

Power System Analysis I

Sample Problems

- 5.14** A 500-km, 500-kV, 60-Hz uncompensated three-phase line has a positive-sequence series impedance $z = 0.03 + j0.35 \Omega/\text{km}$ and a positive-sequence shunt admittance $y = j4.4 \times 10^{-6} \text{ S/km}$. Calculate: (a) Z_c , (b) (γ/l) , and (c) the exact $ABCD$ parameters for this line.

$$\begin{aligned} \text{5.14} \quad \bar{Z}_c &= \sqrt{\frac{z}{y}} = \sqrt{\frac{0.03 + j0.35}{j4.4 \times 10^{-6}}} = \sqrt{\frac{0.3513 \angle 85.10^\circ}{4.4 \times 10^{-6} \angle 90^\circ}} \\ \text{(a)} \quad \bar{Z}_c &= \sqrt{79837 \angle -4.899^\circ} = \underline{282.6 \angle -2.450^\circ \Omega} \end{aligned}$$

$$\text{(b)} \quad \bar{\gamma}l = \sqrt{\bar{zy}'}(l) = \sqrt{(0.3513 \angle 85.10^\circ)(4.4 \times 10^{-6} \angle 90^\circ)}(500)$$

$$\bar{\gamma}l = \underline{0.6216 \angle 87.55^\circ} = 0.02657 + j0.62105$$

$$\begin{aligned} \text{(c)} \quad \bar{A} = \bar{D} &= \cosh(\bar{\gamma}l) = \cosh(0.02657 + j0.62105) \\ &= \cosh(0.02657) \cos(0.62105) + j \sinh(0.02657) \sin(0.62105) \\ &\quad \text{radians} \\ &= (1.000353)(0.813268) + j(0.0265731)(0.581889) \\ &= 0.813555 + j0.015463 = \underline{0.8137 \angle 1.089^\circ} \\ &\quad \text{per unit} \end{aligned}$$

$$\begin{aligned} \sinh(\bar{\gamma}l) &= \sinh(0.02657 + j0.62105) \text{ radians} \\ &= \sinh(0.02657) \cos(0.62105) + j \cosh(0.02657) \sin(0.62105) \\ &= (0.02657)(0.81327) + j(1.000353)(0.581889) \\ &= 0.021609 + j0.582094 = \underline{0.5825 \angle 87.87^\circ} \end{aligned}$$

$$\bar{B} = \bar{Z}_c \sinh(\bar{\gamma}l) = (282.6 \angle -2.450^\circ)(0.5825 \angle 87.87^\circ)$$

$$\bar{B} = \underline{164.6 \angle 85.42^\circ \Omega}$$

$$\bar{C} = \left(\frac{1}{\bar{Z}_c}\right) \sinh(\bar{\gamma}l) = \frac{0.5825 \angle 87.87^\circ}{282.6 \angle -2.450^\circ}$$

$$\bar{C} = \underline{2.061 \times 10^{-3} \angle 90.32^\circ \text{ S}}$$

- 5.15** At full load the line in Problem 5.14 delivers 1000 MW at unity power factor and at 480 kV. Calculate: (a) the sending-end voltage, (b) the sending-end current, (c) the sending-end power factor, (d) the full-load line losses, and (e) the percent voltage regulation.

$$\underline{\bar{V}}_R = \frac{480}{\sqrt{3}} \angle 0^\circ = 277.1 \angle 0^\circ \text{ kV}_{LN}$$

$$\underline{\bar{I}}_R = \frac{P_R}{\sqrt{3} V_{RLL} (\text{pf})} \angle 0^\circ = \frac{1000 \angle 0^\circ}{\sqrt{3} 480 (1)} = 1.202 \angle 0^\circ \text{ kA}$$

$$\begin{aligned} \text{(a)} \quad \underline{\bar{V}}_S &= \underline{\bar{A}} \underline{\bar{V}}_R + \underline{\bar{B}} \underline{\bar{I}}_R = 0.8137 \angle 1.089^\circ (277.1) + 164.6 \angle 85.42^\circ (1.202) \\ &= 314.4 \angle 39.9^\circ \text{ kV}_{LN} ; V_S = 314.4 \sqrt{3} = 544.5 \text{ kV}_{LL} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \underline{\bar{I}}_S &= \underline{\bar{C}} \underline{\bar{V}}_R + \underline{\bar{D}} \underline{\bar{I}}_R = 2.061 \times 10^{-3} \angle 90.32^\circ (277.1) + 0.8137 \angle 1.089^\circ (1.202) \\ &= 1.139 \angle 31.17^\circ \text{ kA} ; I_S = 1.139 \text{ kA} \end{aligned}$$

$$\text{(c)} \quad (\text{pf})_S = \cos(39.9^\circ - 31.17^\circ) = \cos(8.73^\circ) = 0.9884 \text{ LAGGING}$$

$$\begin{aligned} \text{(d)} \quad P_S &= \sqrt{3} V_{SLL} I_S (\text{pf})_S = \sqrt{3} 544.5 (1.139) 0.9884 \\ &= 1061.7 \text{ MW} \end{aligned}$$

$$\text{FULL-LOAD LINE LOSSES} = P_S - P_R = 1061.7 - 1000 = 61.7 \text{ MW}$$

$$\text{(e)} \quad V_{RNL} = V_S / A = 544.5 / 0.8137 = 669.2 \text{ kV}$$

$$\begin{aligned} \% VR &= \frac{V_{RNL} - V_{RFL}}{V_{RFL}} \times 100 \\ &= \frac{669.2 - 480}{480} \times 100 = 39.4\% \end{aligned}$$

- 5.26** A 320-km 500-kV, 60-Hz three-phase uncompensated line has a positive-sequence series reactance $x = 0.34 \Omega/\text{km}$ and a positive-sequence shunt admittance $y = j4.5 \times 10^{-6} \text{ S/km}$. Neglecting losses, calculate: (a) Z_c , (b) (γl) , (c) the $ABCD$ parameters, (d) the wavelength λ of the line, in kilometers, and (e) the surge impedance loading in MW.

5.26

$$(a) \quad \bar{Z}_c = \sqrt{\frac{\bar{Z}_{12}}{\bar{Y}_{12}}} = \sqrt{\frac{j0.34}{j4.5 \times 10^{-6}}} = 274.9 \Omega$$

$$(b) \quad \bar{V}l = \sqrt{\bar{Z}\bar{Y}}l = \sqrt{(j0.34)(j4.5 \times 10^{-6})}(320) = j0.3958 \text{ pu}$$

$$(c) \quad \bar{V}l = j\beta l \quad ; \quad \beta l = 0.3958 \text{ pu}$$

$$\bar{A} = \bar{D} = \cos \beta l = \cos(0.3958 \text{ radians}) = 0.9227 \angle 0^\circ \text{ pu}$$

$$\begin{aligned} \bar{B} &= j\bar{Z}_c \sin \beta l = j(274.9) \sin(0.3958 \text{ radians}) \\ &= j108.81 \Omega \end{aligned}$$

$$\begin{aligned} \bar{C} &= j\left(\frac{1}{\bar{Z}_c}\right) \sin \beta l = j \frac{1}{274.9} \sin(0.3958 \text{ radians}) \\ &= j1.44 \times 10^{-3} \text{ S} \end{aligned}$$

$$(d) \quad \beta = \beta l / l = 0.3958 / 320 = 1.319 \times 10^{-3} \text{ radians/km}$$

$$\lambda = 2\pi / \beta = 4766 \text{ km}$$

$$(e) \quad \text{SIL} = \frac{V_{\text{rated L-L}}^2}{\bar{Z}_c} = \frac{(500)^2}{274.9} = 909.4 \text{ MW (3}\phi\text{)}$$

5.27 Determine the equivalent π circuit for the line in Problem 5.26.

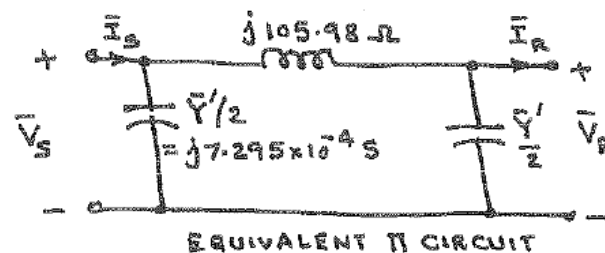
5.27

$$\bar{Z}' = \bar{B} = j 108.81 \Omega$$

ALTERNATIVELY:

$$\begin{aligned}\bar{Z}' = \bar{Z} \bar{F}_1 &= \left(\frac{Z}{j}\right) \frac{\sin \beta l}{\beta l} = (j 0.34 \times 320) \frac{\sin(0.3958 \text{ radians})}{0.3958} \\ &= j 108.8 (0.9741) = j 105.98 \Omega\end{aligned}$$

$$\begin{aligned}\frac{\bar{Y}'}{2} = \frac{\bar{Y}}{2} \bar{F}_2 &= \left(\frac{Y}{2}\right) \frac{\tan(\beta l/2)}{\beta l/2} = j \frac{4.5 \times 10^{-6}}{2} \times 320 \frac{\tan(0.1979 \text{ radians})}{0.1979} \\ &= j 7.2 \times 10^{-4} (1.0133) = j 7.295 \times 10^{-4} \text{ S}\end{aligned}$$



- 5.28** Rated line voltage is applied to the sending end of the line in Problem 5.26. Calculate the receiving-end voltage when the receiving end is terminated by (a) an open circuit, (b) the surge impedance of the line, and (c) one-half of the surge impedance. (d) Also calculate the theoretical maximum real power that the line can deliver when rated voltage is applied to both ends of the line.

5.28

$$(a) \quad V_R = V_S / A = 500 / 0.9227 = 541.9 \text{ kV}$$

$$(b) \quad V_R = V_S = 500 \text{ kV}$$

$$\begin{aligned}(c) \quad \bar{V}_S &= \cos \beta l \bar{V}_R + j Z_c \sin \beta l \left[\bar{V}_R / \left(\frac{1}{2} Z_c \right) \right] \\ &= [\cos \beta l + j 2 \sin \beta l] \bar{V}_R \\ V_S &= |\cos \beta l + j 2 \sin \beta l| V_R \\ &= \frac{500}{|\cos 0.3958 \text{ rad} + j 2 \sin 0.3958 \text{ rad}|} = \frac{500}{1.202} = 416 \text{ kV}\end{aligned}$$

$$(d) \quad P_{\max 3\phi} = \frac{V_S V_R}{X'} = \frac{500 \times 500}{105.98} = 2359 \text{ MW}$$

- 5.31 A 500-kV, 300-km, 60-Hz three-phase overhead transmission line, assumed to be lossless, has a series inductance of 0.97 mH/km per phase and a shunt capacitance of 0.115 μ F/km per phase. (a) Determine the phase constant β , the surge impedance Z_c , velocity of propagation v , and the wavelength λ of the line. (b) Determine the voltage, current, real and reactive power at the sending end, and the percent voltage regulation of the line if the receiving-end load is 800 MW at 0.8 power factor lagging and at 500 kV.

5.31

$$(a) \text{ FOR A LOSSLESS LINE, } \beta = \omega \sqrt{LC} = 2\pi(60) \sqrt{0.97 \times 0.115 \times 10^{-9}} \\ = 0.001259 \text{ rad/km}$$

$$\bar{Z}_c = \sqrt{L/C} = \sqrt{\frac{0.97 \times 10^{-3}}{0.115 \times 10^{-6}}} = 290.43 \Omega$$

$$\text{VELOCITY OF PROPAGATION } v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.97 \times 0.115 \times 10^{-9}}} = 2.994 \times 10^5 \text{ km/s}$$

$$\text{AND THE LINE WAVE LENGTH IS } \lambda = v/f = \frac{1}{60} (2.994 \times 10^5) = 4990 \text{ km}$$

$$(b) \bar{V}_R = \frac{500}{\sqrt{3}} \angle 0^\circ \text{ kV} = 288.675 \angle 0^\circ \text{ kV}$$

$$\bar{S}_{R(3\phi)} = \frac{800}{0.8} \angle \cos^{-1} 0.8 = 800 + j600 \text{ MVA} = 1000 \angle 36.87^\circ \text{ MVA}$$

$$\bar{I}_R = \bar{S}_{R(3\phi)}^* / 3\bar{V}_R^* = \frac{(1000 \angle -36.87^\circ) 10^3}{3 \times 288.675 \angle 0^\circ} = 1154.7 \angle -36.87^\circ \text{ A}$$

$$\text{SENDING END VOLTAGE } \bar{V}_s = \cos \beta l \bar{V}_R + j \bar{Z}_c \sin \beta l \bar{I}_R$$

$$\beta l = 0.001259 \times 300 = 0.3777 \text{ rad} = 21.641^\circ$$

$$\therefore \bar{V}_s = 0.9295 (288.675 \angle 0^\circ) + j(290.43) 0.3688 (1154.7 \angle -36.87^\circ) (10^3) \\ = 356.53 \angle 16.1^\circ \text{ kV}$$

$$\text{SENDING END LINE-TO-LINE VOLTAGE MAGNITUDE} = \sqrt{3} \times 356.53$$

$$= 617.53 \text{ kV}$$

$$\bar{I}_s = j \frac{1}{\bar{Z}_c} \sin \beta l \bar{V}_R + \cos \beta l \bar{I}_R$$

$$= j \frac{1}{290.43} 0.3688 (288.675 \angle 0^\circ) 10^3 + 0.9295 (1154.7 \angle -36.87^\circ)$$

$$= 902.3 \angle -17.9^\circ \text{ A} ; \text{ LINE CURRENT} = 902.3 \text{ A}$$

$$\bar{S}_{S(3\phi)} = 3\bar{V}_s \bar{I}_s^* = 3(356.53 \angle 16.1^\circ) (902.3 \angle -17.9^\circ) 10^3$$

$$= 800 \text{ MW} + j539.672 \text{ MVAR}$$

$$\text{PERCENT VOLTAGE REGULATION} = \frac{(356.53 / 0.9295) - 288.675}{288.675} \times 100$$

$$= 32.87\%$$

- 5.32** The following parameters are based on a preliminary line design: $V_S = 1.0$ per unit, $V_R = 0.9$ per unit; $\lambda = 5000$ km, $Z_C = 320 \Omega$, $\delta = 36.8^\circ$. A three-phase power of 700 MW is to be transmitted to a substation located 315 km from the source of power. (a) Determine a nominal voltage level for the three-phase transmission line, based on the practical line-loadability equation. (b) For the voltage level obtained in (a), determine the theoretical maximum power that can be transferred by the line.

Answers:

- a) 400 kVLN
b) 1167 MW

- 5.38** The line in Problem 5.14 has three ACSR 1113-kcmil conductors per phase. Calculate the theoretical maximum real power that this line can deliver and compare with the thermal limit of the line. Assume $V_S = V_R = 1.0$ per unit and unity power factor at the receiving end.

5.38

From Problem 5.14

$$\bar{A} = 0.8137 \angle 1.089^\circ \text{ per unit} \quad A = 0.8137 \quad \theta_A = 1.089^\circ$$

$$\bar{B} = \bar{Z}' = 164.6 \angle 85.42^\circ \Omega \quad Z' = 164.6 \Omega \quad \theta_Z = 85.42^\circ$$

Using Eq (5.5.6)

$$P_{R \max} = \frac{(500)(500)}{164.6} - \frac{(0.8137)(500)^2}{164.6} \cos(85.42^\circ - 1.089^\circ)$$

$$P_{R \max} = 1518.8 - 122.1 = \underline{1397 \text{ MW}} \quad (3\text{-phase})$$

For this loading at unity power factor:

$$I_R = \frac{P_{R \max}}{\sqrt{3} V_{RL} (\text{p.f.})} = \frac{1397}{\sqrt{3} (500) (1.0)} = 1.613 \text{ kA/phase}$$

From Table A.3, the thermal limit for three ACSR 1113 kcmil conductors is $3(1.11) = 3.33$ kA/phase. The current 1.613 kA corresponding to the theoretical steady-state stability limit is well below the thermal limit of 3.33 kA.

- 5.45** For the line in Problems 5.14 and 5.38, determine: (a) the practical line loadability in MW, assuming $V_S = 1.0$ per unit, $V_R \approx 0.95$ per unit, and $\delta_{\max} = 35^\circ$; (b) the full-load current at 0.99 p.f. leading, based on the above practical line loadability; (c) the exact receiving-end voltage for the full-load current in (b) above; and (d) the percent voltage regulation. For this line, is loadability determined by the thermal limit, the voltage-drop limit, or steady-state stability?

5.45 (a) Using Eq. (5.5.3) with $\delta = 35^\circ$:

$$P_R = \frac{(500)(.95 \times 500)}{164.6} \cos(85.42^\circ - 35^\circ) - \frac{(.8137)(.95 \times 500)^2}{164.6} \cos(85.42^\circ - 1.0^\circ)$$

$$P_R = 919.3 - 110.2 = \underline{809.1 \text{ MW}} \text{ (three-phase)}$$

$P_R = 809.1 \text{ MW}$ is the practical line loadability provided that the voltage drop limit and thermal limit are not exceeded.

$$(b) \quad I_{RFL} = \frac{P_R}{\sqrt{3} V_{RFL} (\text{p.f.})} = \frac{809}{\sqrt{3} (.95 \times 500) (0.99)} = \underline{0.993 \text{ kA}}$$

$$(c) \quad \bar{V}_S = \bar{A} \bar{V}_{RFL} + \bar{B} \bar{I}_{RFL}$$

$$\frac{500}{\sqrt{3}} \angle 0^\circ = (0.8137 \angle 1.089^\circ) (V_{RFL} \angle 0^\circ) + (164.6 \angle 85.42^\circ) (0.993 \angle 8.11^\circ)$$

$$288.68 \angle 0^\circ = 0.8137 V_{RFL} \angle 1.089^\circ + 163.45 \angle 93.53^\circ$$

$$288.68 \angle 0^\circ = (0.8136 V_{RFL} - 10.06) + j(0.01546 V_{RFL} + 163.14)$$

Taking the squared magnitude of the above equation:

$$83,333. = 0.6622 V_{RFL}^2 - 11.33 V_{RFL} + 26716.$$

Solving the above quadratic equation:

$$V_{RFL} = \frac{11.33 + \sqrt{(11.33)^2 + 4(0.6622)(56617)}}{2(0.6622)} = 301.1 \text{ kV}_{LL}$$

$$V_{RFL} = 301.1 \sqrt{3} = \underline{521.5 \text{ kV}_{LL}} = 1.043 \text{ per unit}$$

$$(d) \quad V_{RNL} = V_S / A = \frac{500}{0.8137} = 614.5 \text{ kV}_{LL}$$

$$\% \text{V.R.} = \frac{614.5 - 521.5}{521.5} \times 100 = \underline{17.8\%}$$

(e)

From Problem 5.27, the thermal limit is 3.33 kA. Since $V_{RFL}/V_S = 521.5/500 = 1.043$ is greater than 0.95 and the thermal limit = 3.33 kA is greater than $I_{RFL} = 0.993$ kA, the voltage drop limit and thermal limit are not exceeded at $P_R = 809$ MW. therefore, loadability is determined by stability.

5.47 Determine the practical line loadability in MW and in per-unit of SIL for the line in Problem 5.14 if the line length is (a) 200 km, (b) 600 km. Assume $V_S = 1.0$ per unit, $V_R = 0.95$ per unit, $\delta_{\max} = 35^\circ$, and 0.99 leading power factor at the receiving end.

5.47 (a) $l = 200$ km. The steady-state stability limit is:

$$P_R = \frac{(500)(.95)(500)}{69.54} \cos(85.15^\circ - 35^\circ) - \frac{(.9694)(.95 \times 500)^2}{69.54} \cos(85.15^\circ - 0.154^\circ)$$

$$P_R = 2188.27\% = 1914 \text{ MW}$$

$$I_{RFL} = \frac{P_R}{\sqrt{3} V_{RFL} (\text{P.F.})} = \frac{1914}{(\sqrt{3})(.95 \times 500)(.99)} = 2.35 \text{ kA}$$

$$\bar{V}_S = \bar{A} \bar{V}_{RFL} + \bar{B} \bar{I}_{RFL}$$

$$\frac{500}{\sqrt{3}} \angle 85 = (.9694 / 0.154^\circ)(V_{RFL} \angle 0^\circ) + (69.54 / 85.15^\circ)(2.35 / 8.11^\circ)$$

$$288.675 \angle 85 = (.9694 V_{RFL} - 9.293) + j(0.0026 V_{RFL} + 163.15)$$

Taking the squared magnitude:

$$83,333. = .9397 V_{RFL}^2 - 17.17 V_{RFL} + 26704.$$

Solving

$$V_{RFL} = \frac{17.17 + \sqrt{(17.17)^2 + 4(.9397)(56629)}}{2(.9397)} = 254.8 \text{ kV}_{LL}$$

$$V_{RFL} = 254.8 \sqrt{3} = 441.3 \text{ kV}_{LL} = 0.8826 \text{ per unit}$$

The voltage drop limit $|V_R/V_S| \geq 0.95$ is not satisfied.
At the voltage drop limit:

$$\bar{V}_S = \bar{A} \bar{V}_{RFL} + \bar{B} \bar{I}_{RFL}$$

$$\frac{500}{\sqrt{3}} \angle \delta_s = \left(\frac{0.9694}{0.154^\circ} \right) \left(\frac{0.95 \times 500}{\sqrt{3}} \angle 0^\circ \right) + \left(\frac{69.54}{85.15^\circ} \right) (I_{RFL} \angle 8.109^\circ)$$

$$288.675 / \delta_s = (265.85 - 3.953 I_{RFL}) + j(0.7146 + 69.43 I_{RFL})$$

Squared magnitudes:

$$83,333 = 4836 I_{RFL}^2 - 2003 I_{RFL} + 70677.$$

Solving

$$I_{RFL} = \frac{2003 + \sqrt{(2003)^2 + (4)(4836)(12656)}}{2(4836)} = 1.84 \text{ kA}$$

The practical line loadability for this 200 km line is:

$$P_{RFL} = \sqrt{3} (0.95 \times 500) (1.84) (0.99) = \underline{1497 \text{ MW}}$$

at $V_{RFL}/V_s = 0.95$ per unit and at 0.99 p.f. leading

(b) $l = 600 \text{ km}$; corresponding $\bar{B} = 191.8 \angle 85.57^\circ$; $\bar{A} \cdot \bar{D} = 0.7356 \angle 1.685^\circ$

$$P_R = \frac{(500)(0.95 \times 500) \cos(85.57^\circ - 35^\circ)}{191.8} - \frac{(0.7356)(0.95 \times 500)^2}{191.8} \cos(85.57^\circ - 1.685^\circ)$$

$$P_R = 786.5 - 92.2 = \underline{694.3 \text{ MW}}$$

The practical line loadability for this 600 km line is 694.3 MW corresponding to the steady-state stability limit

- 5.48** It is desired to transmit 2200 MW from a power plant to a load center located 300 km from the plant. Determine the number of 60-Hz three-phase, uncompensated transmission lines required to transmit this power with one line out of service for the following cases: (a) 345-kV lines, $Z_c = 300 \Omega$, (b) 500-kV lines, $Z_c = 275 \Omega$, (c) 765-kV lines, $Z_c = 260 \Omega$. Assume that $V_s = 1.0$ per unit, $V_R = 0.95$ per unit, and $\delta_{\max} = 35^\circ$.

5.48 (a) $SIL = \frac{(345)^2}{300} = 396.8 \text{ MW}$

Neglecting losses and using Eq (5.4.29):

$$P = \frac{(1)(0.95)(SIL) \sin(35^\circ)}{\sin\left(\frac{2\pi \cdot 300}{5000} \text{ radians}\right)} = 1.480(SIL) = 1.480(396.8)$$

$$P = 587.3 \text{ MW/line}$$

$$\# \text{ 345-kV LINES} = \frac{2200}{587.3} + 1 = 3.7 + 1 \approx 5 \text{ LINES}$$

b) FOR 500-kV LINES, $SIL = \frac{(500)^2}{275} = 909.1 \text{ MW}$

$$P = 1.48 \text{ SIL}$$

$$= 1.48 (909.1) = 1345.5 \text{ MW/LINE}$$

$$\# \text{ 500-kV LINES} = \frac{2200}{1345.5} + 1 = 1.6 + 1 \approx 3 \text{ LINES}$$

(c) FOR 765-kV LINES, $SIL = \frac{(765)^2}{260} = 2250.9 \text{ MW}$

$$P = 1.48 (SIL) = 1.48 (2250.9) = 3331.3 \text{ MW/LINE}$$

$$\# \text{ 765-kV LINES} = \frac{2200}{3331.3} + 1 = 0.66 + 1 \approx 2 \text{ LINES}$$