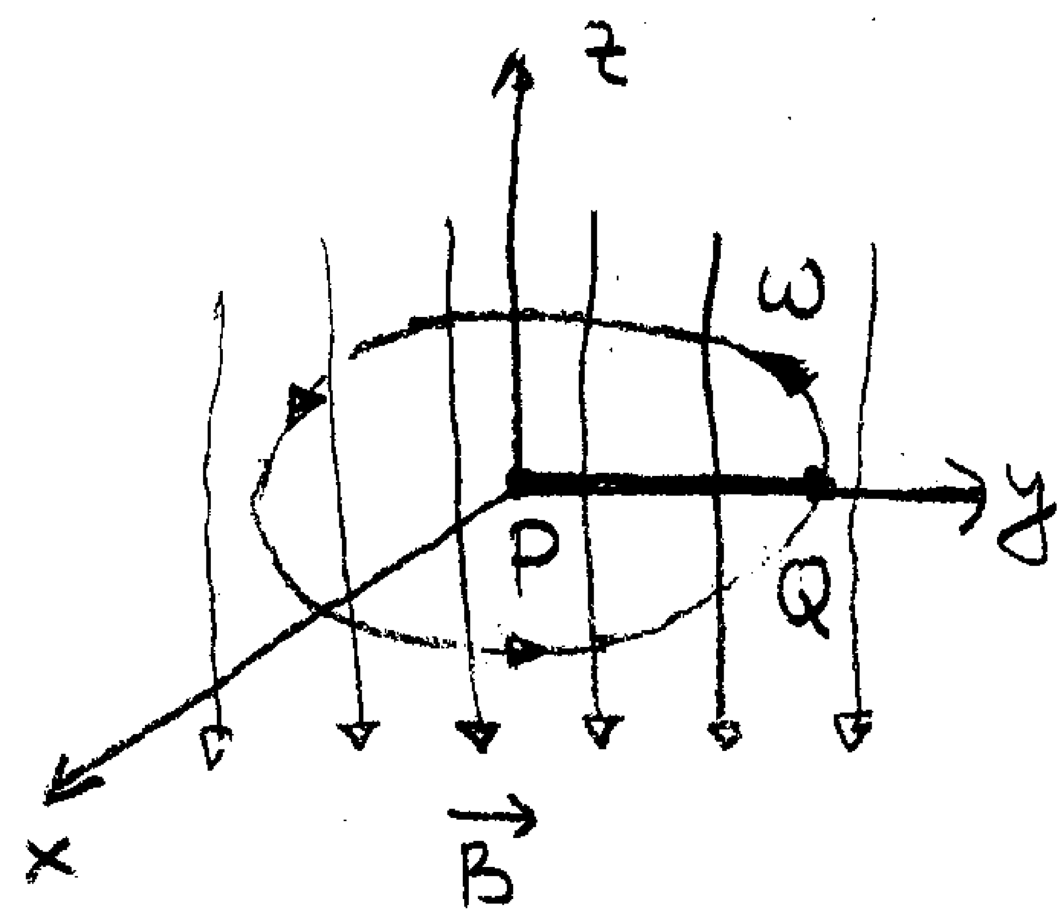
c) If the diameter and length of the rotor are 40 cm and 120 cm, respectively, calculate the B value in the air-gap of the generator.

d) Assume that the number of pole pairs determined in part a, is doubled. The other parameters are remained the same. Draw the cross-sectional view of the generator, label each winding terminal on the rotor as $A-A'$ and $B-B'$ etc., and show the geometric angle between the two consecutive poles in your drawing. If the windings are connected as parallel, draw the induced voltage at the terminal of the brushes. Show also connection of the windings.

#2) A symmetric core of iron with $\mu \to \infty$ has a uniform cross section of $9\,cm^2$. Determine the total magnetic energy stored in the air-gaps. Take $\mu_0 = 4\pi \times 10^{-7}\ H/m$.



#3) A rigid metal rod PQ of length 50 cm, fixed at end P, as shown in the figure, rotates along $\vec{a}_\phi$ inside a uniform field $\vec{B} = -0.8\,\vec{a}_z\ Wb/m^2$. If the induced voltage between P and Q is measured as 80 volt, calculate the speed of the rod as rpm (rotation per minute) and also determine the end of the rod that has positive potential.



PRINCIPLES OF ENERGY CONVERSION FIRST MIDTERM EXAM SOLUTION MANUAL

Dr. Salih FADIL

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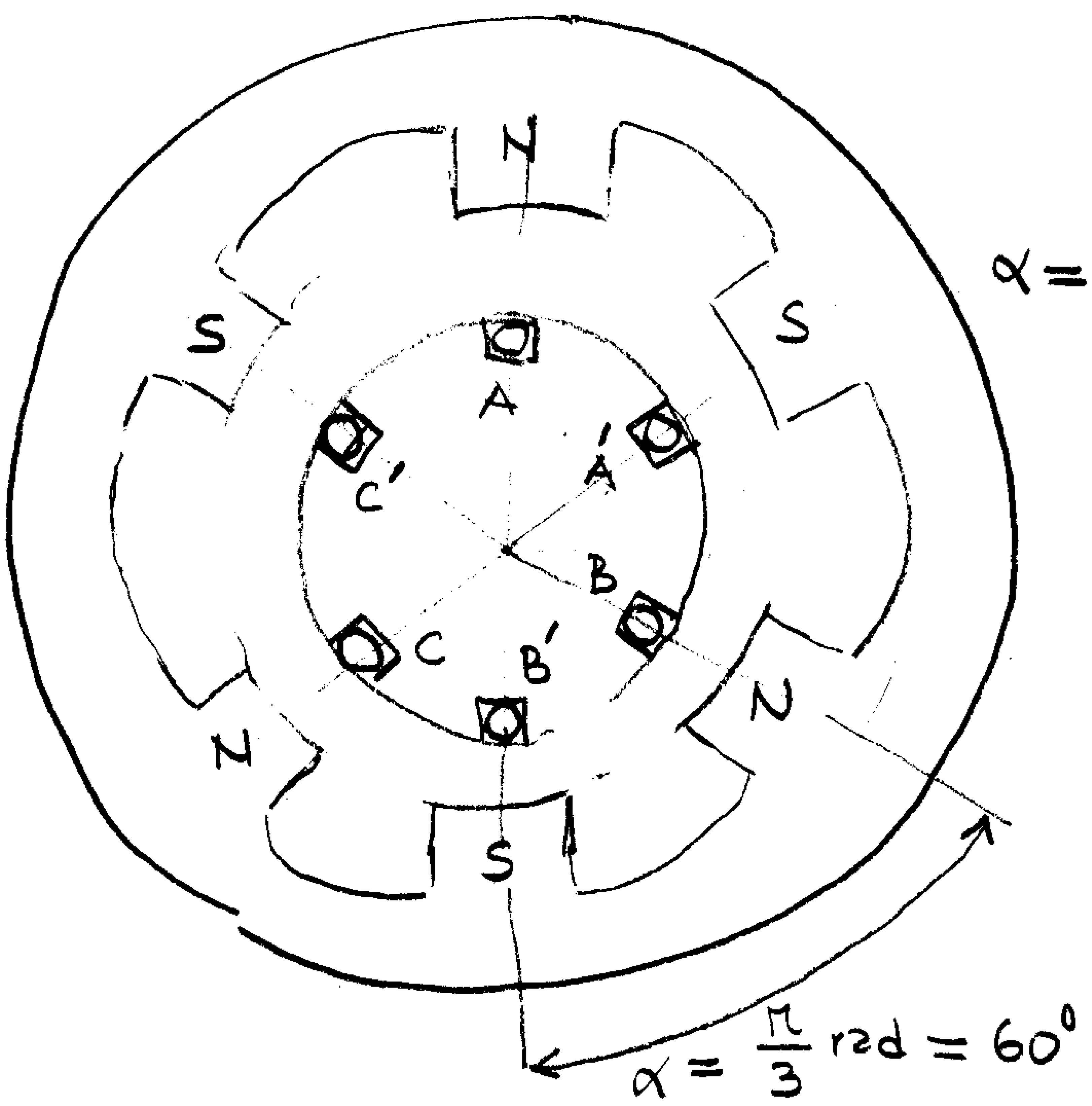
#1) a) # of coils on the rotor $= \frac{150}{50} = 3 =$ # of pole pair on the stator $= P$



$\alpha =$ Angle between two consequtive poles $= \frac{360^0}{6} = 60^\circ$

coils (windings)

A – A'
B – B'
C – C'

A', B', C' have the same (+) polarity.
A, B, C have the same (–) polarity.

b) The induced voltage's frequency $= f = \frac{1}{10\ 10^{-3}} = \frac{1000}{10} = 100$ Hz.

$$n = \frac{f \cdot 60}{P} = \frac{100 \times 60}{3} = \frac{6000}{3} = 2000 \text{ rpm.}$$

$$\omega_{mech} = \frac{2000}{60} 2\pi = \frac{200\pi}{3} = 209.44 \text{ r/s (radian/sec)}$$

c) For a single winding

$e_{max} = 2\, B \cdot l \cdot v$ $\qquad v = \omega_{mech} \cdot r$ $\qquad r =$ radius of the rotor

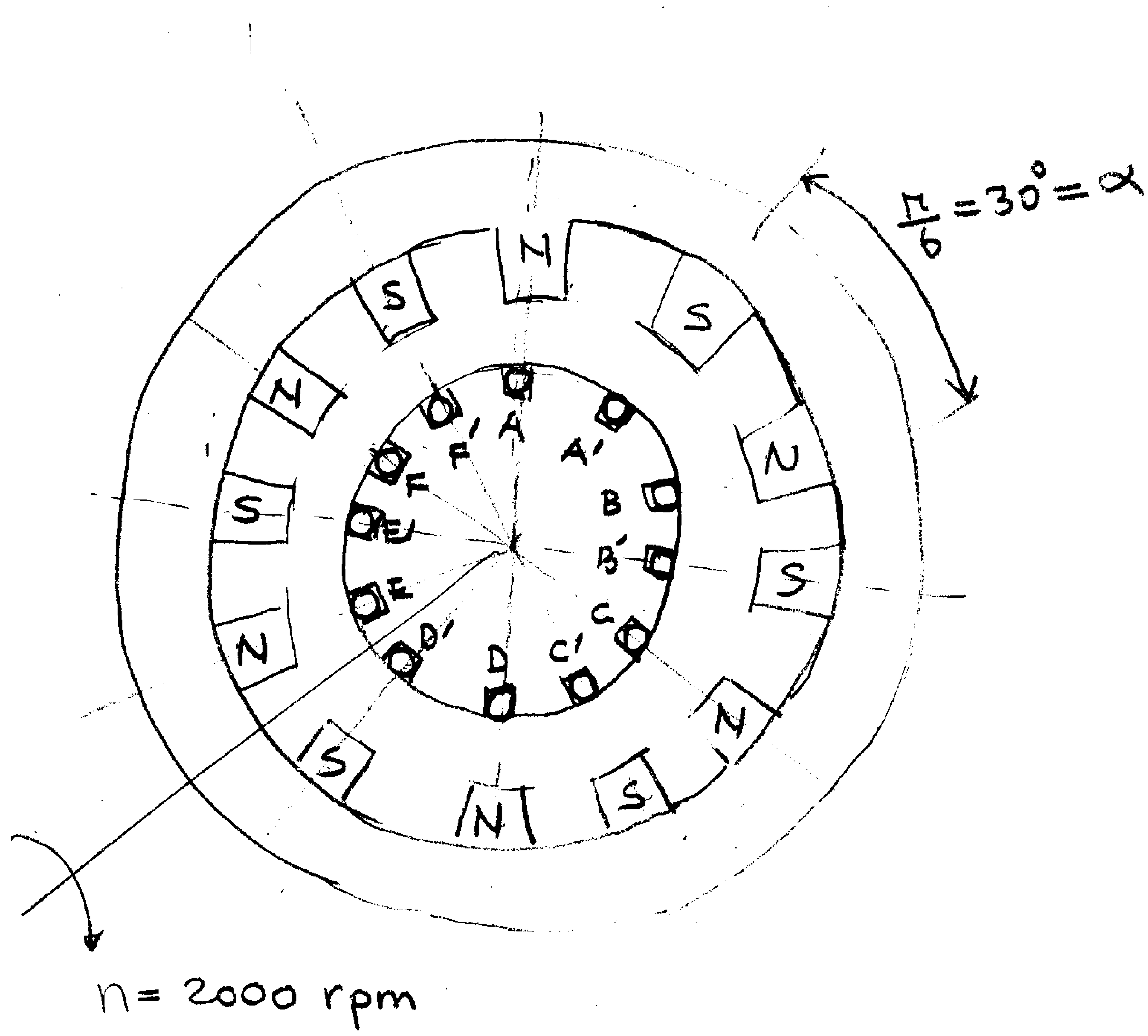
$e_{max} = 2\, B \cdot l\ \omega_{mech} \cdot r$ $\qquad l =$ lenght of the rotor

$$\sqrt{2}\ 50 = (2) \times (B) \times (1{,}2) \times (209.44) \times 20\ 10^{-2}$$

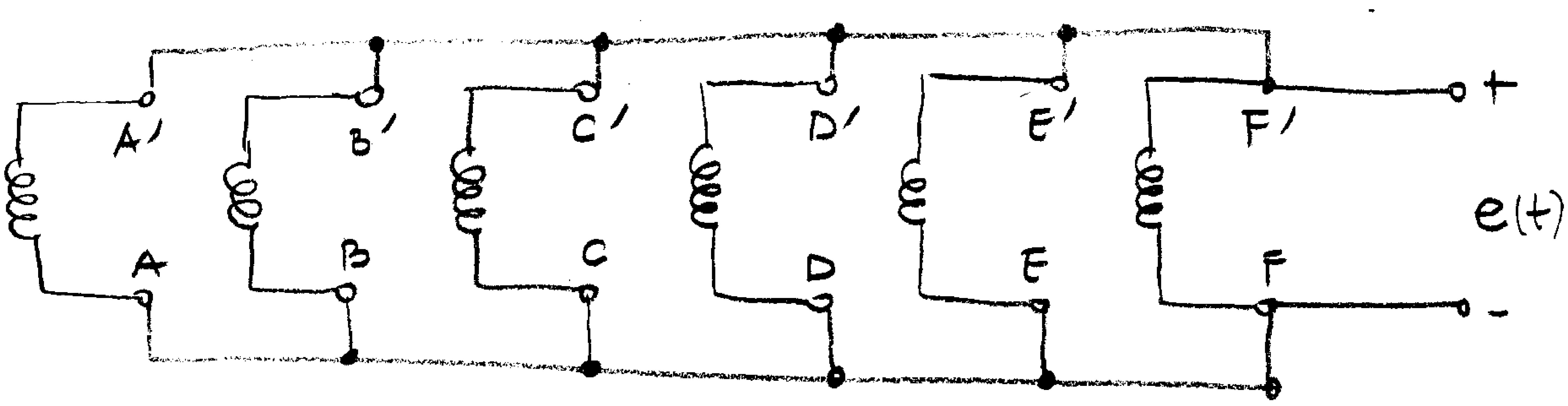
\# of turns = N = 1 is taken for each windings

$$B = \frac{\sqrt{2}\ 50}{2 \times 1{,}2 \times (209.44)\ 20\ 10^{-2}} = 0{,}7033\ T.$$

d)

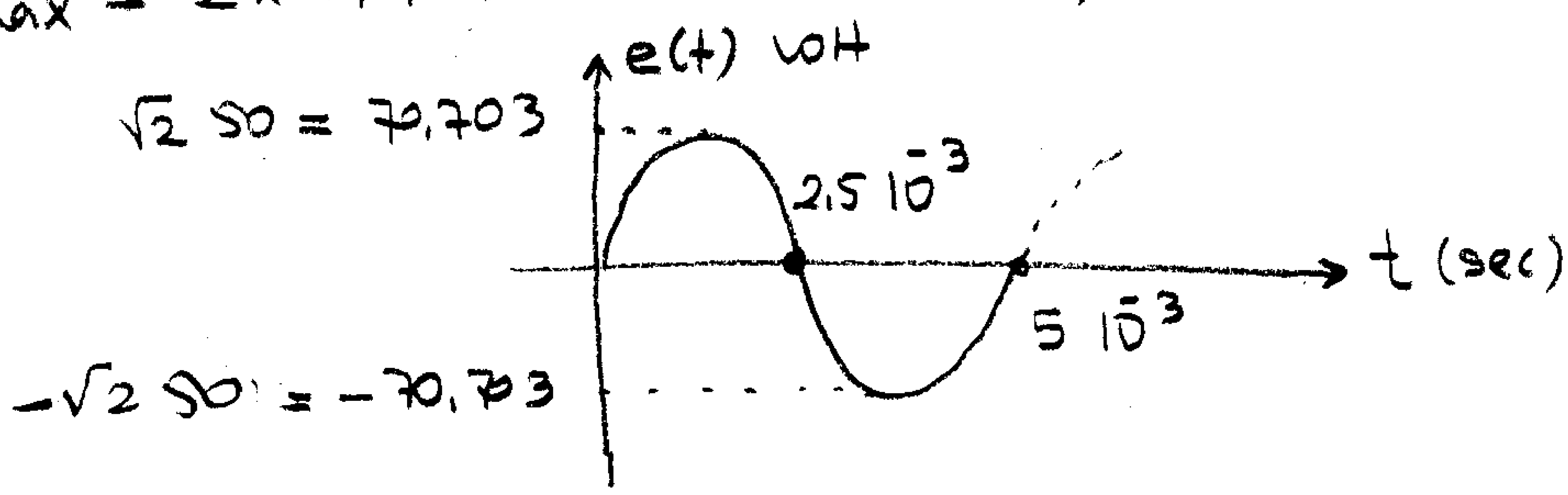


$p = 6, \quad \alpha = \frac{360}{12} = 30^\circ, \quad N = 1.$

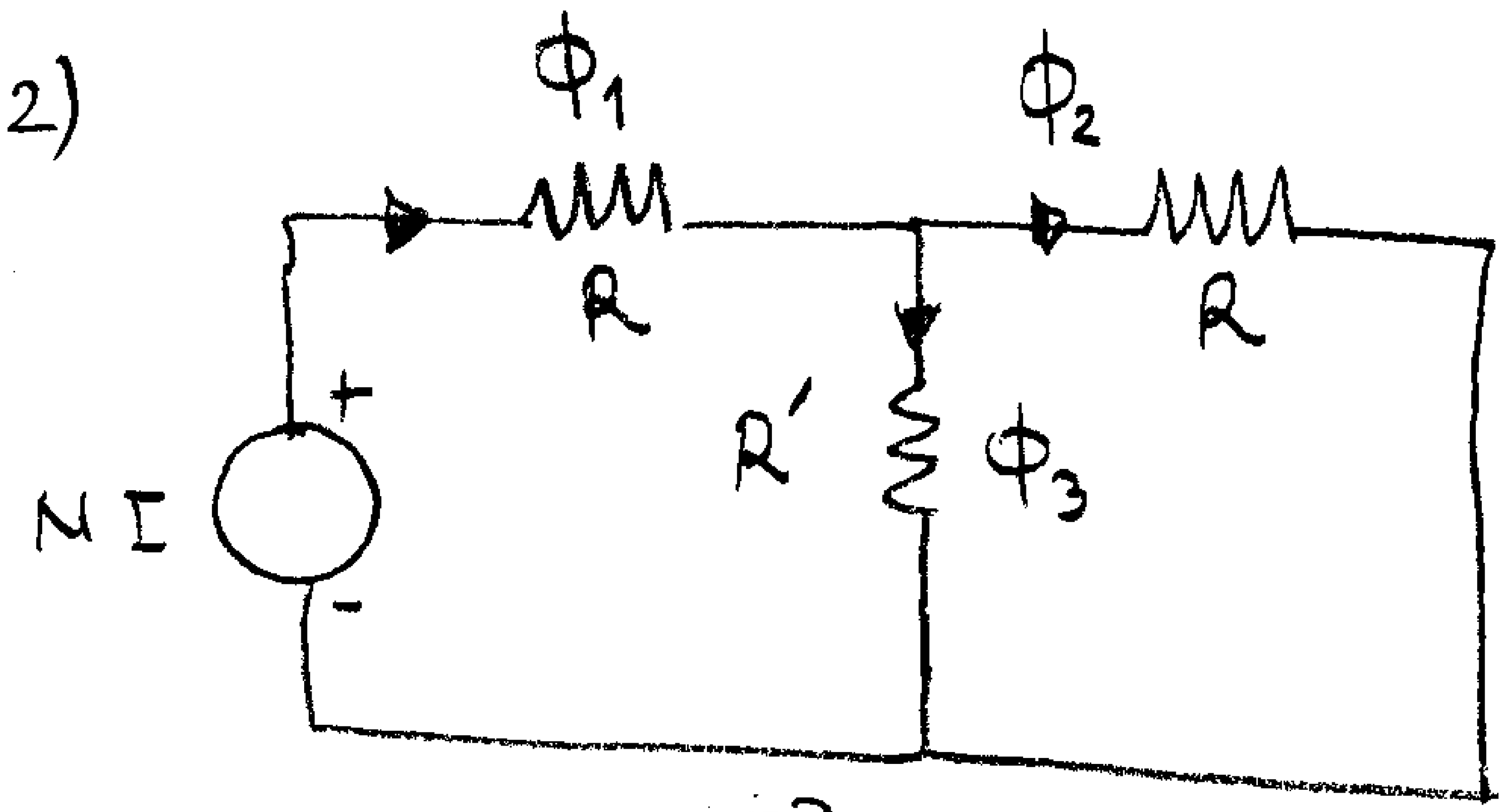


$$n = \frac{f \cdot 60}{P} \rightarrow f = \frac{2000 \times 6}{60} = 200\ Hz. \qquad \omega_{mech} =$$

$$e_{max} = 2 \times 0{,}7033 \times 1{,}2 \times (209.44)\ 20\ 10^{-2} = 70{,}703\ \text{volt}$$



#2)



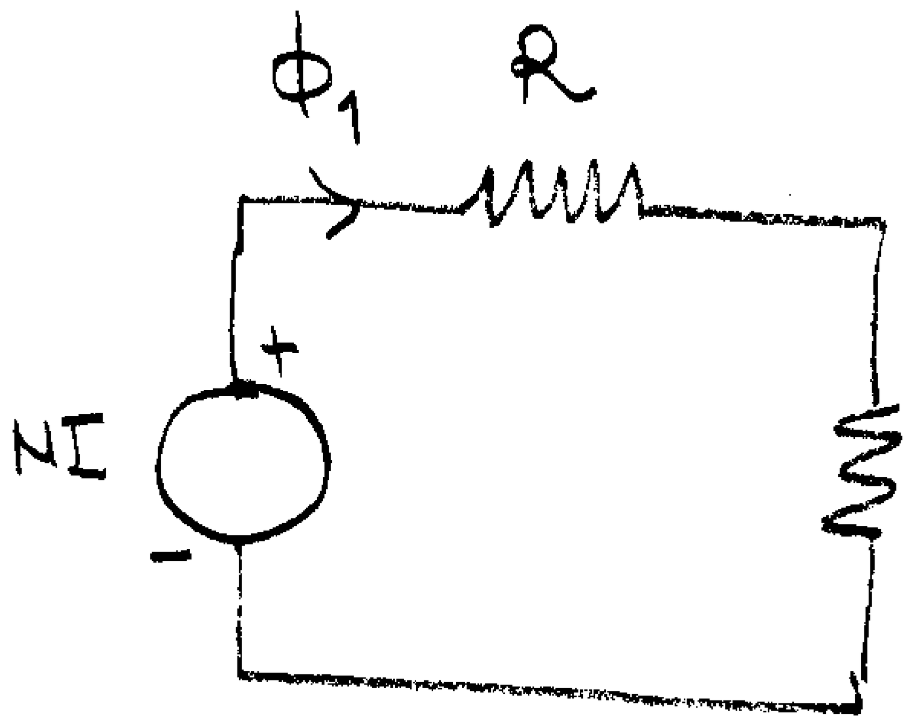
$\mu_0 = 4\pi\, 10^{-7}$ H/m

Since $\mu_i \to \infty \to R_i \to 0$

$NI = 500 \times 10 = 5000$ AT

$$R = \frac{2\, 10^{-3}}{9\, 10^{-4}\, \mu_0} = \frac{20}{9\mu_0} \text{ AT/Wb}$$

$$R' = \frac{4\, 10^{-3}}{9\, 10^{-4}\, \mu_0} = \frac{40}{9\mu_0} = 2R$$



$R_{eq} = \frac{2}{3} R$

$$R_{eq} = R' /\!/ R = \frac{2R \times R}{3R} = \frac{2}{3} R$$

$$\Phi_1 = \frac{NI}{R + \frac{2}{3}R} = \frac{NI}{\frac{5R}{3}} = \frac{3NI}{5R}$$

$$\Phi_2 = \Phi_1 \frac{R'}{R' + R}$$ current (flux) division rule

$$\Phi_2 = \Phi_1 \frac{2R}{3R} = \frac{2}{3}\Phi_1 \qquad \Phi_3 = \Phi_1 \frac{R}{R' + R} = \Phi_1 \frac{R}{3R} = \frac{1}{3}\Phi_1$$

$$\Phi_1 = \frac{3 \times 5000}{5 \frac{20}{9\mu_0}} = \frac{15000 \times 9 \mu_0}{100} = 1350\, \mu_0 \text{ Wb}$$

$$\Phi_2 = \frac{2 \times 1350}{3}\mu_0 = 900\, \mu_0 \text{ Wb} \qquad \Phi_3 = \frac{1350\, \mu_0}{3} = 450\, \mu_0 \text{ Wb}$$

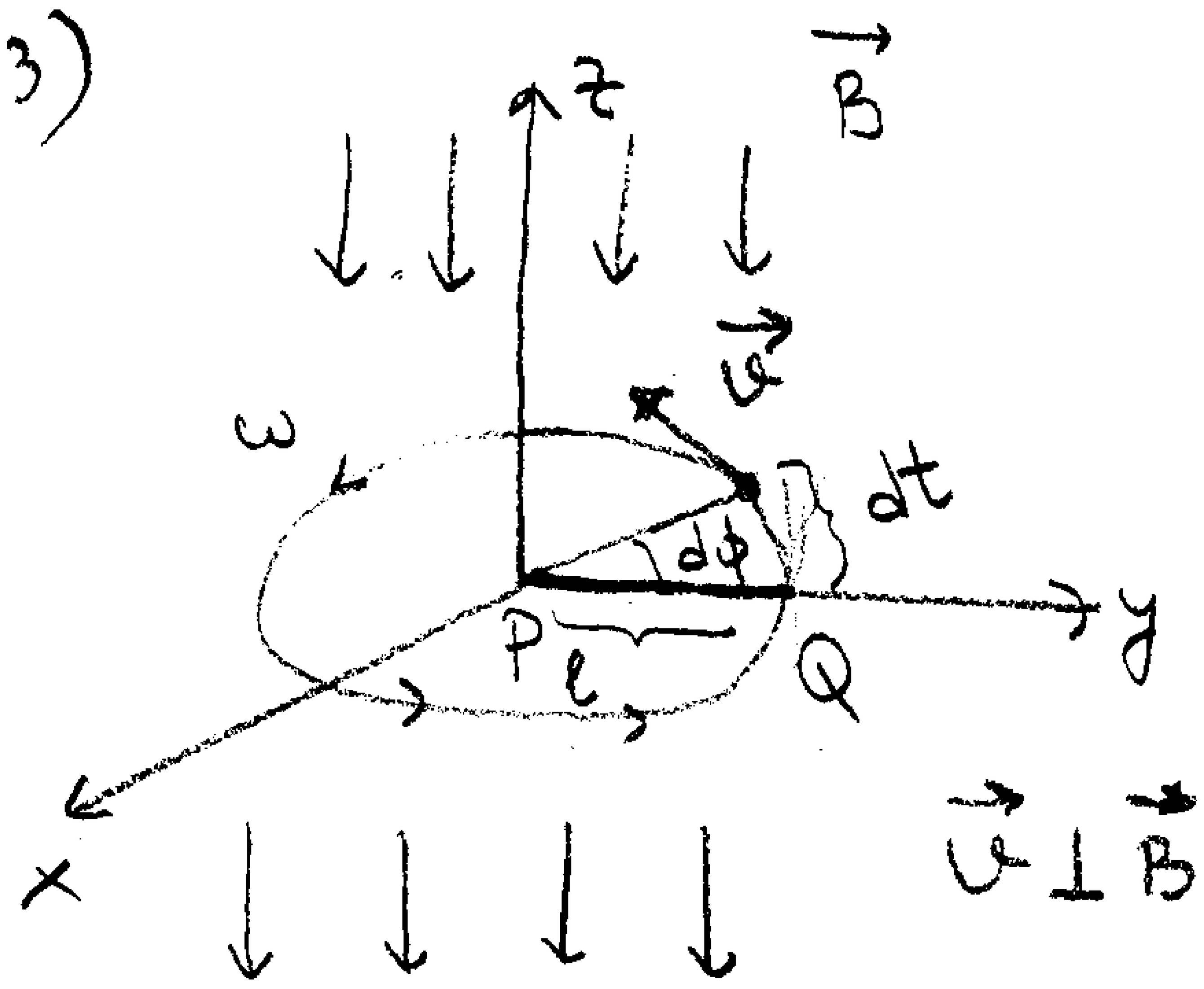
$$B_1 = \frac{1350}{9\, 10^{-4}}\, 4\pi\, 10^{-7} = 1{,}885 \text{ T}, \qquad B_2 = \frac{900 \times 4\pi\, 10^{-7}}{9\, 10^{-4}} = 1{,}2566 \text{ T}$$

$$B_3 = \frac{450 \times 4\pi\, 10^{-7}}{9\, 10^{-4}} = 0{,}6283 \text{ T}$$

$$W/s = \frac{1}{2}\frac{(1{,}885)^2}{4\pi\, 10^{-7}}(2\, 10^{-3} \times 9\, 10^{-4}) + \frac{1}{2}\frac{(1{,}2566)^2}{4\pi\, 10^{-7}}(2\, 10^{-3} \times 9\, 10^{-4})$$

$$+ \frac{1}{2}\frac{(0{,}6283)^2}{4\pi\, 10^{-7}}(4\, 10^{-3} \times 9\, 10^{-4}) = 4.24117 \text{ Joule}$$

#3)



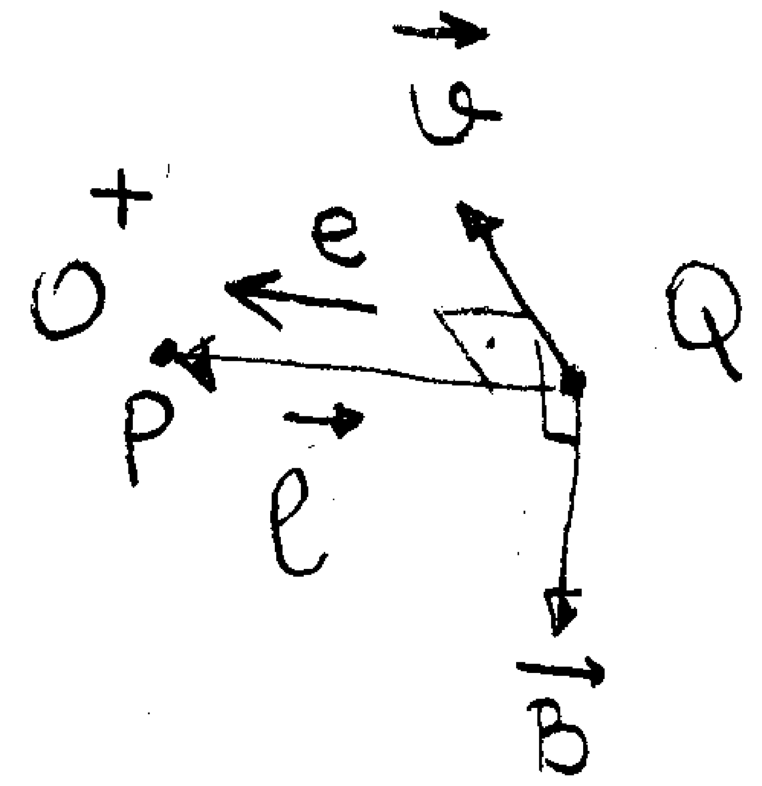
$$e = \vec{\ell} \cdot (\vec{v} \times \vec{B})$$

$$\vec{v} = \frac{\ell \, d\phi}{dt} \vec{a}_\phi = \omega . \ell \, \vec{a}_\phi$$

$$\vec{B} = B\,(-\vec{a}_z)$$

$$\vec{v} \times \vec{B} = B\,\omega . \ell \,(\vec{a}_\phi \times (-\vec{a}_z))$$

$$\vec{v} \times \vec{B} = B . v . \ell \,(-\vec{a}_\rho)$$



$e = v . \ell \, B$ volt <u>P end of the rod has (+) polarity.</u>

$\ell = \overline{PQ} = 50 . 10^{-2}$ m.

$B = 0{,}8$ T

$e = 80$ volt.

$\omega = ?$

$$e = (\omega \ell)\, B . \ell$$

$$\omega = \frac{e}{\ell^2 B}$$

$$\omega = \frac{80}{(50\,10^{-2})^2\, 0{,}8} = \frac{100}{2500 \times 10^{-4}} = 400 \text{ rad/s.}$$

$$n = \frac{400}{2\pi} = \frac{200}{\pi} \text{ rps (rotation per second)}$$

$$n = \frac{200\;60}{\pi} = \frac{12000}{\pi} = \underline{\underline{3819.71}} \text{ rpm (rotation per minute)}$$