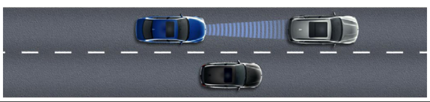
Fuzzy Control Systems

Lecture 10

Fuzzy Control Systems

- Control applications are the kinds of problems for which fuzzy logic has had the greatest success and acclaim. Many of the consumer products that we use today involve fuzzy control.



Fuzzy Control Systems

- Control systems abound in our everyday life.
- For example, economic systems are large, global systems that can be controlled; ecosystems are large, amorphous, and long-term systems that can be controlled.
- Systems that can be controlled have **three** key features: **inputs**, **outputs**, and **control parameters (or actions)** which are used to perturb the system into some desirable state.

Fuzzy Control Systems

- A control system for a physical system is an arrangement of hardware components designed to alter, to regulate, or to command, through a *control action*, another physical system so that it exhibits certain desired characteristics or behaviour.
- Physical control systems are typically of two types: **open-loop control systems**, in which the control action is independent of the physical system output, and **closed-loop control systems** (also known as *feedback control systems*), in which the control action depends on the physical system output

Fuzzy Control Systems

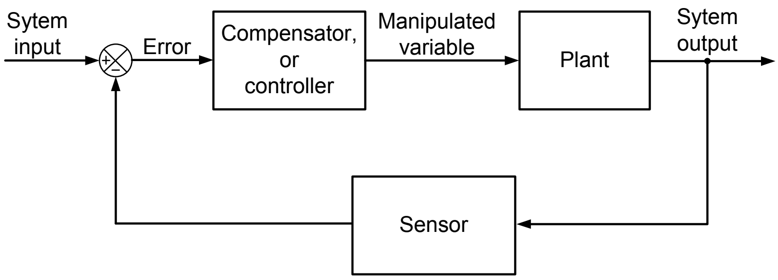
- **Examples of open-loop control systems** are a *toaster*, in which the amount of heat is set by a human.

- **Examples of feedback control** are a *room temperature thermostat*, which senses room temperature and activates the heating or cooling unit when a certain threshold temperature is reached.

Fuzzy Control Systems

- In order to control any physical variable, we must first measure it. The system for measurement of the *controlled signal* is called a ***sensor***.
- The physical system under control is called a **plant**.
- In a closed-loop control system, certain forcing signals of the system (*the inputs*) are determined by the responses of the system (*the outputs*).
- To obtain, satisfactory responses and characteristics for the closed-loop control system, it is necessary to connect an additional system, known as a **compensator**, or a *controller*, into the loop.

Fuzzy Control Systems



A closed-loop control system

Fuzzy Control Systems

- The output, or response, of the physical system under control (plant) is adjusted as required by the **error signal**.
- The error signal **e**(*t*) is the difference between the actual response of the plant, as measured by the sensor, and the desired response, as specified by a **reference input**.

Control System Design Problem

- The general problem of feedback control system design is defined as obtaining a generally nonlinear vector-valued function **h**(), defined for some time, *t*, as follows:

$$\boldsymbol{u}(t) = \boldsymbol{h}[t, \boldsymbol{x}(t), \boldsymbol{r}(t)]$$

- where **u**(*t*) is the **control input** to the plant or process, **r**(*t*) is the **system reference (desired) input**, and **x**(*t*) is the **system state vector**.

Control System Design Problem

- The vast majority of controllers are based on one of the general models given below that is, either full state feedback or output feedback.

$$\boldsymbol{u}(t) = \boldsymbol{h}[\boldsymbol{x}(t)]$$

$$\boldsymbol{u}(t) = \boldsymbol{h}\left[\boldsymbol{y}(t),\ \dot{\boldsymbol{y}}, \int \boldsymbol{y}\, dt\right]$$

- where **y**() is the **system output or response function**.

Control System Design Problem

- In the case of a simple single-input, single-output system and a regulatory type of controller, the function **h** takes one of the following forms:

$$\boldsymbol{u}(t) = K_p \,.\, e(t)$$

for a **proportional, or P, controller**.

- $$\boldsymbol{u}(t) = K_p \,.\, e(t) + K_I \,.\, \int e(t)dt$$

for a **proportional-plus-integral, or PI, controller.**

- $$\boldsymbol{u}(t) = K_p \,.\, e(t) + K_D \,.\, \dot{e}(t)$$

for a **proportional-plus-derivative, or PD, controller.**

Control System Design Problem

- $$\boldsymbol{u}(t) = K_p \,.\, e(t) + K_I \,.\, \int e(t)dt + K_D \,.\, \dot{e}(t)$$

for a **proportional-plus-derivative-plus-integral, or PID, controller.**

- $$\boldsymbol{u}(t) = -[k_1.x_1(t) + k_2.x_2(t) + \cdots + k_n.x_n(t)]$$

for a **full state-feedback controller.**

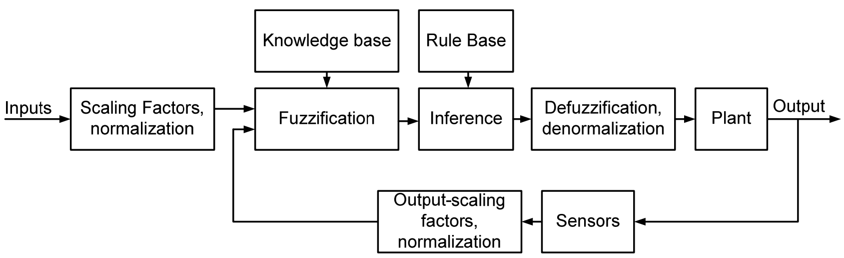
- The problem of control system design is defined as obtaining the generally nonlinear function **h**( ) in the case of nonlinear systems; coefficients K_P, K_I, and K_D in the case of output-feedback systems; and coefficients $k_1, k_2, \ldots k_n$ in the case of a full state-feedback control policy for linear systems.

Assumptions in a Fuzzy Control System Design

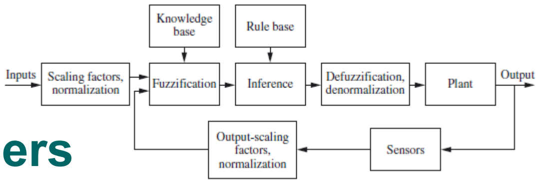
1) The plant is observable and controllable.
2) There exists a body of knowledge.
3) A solution exists.
4) The control engineer is looking for a «good enough» solution, not necessarily the optimum one.
5) The controller will be designed within an acceptable range of precision.
6) The problems of stability and optimality are not addressed explicitly; such issues are still open problems in fuzzy controller design.

Simple Fuzzy Logic Controllers

- First-generation (nonadaptive) simple fuzzy controllers can generally be depicted by a block diagram such as that shown below.



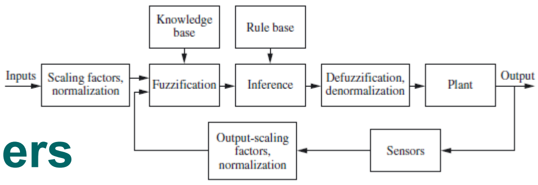
Simple Fuzzy Logic Controllers



- The steps in designing a simple fuzzy control system are as follows:

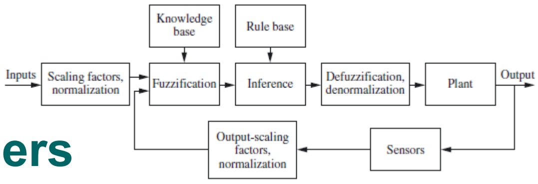
1) Identify the variables (inputs, states, and outputs) of the plant.
2) Partition the universe of discourse or the interval spanned by each variable into a number of fuzzy subsets, assigning each a linguistic label (subsets include all the elements in the universe).
3) Assign or determine a membership function for each fuzzy subset.

Simple Fuzzy Logic Controllers



4) Assign the fuzzy relationships between the inputs' or states' fuzzy subsets on the one hand and the outputs' fuzzy subsets on the other hand, thus forming the rule-base.
5) Choose appropriate scaling factors for the input and output variables in order to normalize the variables to the [0, 1] or the [−1, 1] interval.
6) Fuzzify the inputs to the controller.

Simple Fuzzy Logic Controllers



7) Use fuzzy approximate reasoning to infer the output contributed from each rule.
8) Aggregate the fuzzy outputs recommended by each rule.
9) Apply defuzzification to form a crisp output.

Simple Fuzzy Logic Controllers

- A fuzzy control mechanism includes
  - A set of input data
  - A set of rules that represents the policies and heuristic.
- Strategies of the expert decision maker
  - A method for evaluation of proposed action
  - A method for generating promising actions.

Fuzzy Control System Design

Example: Pressure Control

Suppose an industrial process output is given in terms of the pressure.

- We can calculate the difference between the desired pressure and the output pressure, called the pressure error (**e**), and we can calculate the difference between the desired rate of change of the pressure, dp/dt, and the actual pressure rate, called the pressure error rate, (**ė**).

Example: Pressure Control

Also, assume that knowledge can be expressed in the form of such IF–THEN rules (just one rule as an example):

IF pressure error (e) is "positive big (PB)" or "positive medium (PM)" and

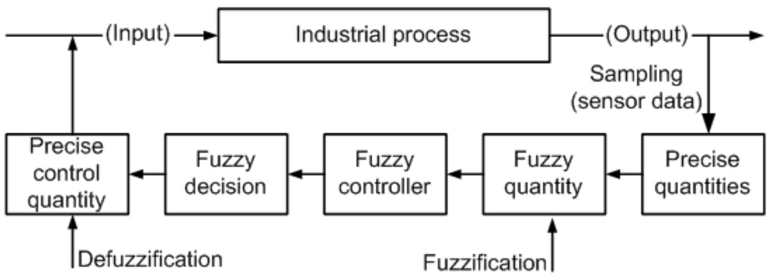
IF pressure error rate ($\dot{e}$) is "negative small (NS),"

THEN heat input change is "negative medium (NM)."

- The linguistic variables defining the pressure error: "PB" and "PM," the pressure error rate: "NS" and heat input change: "NM," are fuzzy, but the measurements of both the pressure and pressure rate as well as the control value for the heat (the control variable) ultimately applied to the system are precise (crisp).

Example: Pressure Control

Typical closed-loop fuzzy control situation for this example can be given as:



Example: Pressure Control

This example is illustrated in four steps.

Step 1. Value assignment for the fuzzy input and output variables:

We will let the error (**e**) be defined by eight linguistic variables, labeled $A_1, A_2, \ldots, A_8$, partitioned on the error space of $[-e_m, +e_m]$, and the error rate ($\dot{\mathbf{e}}$, or $d\mathbf{e}/dt$) be defined by seven variables, labeled $B_1, B_2, \ldots, B_7$, partitioned on the error rate space of $[-\dot{e}_m, \dot{e}_m]$. We will normalize these ranges to the same interval $[-a,+a]$ as

$$e_1 = \left(\frac{a}{e_m}\right) \cdot e, \qquad \dot{e}_1 = \left(\frac{a}{\dot{e}_m}\right) \cdot \dot{e}.$$

Both inputs are defined in $[-a,+a]$

Example: Pressure Control

MFs for INPUTS:

–For the error, **e**, the eight fuzzy variables: NB, NM, NS, N0, P0, PS, PM, PB.

–For the error rate, **ė**, the seven fuzzy variables: NB, NM, NS, 0, PS, PM, PB.

–The membership functions for these quantities will be on the range [−*a*, *a*], and these can be shown in Tables shortly for discrete variables or in figures for continuous variables

(for this examples input-1 can be shown with x = e and input-2 can be shown with y = ė).

Example: Pressure Control

MF Table for input-1:

Membership functions for error $(e)^a$.

A_i	x	−6	−5	−4	−3	−2	−1	−0	0+	1	2	3	4	5	6
A_8	PB	0	0	0	0	0	0	0	0	0	0	0.1	0.4	0.8	1
A_7	PM	0	0	0	0	0	0	0	0	0	0.2	0.7	1	0.7	0.2
A_6	PS	0	0	0	0	0	0	0	0.3	0.8	1	0.5	0.1	0	0
A_5	P0	0	0	0	0	0	0	0	1	0.6	0.1	0	0	0	0
A_4	N0	0	0	0	0	0.1	0.6	1	0	0	0	0	0	0	0
A_3	NS	0	0	0.1	0.5	1	0.8	0.3	0	0	0	0	0	0	0
A_2	NM	0.2	0.7	1	0.7	0.2	0	0	0	0	0	0	0	0	0
A_1	NB	1	0.8	0.4	0.1	0	0	0	0	0	0	0	0	0	0

[a] In the case of crisp control, the membership values in the shaded boxes become unity and all other values become zero.

Example: Pressure Control

MF Table for input-2:

Membership functions for error rate $(de/dt)^a$.

	y	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6
B_j														
B_7	PB	0	0	0	0	0	0	0	0	0	0.1	0.4	0.8	1
B_6	PM	0	0	0	0	0	0	0	0	0.2	0.7	1	0.7	0.2
B_5	PS	0	0	0	0	0	0	0	0.9	1	0.7	0.2	0	0
B_4	0	0	0	0	0	0	0.5	1	0.5	0	0	0	0	0
B_3	NS	0	0	0.2	0.7	1	0.9	0	0	0	0	0	0	0
B_2	NM	0.2	0.7	1	0.7	0.2	0	0	0	0	0	0	0	0
B_1	NB	1	0.8	0.4	0.1	0	0	0	0	0	0	0	0	0

[a]In the case of crisp control, the membership values in the shaded boxes become unity and all other values become zero.

Example: Pressure Control

MFs for the OUTPUT
The fuzzy output variable, the control quantity (z), will use seven fuzzy variables on the normalized universe, z = {−7,−6,−5, . . . ,+7}:

Membership functions for the control quantity $(z)^a$.

	z	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7
C_k																
C_1	PB	0	0	0	0	0	0	0	0	0	0	0	0.1	0.4	0.8	1
C_2	PM	0	0	0	0	0	0	0	0	0	0.2	0.7	1	0.7	0.2	0
C_3	PS	0	0	0	0	0	0	0	0.4	1	0.8	0.4	0.1	0	0	0
C_4	0	0	0	0	0	0	0	0.5	1	0.5	0	0	0	0	0	0
C_5	NS	0	0	0	0.1	0.4	0.8	1	0.4	0	0	0	0	0	0	0
C_6	NM	0	0.2	0.7	1	0.7	0.2	0	0	0	0	0	0	0	0	0
C_7	NB	1	0.8	0.4	0.1	0	0	0	0	0	0	0	0	0	0	0

[a]In the case of crisp control, the membership values in the shaded boxes become unity and all other values become zero.

Example: Pressure Control

Step 2. Summary of control rules:

According to human operator experience, control rules are of the form:

$$\text{If } e \text{ is } \underset{\sim}{A}_1 \text{ and } \dot{e} \text{ is } \underset{\sim}{B}_1, \text{ then } z \text{ is } \underset{\sim}{C}_1.$$

$$\text{If } e \text{ is } \underset{\sim}{A}_1 \text{ and } \dot{e} \text{ is } \underset{\sim}{B}_2, \text{ then } z \text{ is } \underset{\sim}{C}_{12}.$$

$$\text{If } e \text{ is } \underset{\sim}{A}_i \text{ and } \dot{e} \text{ is } \underset{\sim}{B}_j, \text{ then } z \text{ is } \underset{\sim}{C}_k.$$

The complete IF-THEN rules can be shown in a table, called a Fuzzy Associative Memory (FAM) Table.

Example: Pressure Control

IF-THEN Rules shown in the following FAM Table:

Control rules (FAM table).

	A_i							
	NB	**NM**	**NS**	**N0**	**P0**	**PS**	**PM**	**PB**
B_j								
NB	PB	PM	NB	NB	NB	NB		
NM	PB	PM	NM	NM	NS	NM		
NS	PB	PM	NS	NS	NS	NS	NM	NB
0	PB	PM	PS	0	0	NS	NM	NB
PS	PB	PM	PS	PS	PS	PS	NM	NB
PM			PS	PS	PM	PM	NM	NB
PB			PB	PB	PB	PB	NM	NB

Example: Pressure Control

Step 3. Conversion between fuzzy variables and precise quantities:

From the output of the system we can use an instrument to measure the error (**e**) and calculate the error rate (**ė**), both of which are precise numbers.

– We can convert precise numbers into fuzzy by using the defined MFs

– We can convert fuzzy quantities into crisp by using any defuzzification method mentioned in previous chapters.

Example: Pressure Control

Step 4. Development of control table:

–When the procedures in step 3 are used for all **e** and all **ė**, we obtain a control table as shown in the following table.

–This table now contains precise numerical quantities for use by the industrial system hardware.

Example: Pressure Control

Control actions.

	y												
	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6
x													
−6	7	6	7	6	7	7	7	4	4	2	0	0	0
−5	6	6	6	6	6	6	6	4	4	2	0	0	0
−4	7	6	7	6	7	7	7	4	4	2	0	0	0
−3	6	6	6	6	6	6	6	3	2	0	−1	−1	−1
−2	4	4	4	5	4	4	4	1	0	0	−1	−1	−1
−1	4	4	4	5	4	4	1	0	0	0	−3	−2	−1
0+	4	4	4	5	1	1	0	−1	−1	−1	−4	−4	−4
0+	4	4	4	5	1	1	0	−1	−1	−1	−4	−4	−4
1	2	2	2	2	0	0	−1	−4	−4	−3	−4	−4	−4
2	1	1	1	−2	0	−3	−4	−4	−4	−3	−4	−4	−4
3	0	0	0	0	−3	−3	−6	−6	−6	−6	−6	−6	−6
4	0	0	0	−2	−4	−4	−7	−7	−7	−6	−7	−6	−7
5	0	0	0	−2	−4	−4	−6	−6	−6	−6	−6	−6	−6
6	0	0	0	−2	−4	−4	−7	−7	−7	−6	−7	−6	−7

Example: Pressure Control

- If the values in the table are plotted, they represent a control surface.
- The volume under a control surface is proportional to the amount of energy expended by the controller.
- It can be shown that the fuzzy control surface (Figure 13.4) will actually fit underneath the crisp control surface (Figure 13.5), which indicates that the fuzzy control expends less energy than the crisp control.
- Fuzzy control methods, such as this one, have been used for some industrial systems and have achieved significant efficiency (Mamdani, 1974; Pappas and Mamdani, 1976).

Example: Pressure Control

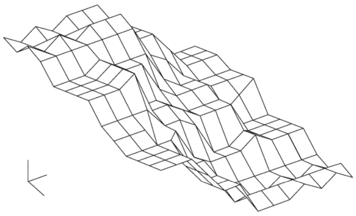


FIGURE 13.4
Control surface for fuzzy process control in Example 13.1.

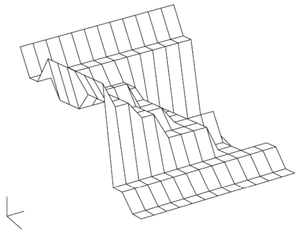
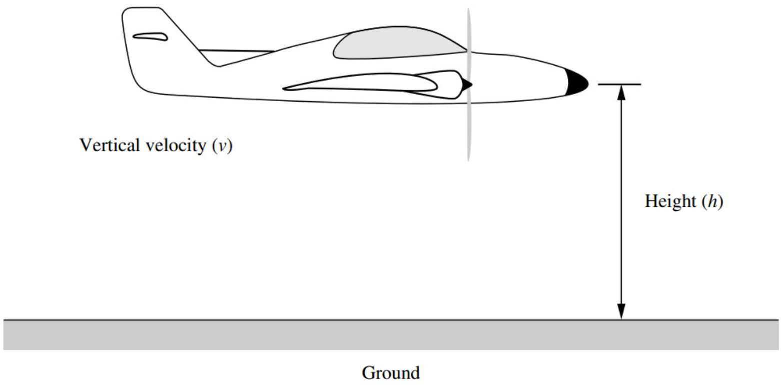


FIGURE 13.5
Control surface for crisp process control in Example 13.1.

Example: Aircraft Landing Control Problem



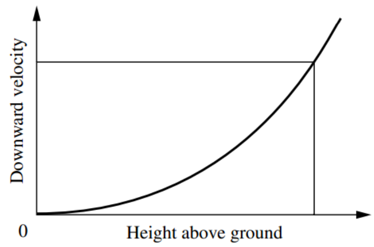
Example: Aircraft Landing Control Problem

- The two state variables for this simulation will be the height above ground, *h*, and the vertical velocity of the aircraft, *v* as inputs.
- The control output will be a force that, when applied to the aircraft, will alter its height, *h*, and velocity, *v*.
- If no external force, velocity will continue in the same direction.
- If 'f' is applied over a time interval Δt, change in velocity Δv.

Example: Aircraft Landing Control Problem

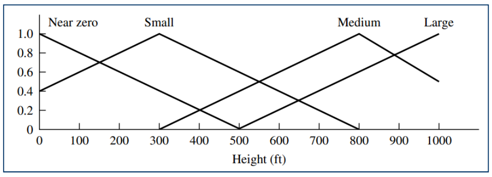
- The differential control equations are loosely derived as follows.

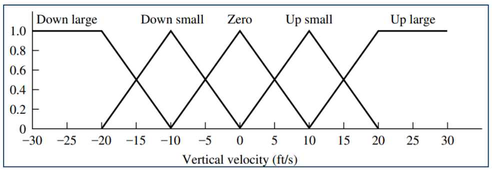
$$v_{i+1} = v_i + f_i$$
$$h_{i+1} = h_i + v_i \,.\Delta t$$



Example: Aircraft Landing Control Problem

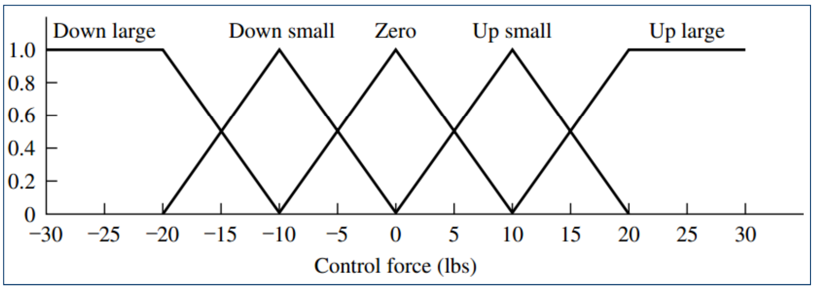
- *Step 1:* Define membership functions for state variables as shown below.





Example: Aircraft Landing Control Problem

- *Step 2:* Define a membership function for the control output, as shown below.



Example: Aircraft Landing Control Problem

- *Step 3:* Define the rules and summarize them in an FAM table.

FAM table

	Velocity				
Height	**DL**	**DS**	**Zero**	**US**	**UL**
L	Z	DS	DL	DL	DL
M	US	Z	DS	DL	DL
S	UL	US	Z	DS	DL
NZ	UL	UL	Z	DS	DS

Example: Aircraft Landing Control Problem

- FAM table is used to represent IF-THEN Rules of the fuzzy system.
- This table is a short representation of the rules.
- IF 'height is L' and 'velocity is DL' THEN 'force is Z'.
- IF 'height is L' and 'velocity is DS' THEN 'force is DS'.

FAM table

	Velocity				
Height	**DL**	**DS**	**Zero**	**US**	**UL**
L	Z	DS	DL	DL	DL
M	US	Z	DS	DL	DL
S	UL	US	Z	DS	DL
NZ	UL	UL	Z	DS	DS

Example: Aircraft Landing Control Problem

- *Step 4:* Define the initial conditions, and conduct a simulation for four cycles. Since the task at hand is to control the aircraft's vertical descent during approach and landing, we will start with the aircraft at an altitude of 1000 feet, with a downward velocity of −20 ft/s. We will use the following equations to update the state variables for each cycle.

$$v_{i+1} = v_i + f_i$$
$$h_{i+1} = h_i + v_i$$

Example: Aircraft Landing Control Problem

- Initial height, h_0 : 1000 ft
- Initial velocity, v_0 : −20 ft/s
- Control f_0 : to be computed

- *Cycle 0:* First calculate f_0 using fuzzy inference.

$$f_0 = 5.8\ lb$$

- Cycle 1: $f_0 = 5.8\ lb$

$$v_1 = v_0 + f_0 = -20 + 5.8 = -14.2\ ft/s$$
$$h_1 = h_0 + v_0 = 1000 + (-20) = 980\ ft$$

Calculate f_1 and $f_1 = -0.5$ lb

Example: Aircraft Landing Control Problem

- Cycle 2: $f_1 = -0.5\ lb$

  $v_2 = v_1 + f_1 = -14.2 + (-0.5) = -14.7\ ft/s$

  $h_2 = h_1 + v_1 = 980 + (-14.2) = 965.8\ ft$

Calculate f_2 and $f_2 = -0.4$ lb

- Cycle 3: $f_2 = -0.4\ lb$

  $v_3 = v_2 + f_2 = -14.7 + (-0.4) = -15.1\ ft/s$

  $h_3 = h_2 + v_2 = 965.8 + (-14.7) = 951.1\ ft$

Calculate f_3 and $f_3 = 0.3$ lb

Example: Aircraft Landing Control Problem

- Cycle 4: $f_3 = 0.3\ lb$

  $v_4 = v_3 + f_3 = -15.1 + (0.3) = -14.8\ ft/s$

  $h_4 = h_3 + v_3 = 951.1 + (-15.1) = 936\ ft$

Calculate f_4 and …

- The summary of the four-cycle simulation results is presented below.

Summary of four-cycle simulation results

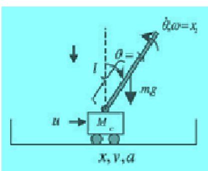
	Cycle 0	Cycle 1	Cycle 2	Cycle 3	Cycle 4
Height, ft	1000.0	980.0	965.8	951.1	936.0
Velocity, ft/s	−20	−14.2	−14.7	−15.1	−14.8
Control force	5.8	−0.5	−0.4	0.3	

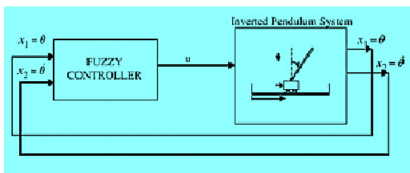
Example: Fuzzy Control of Inverted Pendulum

- In this example, a fuzzy controller for the inverted pendulum system that needs two input items, of which one is the angle(x_1) between the pendulum and the vertical position, and the other is the derivation of the angle(x_2) (angular velocity) of the pendulum, is presented simply for educational purposes.
- The fuzzy controller takes the angle and angular velocity of pendulum from the inverted pendulum system, aggregates inputs with defined IF-THEN rules and derives the obtained force(u) as an output item by means of inference methods.

Example: Fuzzy Control of Inverted Pendulum

- Inverted Pendulum and Fuzzy Control System





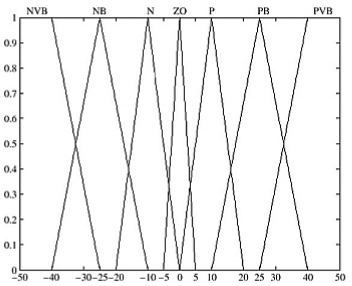
- The dynamic equation of the inverted pendulum system can be expressed as below.

$$\dot{x}_1 = x_2$$

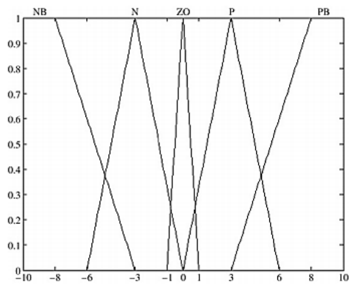
$$\dot{x}_2 = \ddot{\theta} = \dot{\omega} = \alpha = \frac{g\sin\theta - \frac{m_p l \dot{\theta}^2 \cos\theta \sin\theta}{m_c + m_p} + \frac{\cos\theta}{m_c + m_p} u}{l\left(\frac{4}{3} - \frac{m_p \cos^2\theta}{m_c + m_p}\right)}$$

Example: Fuzzy Control of Inverted Pendulum

- *Step 1:* Define membership functions for inputs.



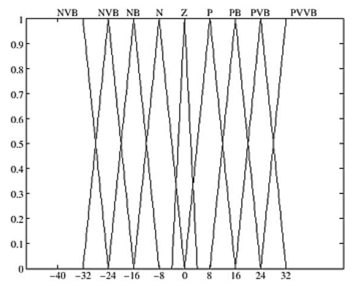
Membership function for state variable x_1



Membership function for state variable x_2

Example: Fuzzy Control of Inverted Pendulum

- *Step 2:* Define a membership function for the control output.



Membership function for output variable u

Example: Fuzzy Control of Inverted Pendulum

- *Step 3:* Define the rules and summarize them in an FAM table.

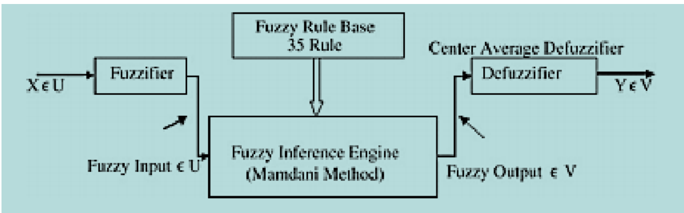
FAM table for inverted pendulum fuzzy control system rule base

x_1	x_2 PB	P	Z	N	NB
PVB	PVVB	PVVB	PVB	PB	P
PB	PVVB	PVB	PB	P	Z
P	PVB	PB	P	Z	N
Z	PB	P	Z	N	NB
N	P	Z	N	NB	NVB
NB	Z	N	NB	NVB	NVVB
NVB	N	NB	NVB	NVVB	NVVB

$$
\begin{aligned}
&\text{R1: IF } (x_1 = \text{PVB}) \text{ and } (x_2 = \text{PB}) \text{ THEN } u = \text{PVVB} \\
&\text{R2: IF } (x_1 = \text{PVB}) \text{ and } (x_2 = \text{P}) \text{ THEN } u = \text{PVVB} \\
&\quad \vdots \\
&\text{R34: IF } (x_1 = \text{NVB}) \text{ and } (x_2 = \text{N}) \text{ THEN } u = \text{NVVB} \\
&\text{R35: IF } (x_1 = \text{NVB}) \text{ and } (x_2 = \text{NB}) \text{ THEN } u = \text{NVVB}
\end{aligned}
$$

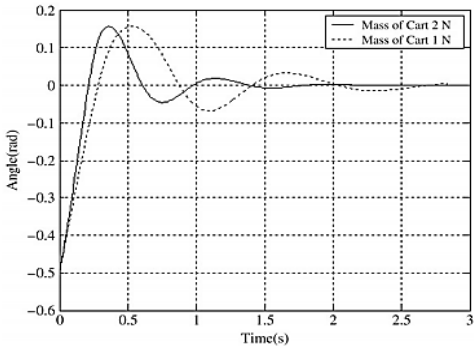
Example: Fuzzy Control of Inverted Pendulum

- *Step 4: The fuzzification method with a Singleton Fuzzifier, inference system with Mamdani Method, and defuzzification method with Center Average for this fuzzy control system as shown below.*



Example: Fuzzy Control of Inverted Pendulum

- Simulation results (mass of the cart is $m_c = 1.0kg$)



Reference : Y. Becerikli, B.K. Celik, "Fuzzy control of inverted pendulum and concept of stability using Java Application",Mathematical and Computer Modeling 46, 2007.