Power System Analysis I Sample Problems 03.01.2015

5.14 A 500-km, 500-kV, 60-Hz uncompensated three-phase line has a positive-sequence series impedance $z = 0.03 + j0.35$ Ω/km and a positive-sequence shunt admittance $y = j4.4 \times 10^{-6}$ S/km. Calculate: (a) Z_c, (b) (γl), and (c) the exact $ABCD$ parameters for this line.

5.14

(a) $\bar{Z}_c = \sqrt{\frac{\bar{z}}{\bar{y}}} = \sqrt{\frac{0.03+j.35}{j\,4.4\times10^{-6}}} = \sqrt{\frac{0.3513\,\angle 85.10^\circ}{4.4\times10^{-6}\,\angle 90^\circ}}$

$\bar{Z}_c = \sqrt{79837.\,\angle -4.899^\circ} = \underline{\underline{282.6\,\angle -2.450^\circ}}\ \Omega$

(b) $\bar{\gamma} l = \sqrt{\bar{z}\bar{y}}\,(l) = \sqrt{(.3513\angle 85.10^\circ)(4.4\times10^{-6}\angle 90^\circ)}\,(500)$

$\bar{\gamma} l = \underline{\underline{0.6216\,\angle 87.55^\circ}} = 0.02657 + j\,0.62105$

(c). $\bar{A} = \bar{D} = \cosh(\bar{\gamma} l) = \cosh(.02657 + j.62105)$

$= \cosh(.02657)\cos(.62105) + j\sinh(.02657)\sin(.62105)$

radians

$= (1.000353)(0.813268) + j(.0265731)(.581889)$

$= 0.813555 + j0.015463 = \underline{\underline{0.8137\,\angle 1.089^\circ}}$

per unit

$\sinh(\bar{\gamma} l) = \sinh(.02657 + j.62105)$ radians

$= \sinh(.02657)\cos(.62105) + j\cosh(.02657)\sin(.62105)$

$= (.02657)(.81327) + j(1.000353)(.581889)$

$= 0.021609 + j\,0.582094 = 0.5825\,\angle 87.87^\circ$

$\bar{B} = \bar{Z}_c \sinh(\bar{\gamma} l) = (282.6\,\angle -2.450^\circ)(.5825\,\angle 87.87^\circ)$

$\bar{B} = \underline{\underline{164.6\,\angle 85.42^\circ}}\ \Omega$

$\bar{C} = \left(\frac{1}{\bar{Z}_c}\right)\sinh(\bar{\gamma} l) = \frac{0.5825\,\angle 87.87^\circ}{282.6\,\angle -2.450^\circ}$

$\bar{C} = \underline{\underline{2.061\times10^{-3}\,\angle 90.32^\circ}}\ S$

5.15 At full load the line in Problem 5.14 delivers 1000 MW at unity power factor and at 480 kV. Calculate: (a) the sending-end voltage, (b) the sending-end current, (c) the sending-end power factor, (d) the full-load line losses, and (e) the percent voltage regulation.

5.15

$$\bar{V}_R = \frac{480}{\sqrt{3}} \angle 0^\circ = 277.1 \angle 0^\circ \ kV_{LN}$$

$$\bar{I}_R = \frac{P_R}{\sqrt{3}\, V_{R\,LL}\,(pf)} \angle 0^\circ = \frac{1000 \angle 0^\circ}{\sqrt{3}\; 480\; (1)} = 1.202 \angle 0^\circ \ kA$$

(a) $\bar{V}_S = \bar{A}\bar{V}_R + \bar{B}\bar{I}_R = 0.8137 \angle 1.089^\circ\,(277.1) + 164.6 \angle 85.42^\circ\,(1.202)$

$= 314.4 \angle 39.9^\circ \ kV_{LN}$; $V_S = 314.4\sqrt{3} = 544.5 \ kV_{LL}$

(b) $\bar{I}_S = \bar{C}\bar{V}_R + \bar{D}\bar{I}_R = 2.061 \times 10^{-3} \angle 90.32^\circ\,(277.1) + 0.8137 \angle 1.089^\circ\,(1.202)$

$= 1.139 \angle 31.17^\circ \ kA$; $I_S = 1.139 \ kA$

(c) $(pf)_S = \cos(39.9^\circ - 31.17^\circ) = \cos(8.73^\circ) = 0.9884$ LAGGING

(d) $P_S = \sqrt{3}\, V_{S\,LL}\, I_S\, (pf)_S = \sqrt{3}\; 544.5\,(1.139)\,0.9884$

$= 1061.7 \ MW$

FULL-LOAD LINE LOSSES $= P_S - P_R = 1061.7 - 1000 = 61.7 \ MW$

(e) $V_{R\,NL} = V_S / A = 544.5 / 0.8137 = 669.2 \ kV$

$$\%\ VR = \frac{V_{R\,NL} - V_{R\,FL}}{V_{R\,FL}} \times 100$$

$$= \frac{669.2 - 480}{480} \times 100 = 39.4\%$$

5.26 A 320-km 500-kV, 60-Hz three-phase uncompensated line has a positive-sequence series reactance $x = 0.34\ \Omega$/km and a positive-sequence shunt admittance $y = j4.5 \times 10^{-6}$ S/km. Neglecting losses, calculate: (a) Z_c, (b) (γl), (c) the *ABCD* parameters, (d) the wavelength λ of the line, in kilometers, and (e) the surge impedance loading in MW.

5.26

(a) $\bar{Z}_c = \sqrt{\frac{\bar{z}}{\bar{y}}} = \sqrt{\frac{j0.34}{j4.5\times10^{-6}}} = 274.9\ \Omega$

(b) $\bar{\gamma}\ell = \sqrt{\bar{z}\bar{y}}\ \ell = \sqrt{(j0.34)(j4.5\times10^{-6})}\ (320) = j0.3958$ pu

(c) $\bar{\gamma}\ell = j\beta\ell$; $\beta\ell = 0.3958$ pu

$\bar{A} = \bar{D} = \cos\beta\ell = \cos(0.3958 \text{ radians}) = 0.9227\angle 0^\circ$ pu

$\bar{B} = j\bar{Z}_c \sin\beta\ell = j(274.9)\sin(0.3958 \text{ radians})$

$= j108.81\ \Omega$

$\bar{C} = j\left(\frac{1}{Z_c}\right)\sin\beta\ell = j\ \frac{1}{274.9}\ \sin(0.3958 \text{ radians})$

$= j1.44\times10^{-3}$ S

(d) $\beta = \beta\ell/\ell = 0.3958/300 = 1.319\times10^{-3}$ radians/km

$\lambda = 2\pi/\beta = 4766$ km

(e) $SIL = \frac{V^2_{\text{rated L-L}}}{Z_c} = \frac{(500)^2}{274.9} = 909.4$ MW (3ϕ)

5.27 Determine the equivalent π circuit for the line in Problem 5.26.

5.27

$$\bar{Z}' = \bar{B} = j\,108.81\ \Omega$$

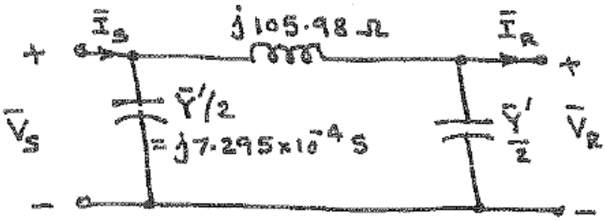
ALTERNATIVELY:

$$\bar{Z}' = \bar{Z}\bar{F}_1 = (\bar{z}\ell)\frac{\sin\beta\ell}{\beta\ell} = (j0.34\times320)\frac{\sin(0.3958\text{ radians})}{0.3958}$$

$$= j\,108.8\,(0.9741) = j\,105.98\ \Omega$$

$$\frac{\bar{Y}'}{2} = \frac{\bar{Y}}{2}\bar{F}_2 = \left(\frac{\bar{y}}{2}\ell\right)\frac{\tan(\beta\ell/2)}{\beta\ell/2} = j\,\frac{4.5\times10^{-6}}{2}\times320\ \frac{\tan(0.1979\text{ radians})}{0.1979}$$

$$= j\,7.2\times10^{-4}\,(1.0133) = j\,7.295\times10^{-4}\ \text{S}$$



EQUIVALENT π CIRCUIT

5.28 Rated line voltage is applied to the sending end of the line in Problem 5.26. Calculate the receiving-end voltage when the receiving end is terminated by (a) an open circuit, (b) the surge impedance of the line, and (c) one-half of the surge impedance. (d) Also calculate the theoretical maximum real power that the line can deliver when rated voltage is applied to both ends of the line.

5.28

(a) $V_R = V_S/A = 500/0.9227 = 541.9\text{ kV}$

(b) $V_R = V_S = 500\text{kV}$

(c) $\bar{V}_S = \cos\beta\ell\ \bar{V}_R + j Z_c \sin\beta\ell\left[\bar{V}_R/(\tfrac{1}{2}Z_c)\right]$

$$= [\cos\beta\ell + j\,2\sin\beta\ell]\ \bar{V}_R$$

$$V_S = |\cos\beta\ell + j\,2\sin\beta\ell|\ V_R$$

$$= \frac{500}{|\cos\underset{\text{rad.}}{0.3958} + j\,2\sin\underset{\text{rad.}}{0.3958}|} = \frac{500}{1.202} = 416\text{kV}$$

(d) $P_{\max 3\phi} = \dfrac{V_S V_R}{X'} = \dfrac{500\times500}{105.98} = 2359\text{ MW}$

5.31 A 500-kV, 300-km, 60-Hz three-phase overhead transmission line, assumed to be lossless, has a series inductance of 0.97 mH/km per phase and a shunt capacitance of 0.115 μF/km per phase. (a) Determine the phase constant β, the surge impedance Z_C, velocity of propagation v, and the wavelength λ of the line. (b) Determine the voltage, current, real and reactive power at the sending end, and the percent voltage regulation of the line if the receiving-end load is 800 MW at 0.8 power factor lagging and at 500 kV.

5.31

(a) FOR A LOSSLESS LINE, $\beta = \omega\sqrt{LC} = 2\pi(60)\sqrt{0.97 \times 0.0115 \times 10^{-9}}$

$= 0.001259$ rad/km

$\bar{Z}_c = \sqrt{L/C} = \sqrt{\dfrac{0.97\times10^{-3}}{0.0115\times10^{-6}}} = 290.43\ \Omega$

VELOCITY OF PROPAGATION $v = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{0.97\times0.0115\times10^{-9}}} = 2.994\times10^{5}$ km/s

AND THE LINE WAVE LENGTH IS $\lambda = v/f = \frac{1}{60}(2.994\times10^{5}) = 4990$ km

(b) $\bar{V}_R = \dfrac{500}{\sqrt{3}}\angle 0^\circ$ kV $= 288.675\angle 0^\circ$ kV

$\bar{S}_{R(3\phi)} = \dfrac{800}{0.8}\angle\cos^{-1}0.8 = 800 + j600$ MVA $= 1000\angle 36.87^\circ$ MVA

$\bar{I}_R = \bar{S}^*_{R(3\phi)} / 3\bar{V}^*_R = \dfrac{(1000\angle -36.87^\circ)\,10^3}{3\times288.675\angle 0^\circ} = 1154.7\angle -36.87^\circ$ A

SENDING END VOLTAGE $\bar{V}_S = \cos\beta l\,\bar{V}_R + j\,\bar{Z}_c \sin\beta l\,\bar{I}_R$

$\beta l = 0.001259\times300 = 0.3777$ rad $= 21.641^\circ$

$\therefore \bar{V}_S = 0.9295(288.675\angle 0^\circ) + j(290.43)\,0.3688\,(1154.7\angle -36.87^\circ)(10^{-3})$

$= 356.53\angle 16.1^\circ$ kV

SENDING END LINE-TO-LINE VOLTAGE MAGNITUDE $= \sqrt{3}\ 356.53$

$= 617.53$ kV

$\bar{I}_S = j\dfrac{1}{\bar{Z}_c}\sin\beta l\,\bar{V}_R + \cos\beta l\,\bar{I}_R$

$= j\dfrac{1}{290.43}\,0.3688\,(288.675\angle 0^\circ)\,10^3 + 0.9295(1154.7\angle -36.87^\circ)$

$= 902.3\angle -17.9^\circ$ A ; LINE CURRENT $= 902.3$ A

$\bar{S}_{S(3\phi)} = 3\bar{V}_S\bar{I}^*_S = 3(356.53\angle 16.1^\circ)(902.3\angle -17.9^\circ)\,10^{-3}$

$= 800$ MW $+ j\,539.672$ MVAR

PERCENT VOLTAGE REGULATION $= \dfrac{(356.53/0.9295) - 288.675}{288.675}\times100$

$= 32.87\%$

5.32 The following parameters are based on a preliminary line design: $V_S = 1.0$ per unit, $V_R = 0.9$ per unit; $\lambda = 5000$ km, $Z_C = 320\ \Omega$, $\delta = 36.8°$. A three-phase power of 700 MW is to be transmitted to a substation located 315 km from the source of power. (a) Determine a nominal voltage level for the three-phase transmission line, based on the practical line-loadability equation. (b) For the voltage level obtained in (a), determine the theoretical maximum power that can be transferred by the line.

Answers:

a) 400 kVLN
b) 1167 MW

5.38 The line in Problem 5.14 has three ACSR 1113-kcmil conductors per phase. Calculate the theoretical maximum real power that this line can deliver and compare with the thermal limit of the line. Assume $V_S = V_R = 1.0$ per unit and unity power factor at the receiving end.

5.38

From Problem 5.14

$\bar{A} = 0.8137\ \angle 1.089°$ per unit $\qquad A = 0.8137 \quad \theta_A = 1.089°$

$\bar{B} = \bar{Z}' = 164.6\ \angle 85.42°\ \Omega \qquad Z' = 164.6\ \Omega \quad \theta_Z = 85.42°$

Using Eq (5.5.6)

$$P_{R\,MAX} = \frac{(500)(500)}{164.6} - \frac{(0.8137)(500)^2}{164.6}\cos(85.42° - 1.089°)$$

$P_{R\,MAX} = 1518.8 - 122.1 = \underline{1397.}$ MW (3-phase)

For this loading at unity power factor:

$$I_R = \frac{P_{R\,MAX}}{\sqrt{3}\, V_{RLL}\,(P.F.)} = \frac{1397}{\sqrt{3}\,(500)(1.0)} = 1.613 \text{ kA/phase}$$

From Table A.3, the thermal limit for three ACSR 1113 kcmil conductors is 3(1.11) = 3.33 kA/phase. The current 1.613 kA corresponding to the theoretical steady-state stability limit is well below the thermal limit of 3.33 kA.

5.45 For the line in Problems 5.14 and 5.38, determine: (a) the practical line loadability in MW, assuming $V_S = 1.0$ per unit, $V_R \approx 0.95$ per unit, and $\delta_{max} = 35°$; (b) the full-load current at 0.99 p.f. leading, based on the above practical line loadability; (c) the exact receiving-end voltage for the full-load current in (b) above; and (d) the percent voltage regulation. For this line, is loadability determined by the thermal limit, the voltage-drop limit, or steady-state stability?

5.45 (a) Using Eq(5.5.3) with $\delta = 35°$:

$$P_R = \frac{(500)(.95\times 500)}{164.6}\cos(85.42° - 35°) - \frac{(.8137)(.95\times 500)^2}{164.6}\cos(85.42° - 1.09°)$$

$$P_R = 919.3 - 110.2 = \underline{\underline{809.\ \text{MW}}} \quad \text{(three-phase)}$$

$P_R = 809.$ MW is the practical line loadability provided that the voltage drop limit and thermal limit are not exceeded.

(b) $$I_{RFL} = \frac{P_R}{\sqrt{3}\,V_{RLL}\,(\text{P.F.})} = \frac{809}{\sqrt{3}(0.95\times 500)(0.99)} = \underline{\underline{0.993\ \text{kA}}}$$

(c) $$\bar{V}_S = \bar{A}\bar{V}_{RFL} + \bar{B}\bar{I}_{RFL}$$

$$\frac{500}{\sqrt{3}}\angle\delta = (0.8137\angle 1.089°)(V_{RFL}\angle 0°) + (164.6\angle 85.42°)(0.993\angle 8.11°)$$

$$288.68\angle\delta = .8137 V_{RFL}\angle 1.089° + 163.45\angle 93.53°$$

$$288.68\angle\delta = (0.8136 V_{RFL} - 10.06) + j(0.01546 V_{RFL} + 163.14)$$

Taking the squared magnitude of the above equation:

$$83{,}333. = 0.6622 V_{RFL}^2 - 11.33 V_{RFL} + 26716.$$

Solving the above quadratic equation:

$$V_{RFL} = \frac{11.33 + \sqrt{(11.33)^2 + 4(.6622)(56617)}}{2(0.6622)} = 301.1\ \text{kV}_{LN}$$

$$V_{RFL} = 301.1\sqrt{3} = \underline{\underline{521.5\ \text{kV}_{LL}}} = 1.043 \text{ per unit}$$

(d) $$V_{RNL} = V_S/A = \frac{500}{0.8137} = 614.5\ \text{kV}_{LL}$$

$$\%\text{V.R.} = \frac{614.5 - 521.5}{521.5}\times 100 = \underline{\underline{17.8\ \%}}$$

(e)

From Problem 5.27, the thermal limit is 3.33 kA. Since $V_{RFL}/V_S = 521.5/500 = 1.043$ is greater than 0.95 and the thermal limit $= 3.33$ kA is greater than $I_{RFL} = 0.993$ kA, the voltage drop limit and thermal limit are not exceeded at $P_R = 809.$ MW. Therefore, loadability is determined by stability.

5.47 Determine the practical line loadability in MW and in per-unit of SIL for the line in Problem 5.14 if the line length is (a) 200 km, (b) 600 km. Assume $V_S = 1.0$ per unit, $V_R = 0.95$ per unit, $\delta_{max} = 35°$, and 0.99 leading power factor at the receiving end.

5.47 (a) $\ell = 200$ km. The steady-state stability limit is:

$$P_R = \frac{(500)(.95)(500)}{69.54}\cos(85.15°-35°) - \frac{(.9694)(.95\times 500)^2}{69.54}\cos(85.15°-0.154°)$$

$$P_R = 2188. - 274. = 1914. \text{ MW}$$

$$I_{RFL} = \frac{P_R}{\sqrt{3}\, V_{RFL}\,(\text{P.F.})} = \frac{1914}{(\sqrt{3})(.95\times 500)(.99)} = 2.35 \text{ kA}$$

$$\bar{V}_S = \bar{A}\bar{V}_{RFL} + \bar{B}\bar{I}_{RFL}$$

$$\frac{500}{\sqrt{3}}\angle\delta_S = (.9694\angle 0.154°)(V_{RFL}\angle 0°) + (69.54\angle 85.15°)(2.35\angle 8.11°)$$

$$288.675\angle\delta_S = (.9694\, V_{RFL} - 9.293) + j(0.0026\, V_{RFL} + 163.15)$$

Taking the squared magnitude:

$$83{,}333. = .9397\, V_{RFL}^2 - 17.17\, V_{RFL} + 26704.$$

Solving

$$V_{RFL} = \frac{17.17 + \sqrt{(17.17)^2 + 4(.9397)(56629)}}{2(.9397)} = 254.8 \text{ kV}_{LN}$$

$$V_{RFL} = 254.8\sqrt{3} = 441.3 \text{ kV}_{LL} = 0.8826 \text{ per unit}$$

The voltage drop limit $\left|V_R/V_S\right| \geq 0.95$ is not satisfied.

At the voltage drop limit:

$$\bar{V}_S = \bar{A}\bar{V}_{RFL} + \bar{B}\bar{I}_{RFL}$$

$$\frac{500}{\sqrt{3}}\angle\delta_S = (0.9694\angle 0.154^\circ)\left(\frac{.95\times 500}{\sqrt{3}}\angle 0^\circ\right) + (69.54\angle 85.15^\circ)(I_{RFL}\angle 8.109^\circ)$$

$$288.675\angle\delta_S = (265.85 - 3.953\, I_{RFL}) + j(0.7146 + 69.43\, I_{RFL})$$

squared magnitudes:

$$83{,}333 = 4836\, I_{RFL}^2 - 2003.\, I_{RFL} + 70677.$$

Solving

$$I_{RFL} = \frac{2003 + \sqrt{(2003)^2 + (4)(4836)(12656)}}{2(4836)} = 1.84\ kA$$

The practical line loadability for this 200 km line is:

$$P_{RFL} = \sqrt{3}\,(.95\times 500)(1.84)(.99) = \underline{1497.\ MW}$$

at $V_{RFL}/V_S = 0.95$ per unit and at 0.99 p.f. leading

(b) $\ell = 600$ km ; CORRESPONDING $\bar{B} = 191.8\angle 85.57^\circ$; $\bar{A} = \bar{D} = 0.7356\angle 1.685^\circ$

$$P_R = \frac{(500)(.95\times 500)}{191.8}\cos(85.57^\circ - 35^\circ) - \frac{(.7356)(.95\times 500)^2}{191.8}\cos(85.57^\circ - 1.685^\circ)$$

$$P_R = 786.5 - 92.2 = \underline{694.3\ MW}$$

The practical line loadability for this 600 km line is 694.3 MW corresponding to the steady-state stability limit

5.48 It is desired to transmit 2200 MW from a power plant to a load center located 300 km from the plant. Determine the number of 60-Hz three-phase, uncompensated transmission lines required to transmit this power with one line out of service for the following cases: (a) 345-kV lines, $Z_c = 300\ \Omega$, (b) 500-kV lines, $Z_c = 275\ \Omega$, (c) 765-kV lines, $Z_c = 260\ \Omega$. Assume that $V_S = 1.0$ per unit, $V_R = 0.95$ per unit, and $\delta_{max} = 35°$.

5.48 (a) $SIL = \frac{(345)^2}{300} = 396.8$ MW

Neglecting losses and using Eq (5.4.29):

$$P = \frac{(1)(.95)(SIL)\sin(35^\circ)}{\sin\left(\frac{2\pi\, 300}{5000}\text{ radians}\right)} = 1.480(SIL) = 1.480(396.8)$$

$P = 587.3$ MW/line

$\#$ 345-kV LINES $= \dfrac{2200}{587.3} + 1 = 3.7 + 1 \simeq 5$ LINES

b) FOR 500kV LINES, $SIL = \dfrac{(500)^2}{275} = 909.1$ MW

$P = 1.48$ SIL

$= 1.48(909.1) = 1345.5$ MW/LINE

$\#$ 500-kV LINES $= \dfrac{2200}{1345.5} + 1 = 1.6 + 1 \simeq 3$ LINES

(c) FOR 765-kV LINES, $SIL = \dfrac{(765)^2}{260} = 2250.9$ MW

$P = 1.48(SIL) = 1.48(2250.9) = 3331.3$ MW/LINE

$\#$ 765-kV LINES $= \dfrac{2200}{3331.3} + 1 = 0.66 + 1 \simeq 2$ LINES