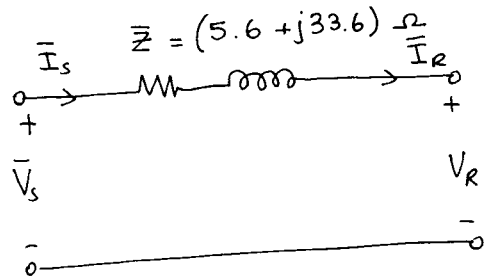


Power System Analysis I *** Midterm II *** Fall**** December 17, 2010

SOLUTIONS

1(35): A 70-km, 230-kV, 60Hz three-phase overhead transmission line has a positive-sequence series impedance of $\bar{z} = 0.08 + j0.48 \Omega/\text{km}$ and positive sequence shunt admittance $\bar{y} = j3.33 \times 10^{-6} \text{ S/km}$. At full load, the line delivers 250 MW at 0.99 power factor lagging and at 220 kV. Using the short line model, calculate the following:

- ABCD parameters of the line
- The sending-end voltage and current
- The percent voltage regulation
- The sending-end real power and reactive power
- The real and reactive power lost in the line.



$$\bar{V}_s = \bar{A} \cdot \bar{V}_R + \bar{B} \cdot \bar{I}_R$$

$$\bar{I}_s = \bar{C} \cdot \bar{V}_R + \bar{D} \cdot \bar{I}_R$$

a) $\bar{A} = \bar{D} = 1.0$ $\bar{C} = 0$ $\bar{B} = \bar{Z} = 34 \angle 80.54^\circ \Omega$

b) $\bar{V}_s = (1.0) \left(\frac{220}{\sqrt{3}} \angle 0^\circ \right) + (34 \angle 80.54^\circ) \bar{I}_R$

let's find $\bar{I}_R$

$$\bar{I}_R = \frac{250 \times 10^6}{\sqrt{3} (220000)} \angle -\cos^{-1}(0.99) = 663 \angle -8.11^\circ \text{ A}$$

$$\bar{V}_s = 127 \text{ kV} + (34 \angle 80.54^\circ) (0.663 \angle -8.11^\circ)$$

$$\bar{V}_s = 127 + 22.542 \angle 72.43^\circ$$

$$\bar{V}_s = 135.52 \angle 9.1^\circ \text{ kV}_{LN}$$

$$\bar{I}_s = \bar{I}_R = 0.663 \angle -8.11^\circ$$

c) $V_{RNL} = \frac{V_s}{A} = \frac{135.52}{1.0} = 135.52 \text{ kV}_{LN}$

$$V_{RFL} = \frac{220}{\sqrt{3}} = 127 \text{ kV}_{LN}$$

$$\%VR = \frac{V_{RNL} - V_{RFL}}{V_{RFL}} = \frac{135.52 - 127}{127} \times 100$$

$$\%VR = 6.7\%$$

d) $\bar{S}_s = 3 \bar{V}_s \bar{I}_s^*$

$$= 3 (135.52 \angle 9.1^\circ) (0.663 \angle 8.11^\circ)$$

$$= 269.55 \angle 17.21^\circ$$

$$= 257.5 + j 79.8$$

$$P_s = 257.5 \text{ MW}$$

$$Q_s = 79.8 \text{ MVar}$$

e) $P_R = 250 \text{ MW}$

$$P_{\text{lost}} = P_s - P_R = 257.5 - 250$$

$$= 7.5 \text{ MW}$$

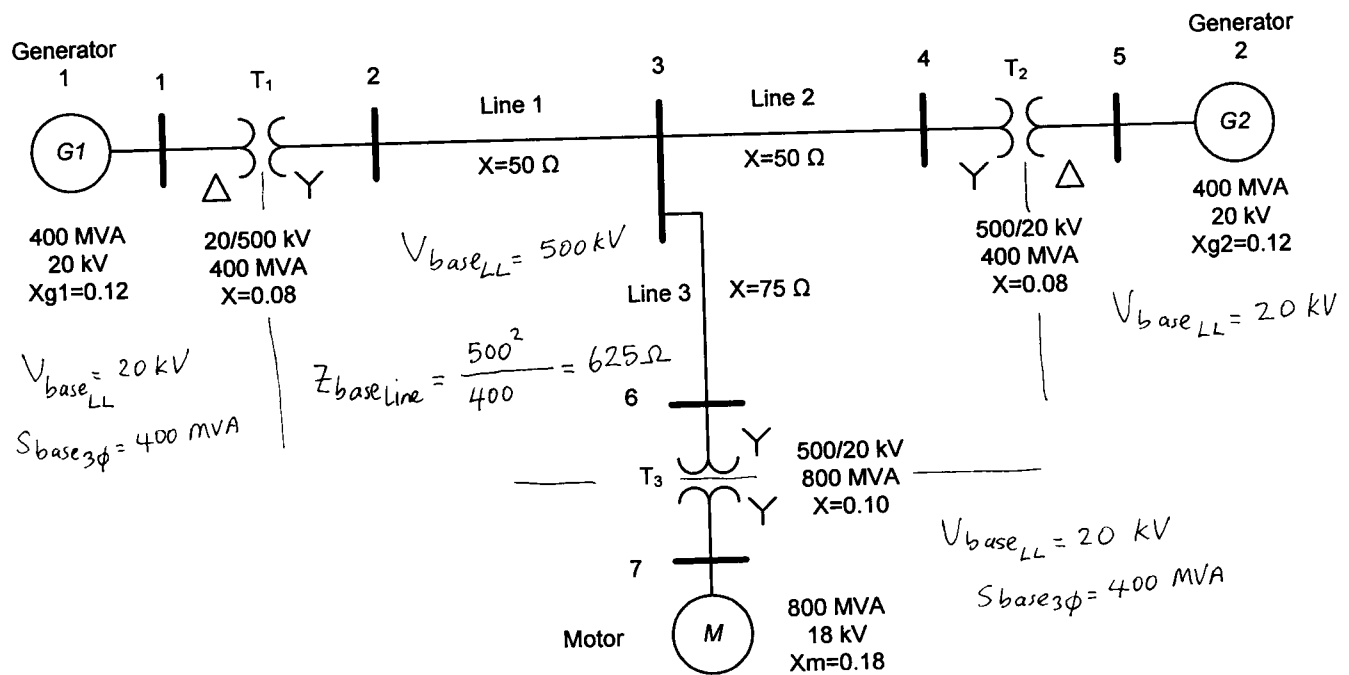
$$Q_R = 250 \tan(8.11^\circ) = 35.62 \text{ MVar}$$

$$Q_{\text{lost}} = 79.8 - 35.62$$

$$= 44.18 \text{ MVar}$$

2(35): The figure below shows the one-line diagram of a three-phase power system.

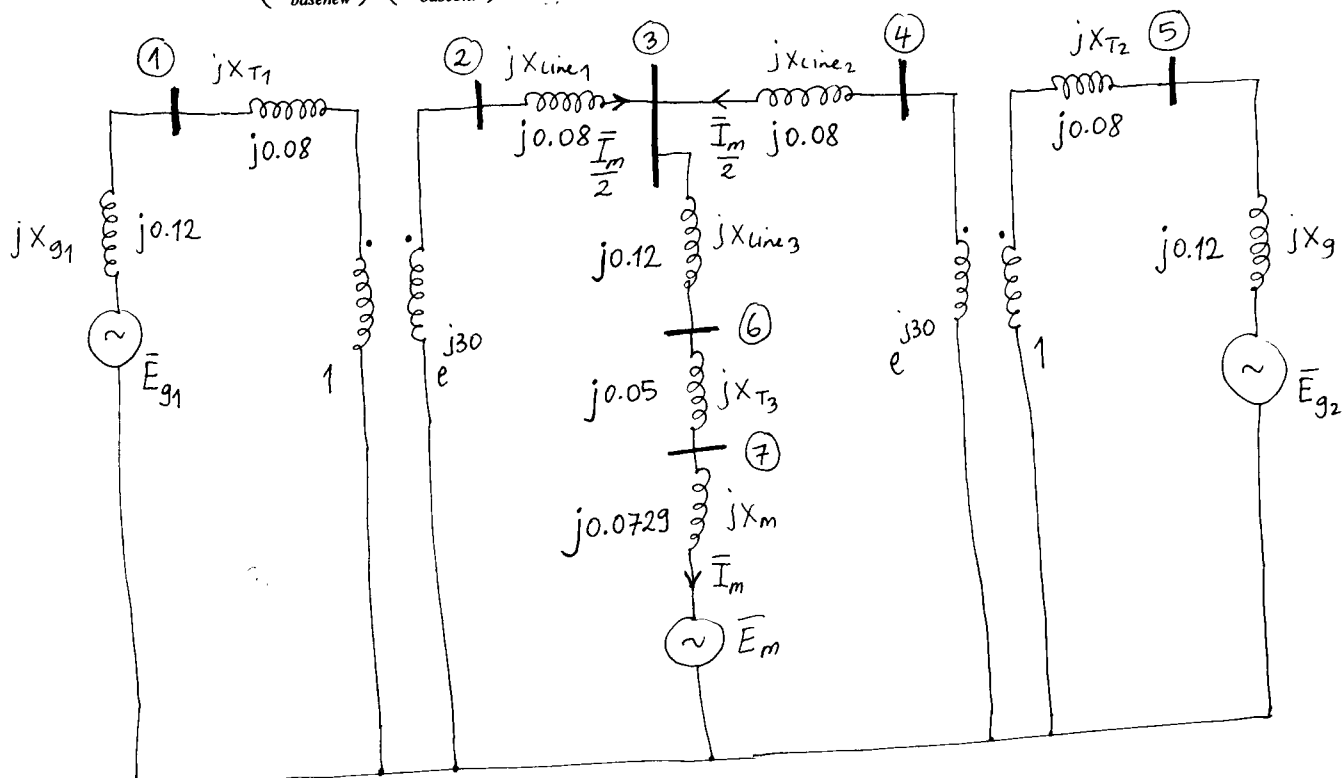
- a) Draw the per-unit impedance diagram of the system including the transformer phase shifts. Use the ratings of the generator 1 as the base values.
- b) The motor is drawing 700 MVA from the system at the rated terminal voltage and at 0.8 power factor lagging. Determine the voltages at bus 1 and bus 5 in per-unit and also in kV. Assume that generators 1 and 2 supply equal real powers and equal reactive powers.



$$Z_{new} = Z_{old} \left(\frac{V_{baseold}}{V_{basenew}} \right)^2 \left(\frac{S_{basenew}}{S_{baseold}} \right)$$

$$I_{base} = \frac{S_{base1\phi}}{V_{baseLN}} = \frac{S_{base3\phi}}{\sqrt{3}V_{baseLL}}$$

$$Z_{base} = \frac{V_{baseLN}}{I_{base}} = \frac{V_{baseLN}^2}{S_{base1\phi}} = \frac{V_{baseLL}^2}{S_{base3\phi}}$$



b) let's first find the ^{per-unit} motor current

$$\bar{I}_m = \frac{700 \times 10^6}{\sqrt{3}(18000)} \angle -\cos^{-1}(0.8) = 22452 \angle -36.87^\circ \text{ A}$$

$$I_{base} = \frac{400 \times 10^6}{\sqrt{3}(20000)} = 11547 \text{ A}$$

$$\bar{I}_{mpu} = 1.9444 \angle -36.87^\circ \text{ pu}$$

$$\bar{V}_7 = 1.0 \text{ pu}$$

$$\bar{V}_3 = \bar{V}_7 + \bar{I}_m (\bar{X}_{T3} + \bar{X}_{line3}) = 1.0 + (1.9444 \angle -36.87^\circ) (0.17 \angle 90^\circ)$$

$$\bar{V}_3 = 1 + 0.33 \angle 53.13^\circ = 1.2272 \angle 12.444^\circ$$

$$\bar{V}_4 = 1.2272 \angle 12.444^\circ + \left(\frac{1.9444 \angle -36.87^\circ}{2} \right) \times (0.08 \angle 90^\circ)$$

$$\bar{V}_4 = 1.2272 \angle 12.444^\circ + 0.0778 \angle 53.13^\circ$$

$$\bar{V}_4 = 1.2872 \angle 14.7^\circ \text{ pu}$$

$$\bar{V}_5 = 1.2872 \angle -15.3^\circ \quad \bar{V}_5 = 25744 \angle -15.3^\circ \text{ Volt}$$

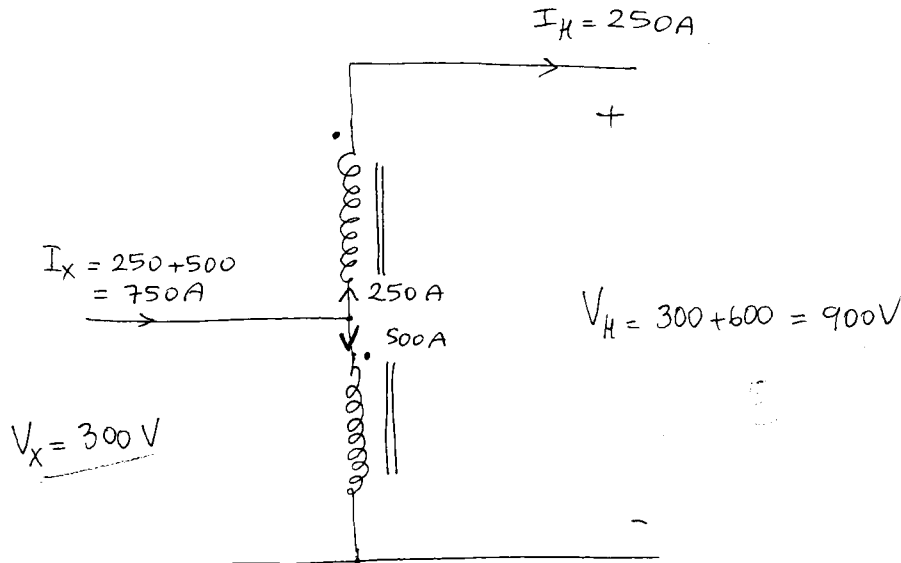
$$\bar{V}_1 \text{ is also equal to } \bar{V}_5$$

3(30): A single-phase two-winding transformer has the following ratings:
 150 kVA, 300 V/600 V, and a total series leakage reactance of 8.4%.

If the given transformer is connected as a step-up autotransformer,

- Determine the voltage and power rating as an autotransformer
- Draw the circuit schematic of the autotransformer.
- Find the per-unit leakage reactance for the high voltage terminal as an autotransformer.

a)



$$S_X = S_H = 750 \times 300 = 250 \times 900$$

$$S_X = S_H = 225 \text{ kVA}$$

$$V_H = 300 + 600 = 900 \text{ V}$$

b)

c) let's first find the actual leakage reactance of the single-phase two-winding transformer.

$$Z_{\text{base old}} = \frac{600^2}{150000} = 2.4 \Omega$$

$$X_{eq} = (0.084)(2.4) = 0.2016 \Omega$$

$$Z_{\text{base H}} = \frac{900^2}{225000} = 3.6 \Omega$$

$$X_{eq} = \left(\frac{0.2016 \Omega}{3.6 \Omega} \right) = 0.056 = 5.6\% \text{ after (as an autotransformer)}$$