PRINCIPLES OF ENERGY CONVERSION SECOND MIDTERM EXAM

Dr. Salih FADIL **May 02, 2009**

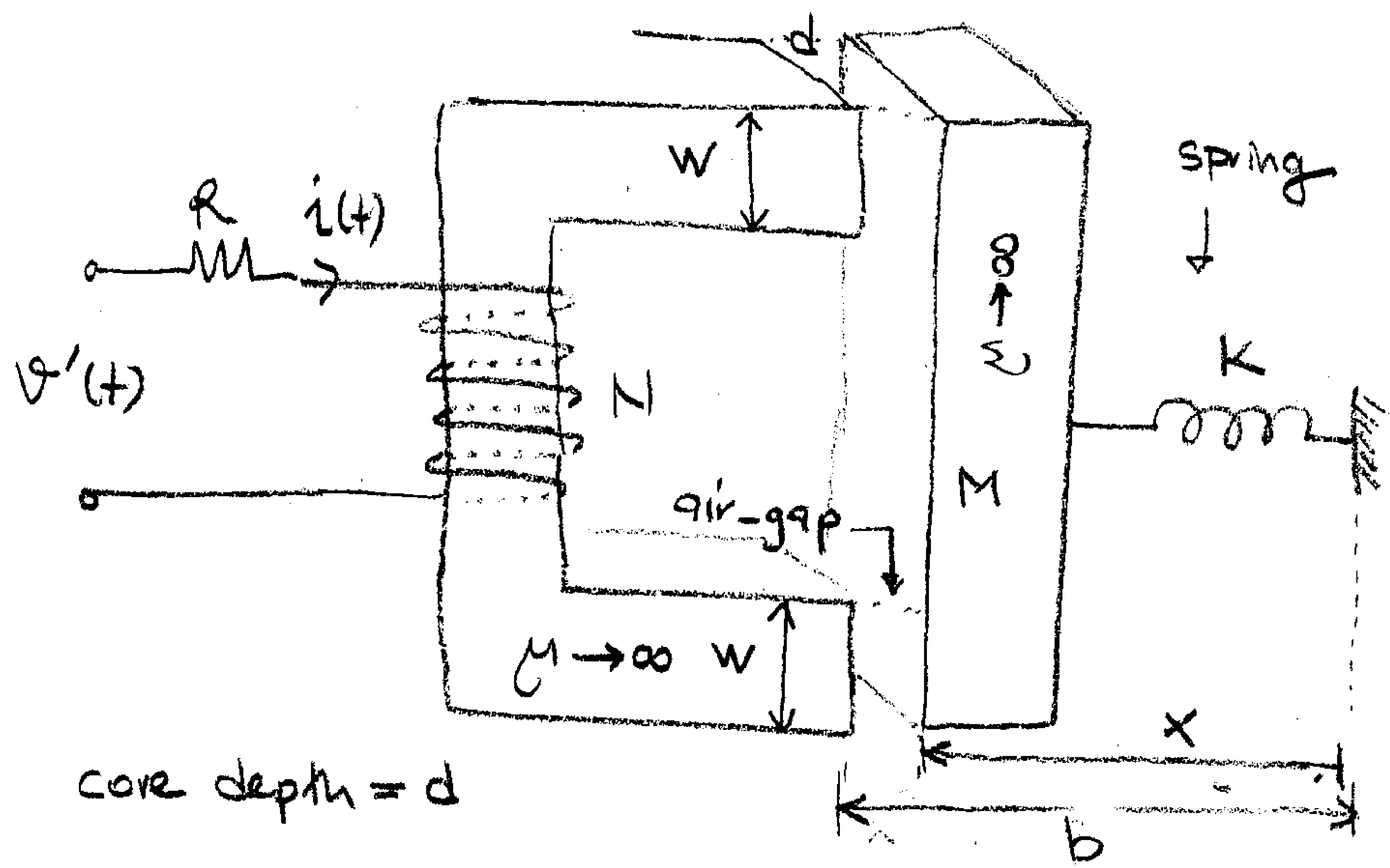
#1) On a single phase transformer 30 kVA, 480/120 V, short circuit and open circuit tests are performed in the following manner.

During the short circuit test where nominal current is applied to 480 V winding, and 40 V and 400 W are read as supplied voltage and active power, 120 V winding is short circuated.

During the open circuit test, rated voltage is applied to 480 V winding and 3A and 300 W are read as supplied current and active power. 120 V winding is open circuated.

Determine the actual equivalent circuit of the transformer *referred to low voltage side.*

#2) Consider the following electromechanical system and write the *first degree* nonlinear differential equation set that describes the motion of the system. Determine the quantities whose initial conditions should be known to solve the differential equations numerically.



#3) The average value of the electrostatic force of attraction between the plates of a parallel plate capacitor that is connected to a voltage source being in the form of $V_{max}\sin(\omega t)$ volt is $12x10^{-4}\ N$. If the plate area is $100\ cm^2$, the dielectric has a permittivity of $\varepsilon = 3\varepsilon_0\ F/m$ and thickness of $1\ mm$, determine the applied voltage's *rms* value.

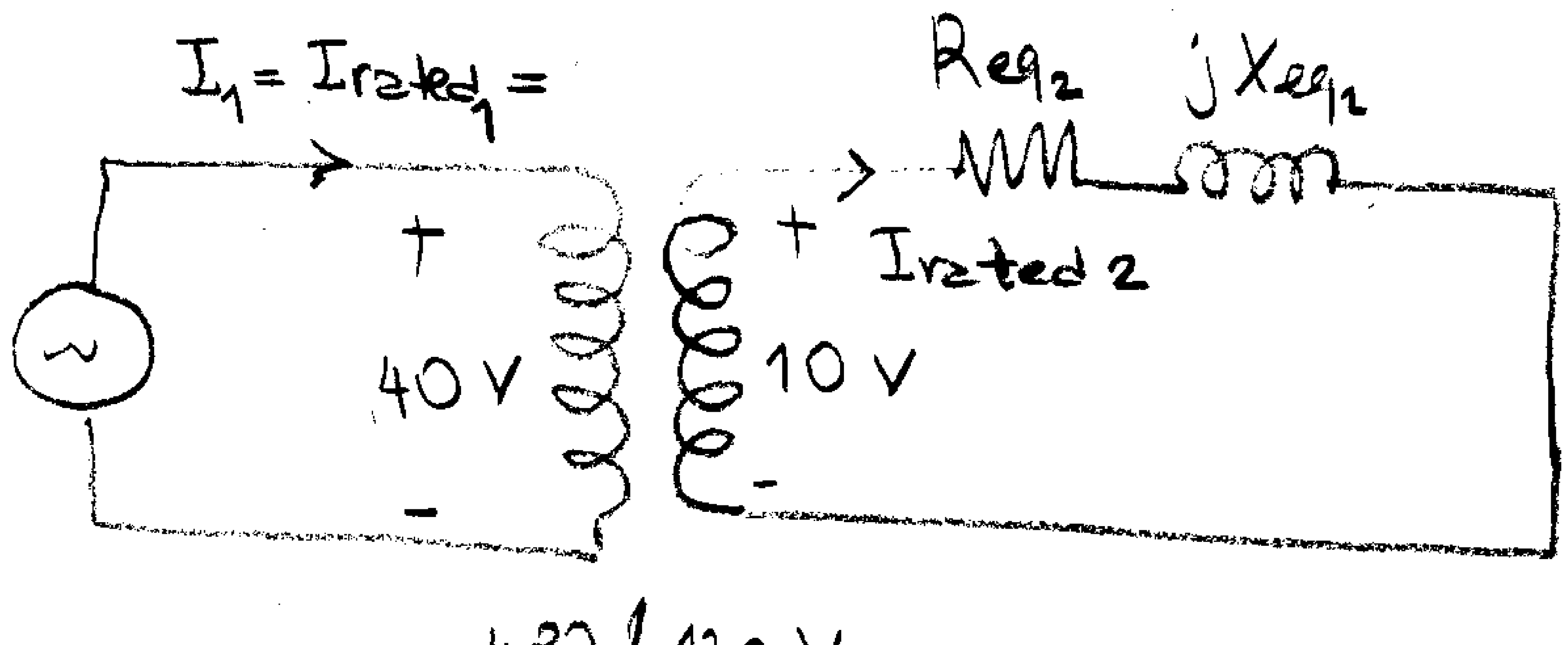
PRINCIPLES OF ENERGY CONVERSION SECOND MIDTERM EXAM SOLUTION MANUAL

Dr. Salih FADIL

May 02, 2009

#1) 30 kVA, 480/120V single phase transformer

short circuit test



$$I_{rated_1} = \frac{30\ 10^3}{480} = 62{,}5A$$

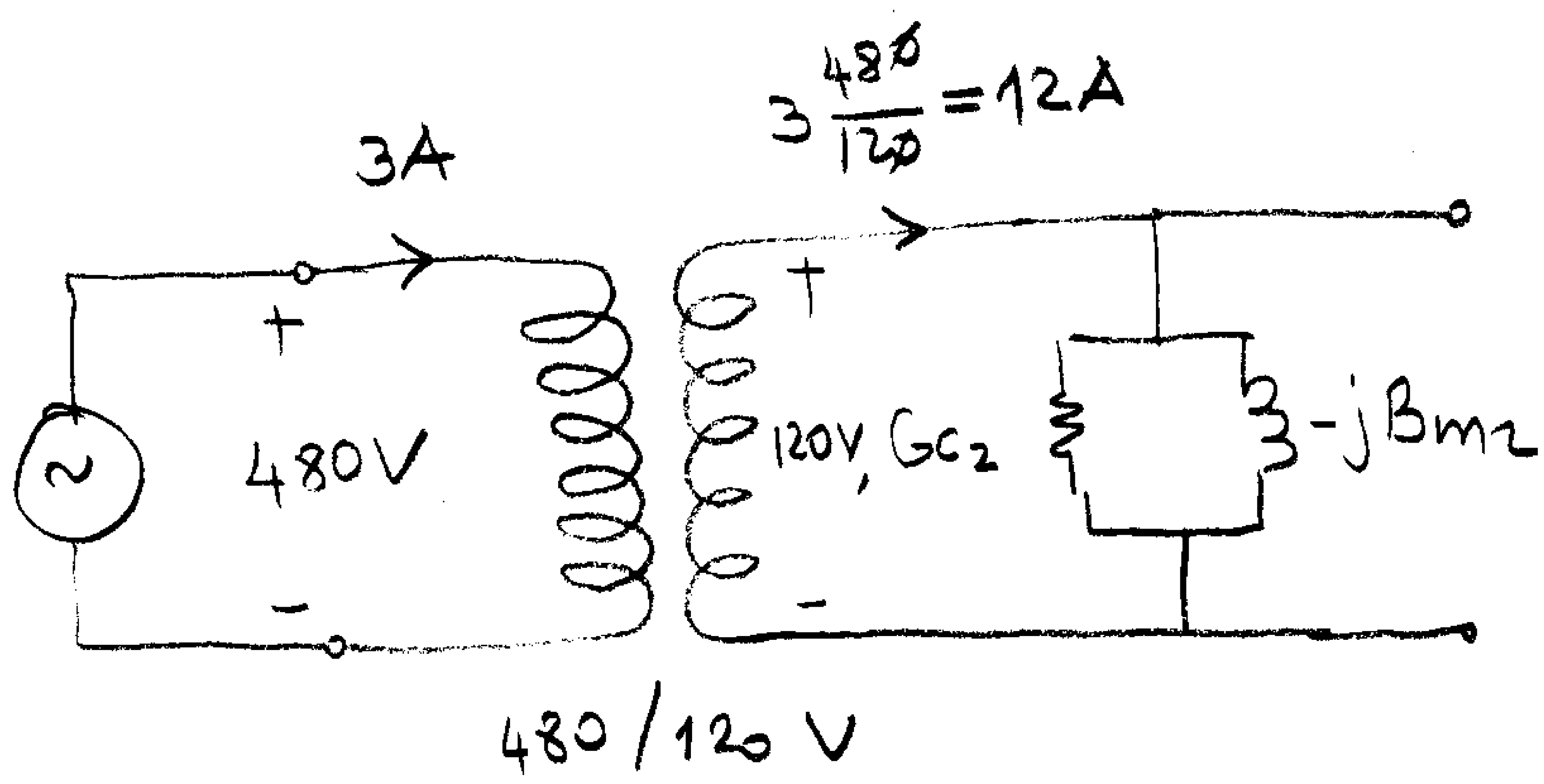
$$I_{rated_2} = \frac{30\ 10^3}{120} = 250A.$$

$$R_{eq_2} = \frac{400}{(250)^2} = 6.4\ 10^{-3}\ \Omega \qquad |Z_{eq_2}| = \frac{10}{250} = 0.04\ \Omega$$

$$X_{eq_2} = \left[(0{,}04)^2 - (6.4\ 10^{-3})^2\right]^{1/2} = 0.0394846\ \Omega$$

$$\bar{Z}_{eq_2} = 6{,}4\ 10^{-3} + j\,0{,}0394846\ \Omega$$

From open circuit test

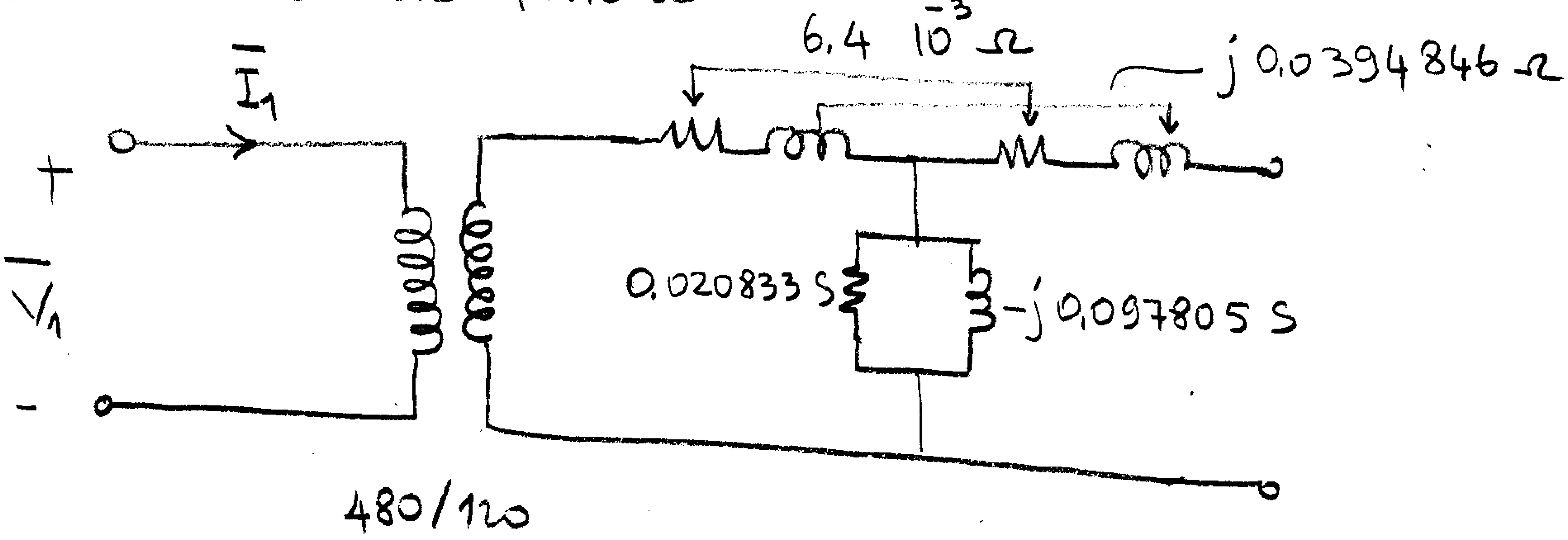


$$G_{c_2}(120)^2 = 300 \qquad G_{c_2} = \frac{300}{(120)^2} = 0{,}020833\ S.$$

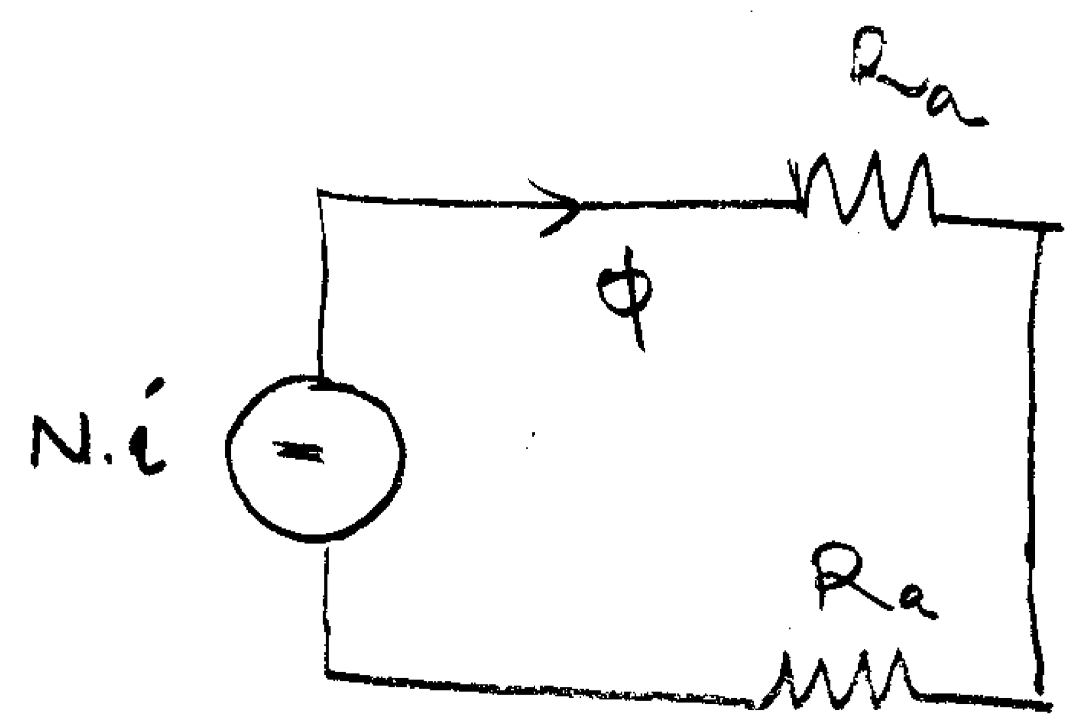
$$|\bar{Y}_{m_2}| = \frac{12}{120} = 0.1\ S \qquad B_{m_2} = \left[(0{,}1)^2 - (0{,}020833)^2\right]^{1/2} S$$

$$B_{m_2} = 0{,}097805\ S$$

The equivalent circuit referred to low voltage (120 V) side will be as follows



#2) Since $\mu \to \infty$ for iron part $R_i \to 0$.



$$R_a = \frac{b-x}{(wd)\mu_0}$$

$$\phi = \frac{N i\,(wd)\mu_0}{2(b-x)}$$

$$\lambda = N\phi = \frac{N^2 (wd)\mu_0}{2(b-x)}\, i = L\, i \qquad L(x) = \frac{N^2 (wd)\mu_0}{2(b-x)}$$

$$f_e = \frac{1}{2} i^2 \frac{dL}{dx} = \frac{1}{2} i^2 \frac{N^2 (wd)\mu_0}{2(b-x)^2} = \frac{1}{4} i^2 \cdot \frac{N^2 (wd)\mu_0}{2(b-x)^2}$$

$$M.\frac{d^2x}{dt^2} + K \cdot x = \frac{1}{2} i^2 \frac{dL}{dx} = f_e \quad \ldots (1)$$

$$v' = R.i + L\frac{di}{dt} + i\frac{dL}{dx}\frac{dx}{dt} \quad \ldots (2)$$

Nonlinear differential equation set (first degree)

$$\frac{dx}{dt} = y$$

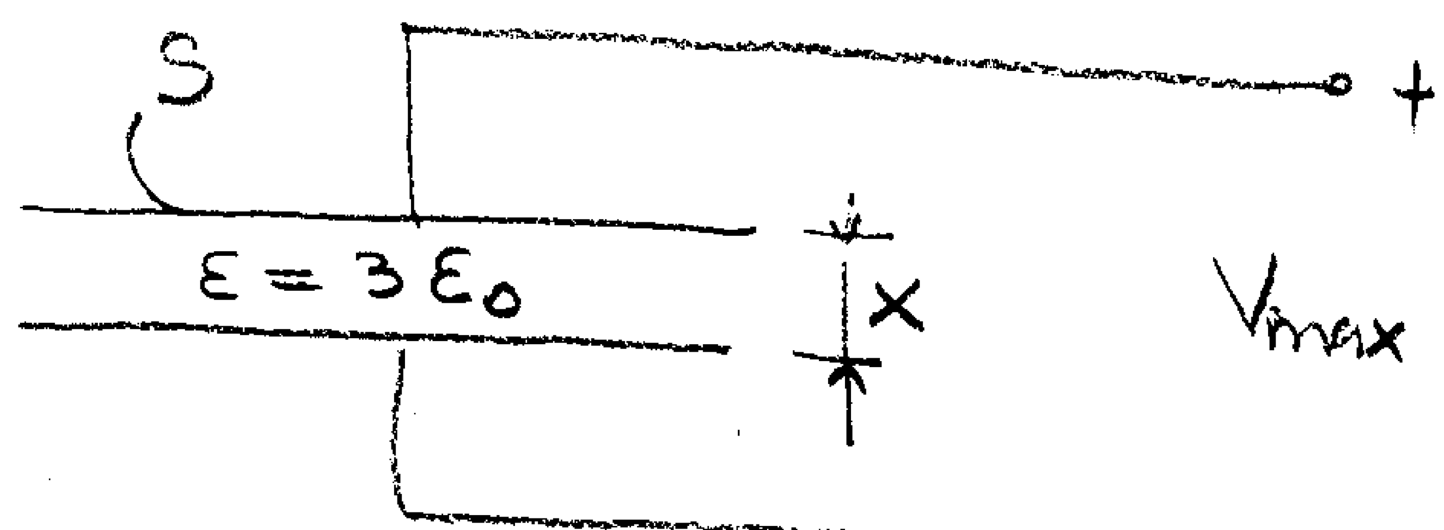
$$\frac{dy}{dt} = \frac{1}{M}\left[\frac{1}{2} i^2 \frac{dL}{dx} - K.x\right], \qquad \frac{dL}{dx} = \frac{N^2 (wd)\mu_0}{2(b-x)^2}$$

$$\frac{di}{dt} = \frac{1}{L}\left[v' - R\,i - i\,\frac{dL}{dx}\,y \right] \quad , \quad L(x) = \frac{N^2 (w.d)\,\mu_0}{2(b-x)}$$

$$\left.\begin{array}{l} x(0) = x_0 \\ \dot{x}(0) = y(0) = y_0 \\ i(0) = i_0 \end{array}\right\}$$

those initial conditions should be know in order to solve the above differential equation set numerically.

#3)



$V_{max} \sin(\omega t) = v(t)$

$x = 1$ mm
$S = 100$ cm^2
$\varepsilon = \frac{3\,10^{-9}}{36\pi}$ F/m

$$W_s' = \frac{1}{2} C v^2 = \frac{1}{2} \frac{\varepsilon S}{x} V_{max}^2 \sin^2 \omega t$$

$$f_e = + \frac{\partial W_s'}{\partial x} = - \frac{\varepsilon S}{2x^2} V_{max}^2 \sin^2 \omega t$$

$$f_e = - \frac{\varepsilon S}{2x^2} \frac{V_{max}^2}{2} \left[1 - \cos(2\omega t) \right] \quad \leftarrow \text{Instantenous force}$$

$$f_{e,avg} = - \frac{\varepsilon S V_{rms}^2}{2x^2} = - \frac{(3\varepsilon_0) S V_{rms}^2}{2x^2}$$

$$|f_{e,avg}| = \frac{(3\varepsilon_0) S V_{rms}^2}{2x^2} = 12\,10^{-4}$$

$$V_{rms} = \left[\frac{12\,10^{-4}\,2\,(10^{-3})^2}{3\,\frac{10^{-9}}{36\pi}\,100\,10^{-4}} \right]^{1/2} = \left[\frac{24 \times 36\pi\,10^{-10}}{300\,10^{-13}} \right]^{1/2} = 95.119 \text{ V}$$