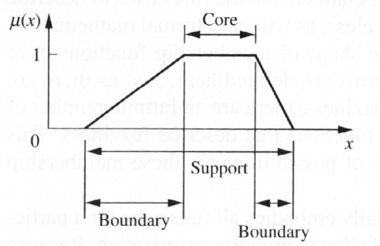
Membership Functions

Lecture 04

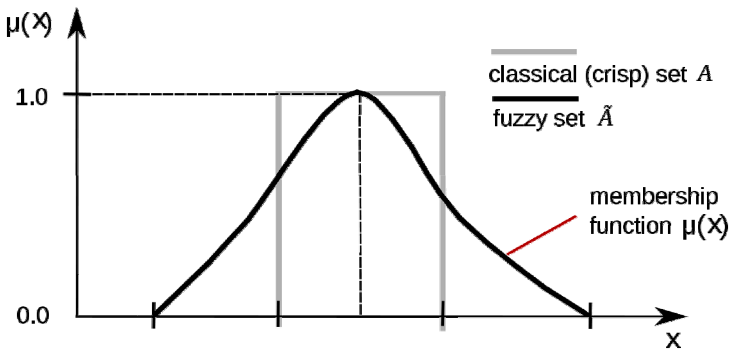
Membership Functions

Membership functions characterize the fuzziness in a fuzzy set.

Core: Elements which have full membership (μ=1)
Boundary: Elements which have membership $0<\mu_{\underset{\sim}{A}}(x)<1$
Support: Elements having nonzero membership (Core U boundary)

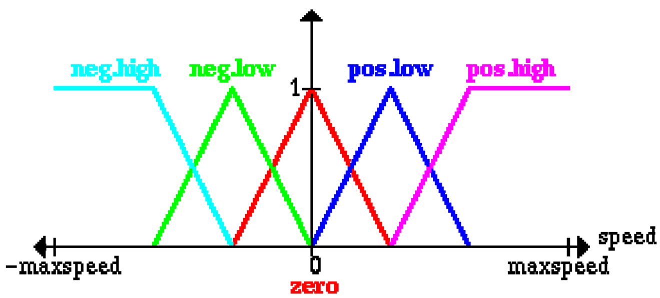


Membership Functions



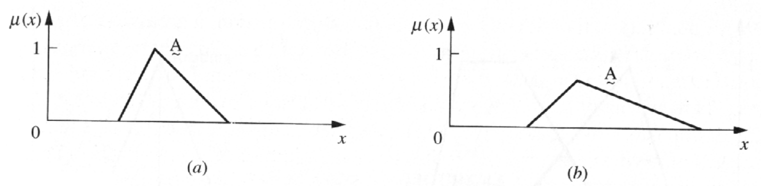
Membership Functions

Example: Membership functions relating the speed of a vehicle.



Membership Functions

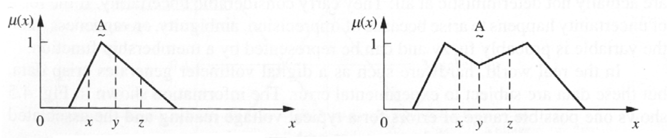
Definition: A **normal fuzzy set** is one whose membership function has at least one element with full membership. Otherwise, the set is called **subnormal**.



Fuzzy sets that are normal (a) and subnormal (b).

Membership Functions

Definition: A **convex fuzzy set** is described by a membership function where membership values are strictly monotonically increasing, monotonically decreasing or first monotonically increasing then monotonically decreasing.



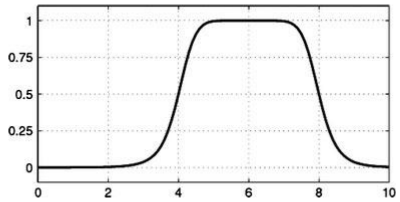
Convex, normal fuzzy set **Non convex, normal fuzzy set**

Membership Functions

The **crossover points** of a membership function are defined as the elements in the universe for which a particular fuzzy set $\underset{\sim}{A}$ has values equal to 0.5, i.e., $\mu_{\underset{\sim}{A}}(x)=0.5$.

The **height** of a fuzzy set $\underset{\sim}{A}$ is the maximum value of the membership function.

Here, points 4 and 8 are crossover points; the height is 1.

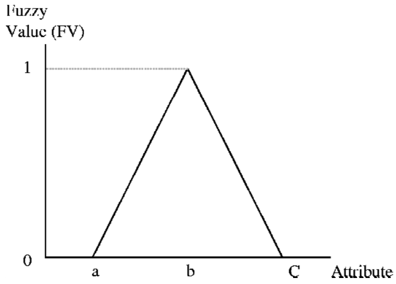


Membership Functions

Some membership functions of one dimension:

1) TRIANGULAR MF

$$\text{triangle}(x;a,b,c) = \begin{cases} 0, & x \le a. \\ \frac{x-a}{b-a}, & a \le x \le b. \\ \frac{c-x}{c-b}, & b \le x \le c. \\ 0, & c \le x. \end{cases}$$



x: input

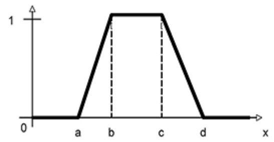
a, *b*, *c*: parameters of

trimf (triangular mf)

Membership Functions

2) TRAPEZOIDAL MF

$$\text{trapezoid}(x;a,b,c,d) = \begin{cases} 0, & x \le a. \\ \frac{x-a}{b-a}, & a \le x \le b. \\ 1, & b \le x \le c. \\ \frac{d-x}{d-c}, & c \le x \le d. \\ 0, & d \le x. \end{cases}$$



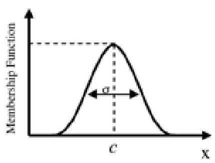
x: input

a, *b*, *c*, *d*: parameters

Membership Functions

3) GAUSSIAN MF

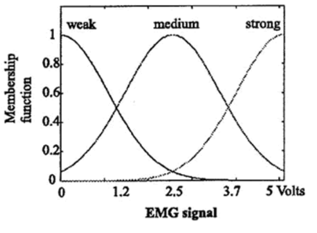
$$\text{gaussian}(x;c,\sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}.$$



x: input

c, σ: parameters

An example:

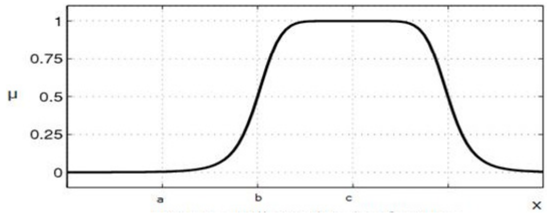


Membership Functions

4) GENERALIZED BELL (or CAUCHY) MF

$$\text{bell}(x; a, b, c) = \frac{1}{1 + \left|\frac{x - c}{a}\right|^{2b}},$$

- *a* defines spread, *b* defines slope
- Negative *b* results in upside down curve



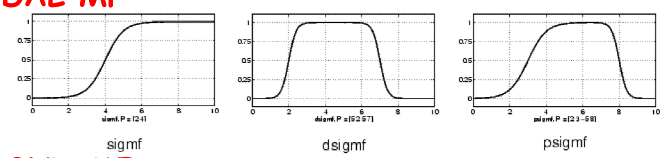
x: input

a, *b*, *c*: parameters

Membership Functions

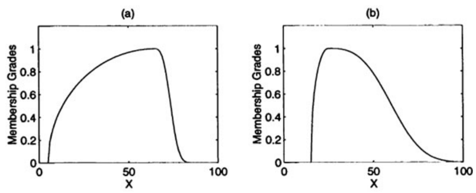
Some other membership functions:

SIGMOIDAL MF



LEFT-RIGHT MF

(a) LR(x; 65, 60, 10);
(b) LR(x; 25, 10, 40)



Membership Functions

MEMBERSHIP VALUE ASSIGNMENTS:

An appropriate question regarding relations is as follows:

Where do the membership values that are contained in a relation come from?

Membership Functions

Some ways to develop the numerical values that characterize a relation:

- Intuition
- Inference
- Rank ordering
- Neural networks
- Genetic algorithms
- Inductive reasoning

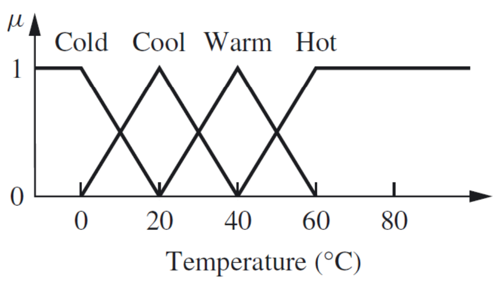
Membership Functions

MEMBERSHIP VALUE ASSIGNMENTS:

Intuition: Membership functions are directly derived from the capacity of humans through their own innate intelligence and understanding. It means we derive membership functions according to us.

Membership Functions

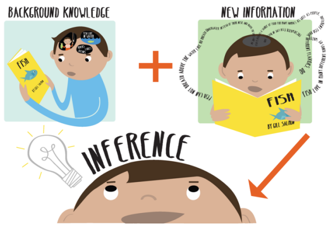
Example: Membership function assignment for the fuzzy variable "temperature".



Membership Functions

MEMBERSHIP VALUE ASSIGNMENTS:

Inference: We use knowledge to perform deductive reasoning. That is, we wish to deduce or infer a conclusion, given a body of facts and knowledge.



Membership Functions

- Example: We want to define I: Approximate isosceles triangle.

Let $A \geq B \geq C > 0$ be the inner angles of triangle. (Fact: $A+B+C=180^\circ$)

$U=\{(A,B,C) \mid A \geq B \geq C > 0;\ A+B+C=180^\circ\}$

$\mu_{I}(A,B,C) = 1 - \frac{1}{60^\circ}\min(A-B,B-C)$.

R: Approximate Right Triangle

$\mu_{R}(A,B,C) = 1 - \frac{1}{90^\circ}\min|A-90^\circ|$.

Note: We assumed A, B, C in degrees in the formulas.

Membership Functions

MEMBERSHIP VALUE ASSIGNMENTS:

Rank Ordering: Preferences are determined first by pairwise comparisons and these determine the ordering of membership.

Membership Functions

- Example: Suppose 1000 people respond to a questionnaire about their pairwise preferences among 5 colors:

X={red,orange,yellow,green,blue}

Define a fuzzy set A on X as the "Best Colour"

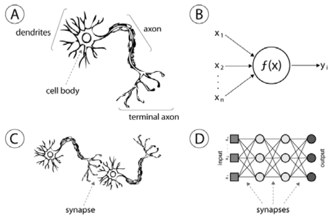
TABLE 6.1
Example in Rank Ordering

	Number who preferred							
	Red	Orange	Yellow	Green	Blue	Total	Percentage	Rank order
Red	–	517	525	545	661	2248	22.5	2
Orange	483	–	841	477	576	2377	23.8	1
Yellow	475	159	–	534	614	1782	17.8	4
Green	455	523	466	–	643	2087	20.9	3
Blue	339	424	386	357	–	1506	15	5
Total						10,000		

Membership Functions

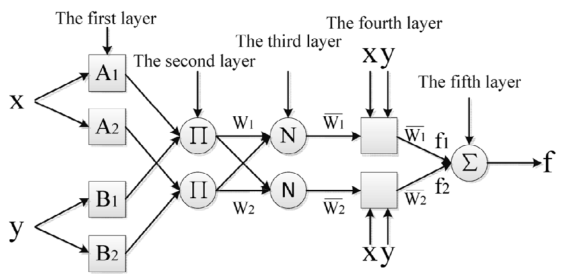
MEMBERSHIP VALUE ASSIGNMENTS:

Neural Networks: A neural network is a technique that seeks to build an intelligent program (to implement intelligence) using models that simulate the working network of the neurons in the human brain.



Membership Functions

We need data to train a neural network. After the training process, the mathemathical model has been obtained.



Membership Functions

MEMBERSHIP VALUE ASSIGNMENTS:

Genetic algorithms: Darwin's theory basically stressed the fact that the existence of all living things is based on the rule of "survival of the fittest". First, different possible solutions to a problem are created. These solutions are then tested for their performance (i.e., how good a solution they provide). Among all possible solutions, a fraction of the good solutions is selected, and the others are eliminated (survival of the fittest).

Membership Functions

MEMBERSHIP VALUE ASSIGNMENTS:

Inductive reasoning:

1. Given a set of irreducible outcomes of an experiment, the induced probabilities are those probabilities consistent with all available information that maximize the entropy of the set.
2. The induced probability of a set of independent observations is proportional to the probability density of the induced probability of a single observation.
3. The induced rule is that rule consistent with all available information of which the entropy is minimum.