Power System Analysis II

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Solution to a sample problem

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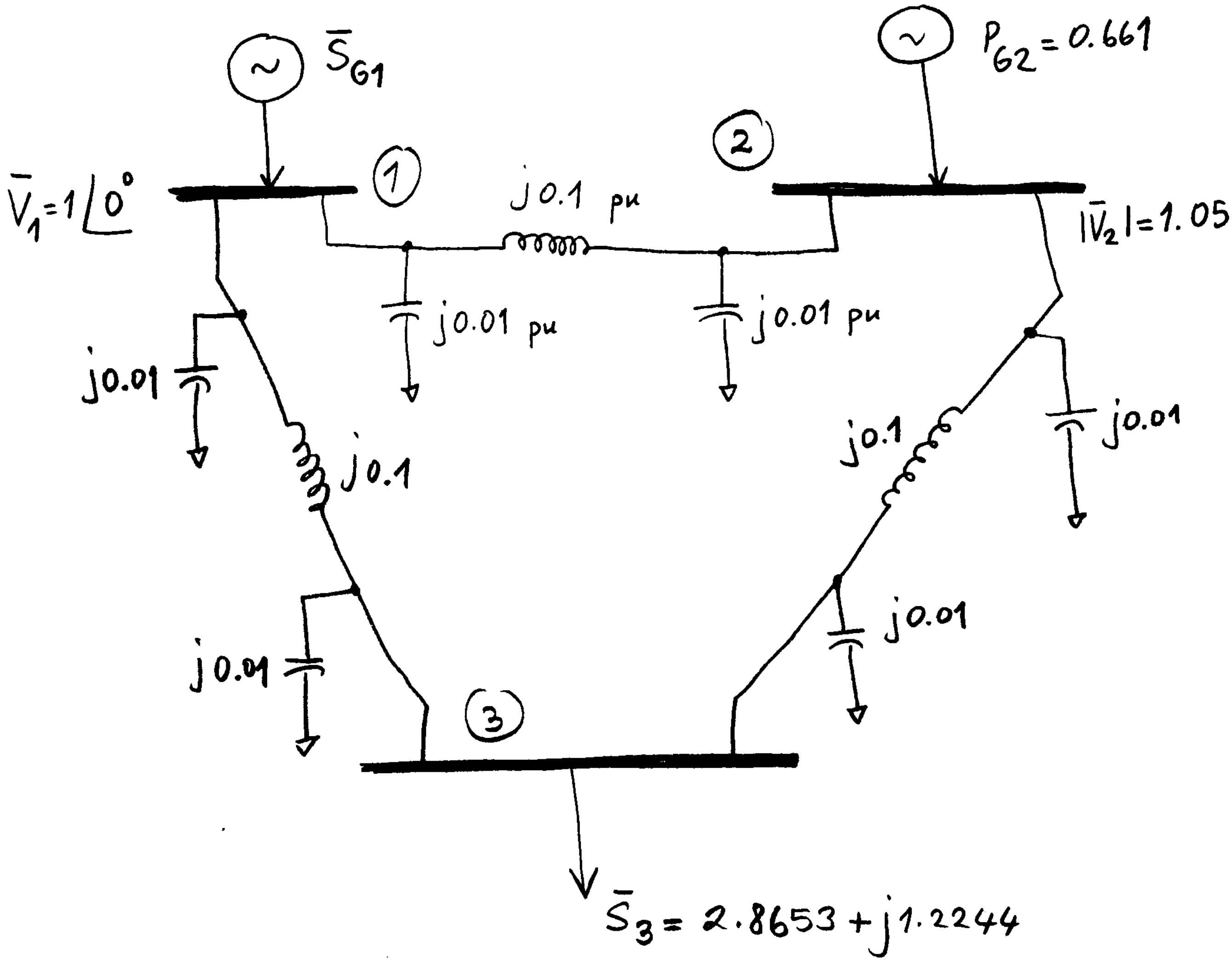
Determine $\delta_2(1)$, $|V_3(1)|$, $\delta_3(1)$, $P_{G1}(1)$, $Q_{G1}(1)$ and $Q_{G2}(1)$ for the power system shown below using Newton-Raphson method.



Take $\bar{V}_3(0) = 1\angle 0°$ and $\delta_2 = 0°$ as the initial guess.

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First, let's determine the input data and unknowns for each bus.

Bus		INPUT	UNKNOWNS	Q_{Gmax}	Q_{Gmin}
1	Swing bus	$V_1 = 1.0$ $\delta_1 = 0°$	P_1 Q_1	—	—
2	Constant voltage bus	$V_2 = 1.05$ $P_2 = 0.661$	δ_2 Q_2	+10 pu	-5 pu
3	Load bus	$P_3 = -2.8653$ $Q_3 = -1.2244$	V_3 δ_3	—	—

Now let's determine $\bar{Y}_{bus}$ for the system.

$$\bar{Y}_{11} = \bar{Y}_{22} = \bar{Y}_{33} = \left(\frac{1}{j0.1} + j0.01 + \frac{1}{j0.1} + j0.01 \right) = -j\,19.98$$

$$= 19.98\angle{-90°} \text{ pu}$$

$$\bar{Y}_{12} = \bar{Y}_{13} = \bar{Y}_{21} = \bar{Y}_{23} = \bar{Y}_{31} = \bar{Y}_{32} = -\left(\frac{1}{j0.1}\right) = +j10 = 10\angle{90°} \text{ pu}$$

Bus 2 is a voltage controlled bus.

V_2 is already known and function $Q_2(X)$ is not needed. So, we will omit the column from the Jacobian Matrix that corresponds to partial derivatives with respect to V_2 and the row corresponding to partial derivatives of $Q_2(x)$.

But, at the end of each iteration, we will compute $Q_2(x)$ using

$$Q_2(x) = V_2 \sum_{n=1}^{N} Y_{2n} V_n \sin(\delta_2 - \delta_n - \theta_{2n}) \qquad \text{where } N = 3$$

and $Q_{G2} = Q_2(x) + Q_{L2}$. If Q_{G2} exceeds its limits, then we will change the bus type to a load bus with Q_{G2} set to its limit value. The power flow program then computes a new value for V_2.

STEP 1

$$P_2(x) = V_2 \left\{ Y_{21} V_1 \cos(\delta_2 - \delta_1 - \theta_{21}) + Y_{22} V_2 \cos(-\theta_{22}) + Y_{23} V_3 \cos(\delta_2 - \delta_3 - \theta_{23}) \right\}$$

$$P_3(x) = V_3 \left\{ Y_{31} V_1 \cos(\delta_3 - \delta_1 - \theta_{31}) + Y_{32} V_2 \cos(\delta_3 - \delta_2 - \theta_{32}) + Y_{33} V_3 \cos(-\theta_{33}) \right\}$$

$$Q_3(x) = V_3 \left\{ Y_{31} V_1 \sin(\delta_3 - \delta_1 - \theta_{31}) + Y_{32} V_2 \sin(\delta_3 - \delta_2 - \theta_{32}) + Y_{33} V_3 \sin(-\theta_{33}) \right\}$$

$$P_2[x(0)] = 1.05 \left[10(1)\cos(-90) + 19.98(1.05)\cos(90) + 10(1.0)\cos(-90) \right]$$

$$= 0$$

$$P_3[x(0)] = 0$$

$$Q_3[x(0)] = (1.0)\left[10(1.0)\sin(-90) + 10(1.05)\sin(-90) + 19.98(1.0)\sin(90) \right]$$

$$= -0.52 \text{ pu}$$

$$\Delta P_2(0) = P_2 - P_2[x(0)] = 0.661 - 0 = 0.661 \quad pu$$

$$\Delta P_3(0) = -2.8653 - 0 = -2.8653 \quad pu$$

$$\Delta Q_3(0) = -1.2244 - (-0.52) = -0.7044 \quad pu$$

STEP 2

$$\frac{\partial P_2}{\partial \delta_2} = -V_2 Y_{21} V_1 \sin(\delta_2 - \delta_1 - \theta_{21}) - V_2 Y_{23} V_3 \sin(\delta_2 - \delta_3 - \theta_{23})$$

$$= -(1.05)(10)\sin(-90) - (1.05)(10)\sin(-90)$$

$$= +10.5 + 10.5 = +21$$

$$\frac{\partial P_2}{\partial \delta_3} = +V_2 Y_{23} V_3 \sin(\delta_2 - \delta_3 - \theta_{23})$$

$$= (1.05)(10)\sin(-90^\circ)$$

$$= -10.5$$

$$\frac{\partial P_2}{\partial V_3} = V_2 Y_{23} \cos(\delta_2 - \delta_3 - \theta_{23})$$

$$= 0$$

$$\frac{\partial P_3}{\partial \delta_2} = V_3 Y_{32} V_2 \sin(\delta_3 - \delta_2 - \theta_{32})$$

$$= (10)(1.05)\sin(-90)$$

$$= -10.5$$

$$\frac{\partial P_3}{\partial \delta_3} = -V_3 Y_{31} V_1 \sin(\delta_3 - \delta_1 - \theta_{31}) - V_3 Y_{32} V_2 \sin(\delta_3 - \delta_2 - \theta_{32})$$

$$= -(10)\sin(-90) - (10)(1.05)\sin(-90) = 20.5$$

$$\frac{\partial P_3}{\partial V_3} = Y_{31}V_1 \cos(-\theta_{31}) + Y_{32}V_2 \cos(-\theta_{32}) + 2Y_{33}V_3 \cos(-\theta_{33})$$

$$= 0$$

$$\frac{\partial Q_3}{\partial \delta_2} = -V_3 Y_{32} V_2 \cos(-\theta_{32})$$

$$= 0$$

$$\frac{\partial Q_3}{\partial \delta_3} = 0$$

$$\frac{\partial Q_3}{\partial V_3} = Y_{31}V_1 \sin(-\theta_{31}) + Y_{32}V_2 \sin(-\theta_{32}) + 2Y_{33}V_3 \sin(-\theta_{33})$$

$$= (10)\sin(-90) + (10)(1.05)\sin(-90) + 2(19.98)\sin(+90)$$

$$= -10 - 10.5 + 39.96$$

$$= 19.46$$

$$J(i) = \begin{bmatrix} 21 & -10.5 & 0 \\ -10.5 & 20.5 & 0 \\ 0 & 0 & 19.46 \end{bmatrix}$$

STEP 3

$$\begin{bmatrix} 21 & -10.5 & 0 \\ -10.5 & 20.5 & 0 \\ 0 & 0 & 19.46 \end{bmatrix} \begin{bmatrix} \Delta\delta_2(0) \\ \Delta\delta_3(0) \\ \Delta V_3(0) \end{bmatrix} = \begin{bmatrix} 0.661 \\ -2.8653 \\ -0.7044 \end{bmatrix}$$

after solving

$\Delta\delta_2(0) = -0.051632$

$\Delta\delta_3(0) = -0.16622$

$\Delta V_3(0) = -0.0362$

STEP 4

$\delta_2(1) = \delta_2(0) + \Delta\delta_2(0) = 0 - 0.051632$

$= -0.051632$ RAD

$\delta_3(1) = -0.16622$ RAD

$V_3(1) = 0.96380$ pu

If you are asked to go to the second iteration, you need to check the value of Q_{G2}.

$$Q_2(x) = V_2 Y_{21} V_1 \sin(\delta_2 - \delta_1 - \theta_{21}) + V_2^2 Y_{22} \sin(-\theta_{22}) + V_2 Y_{23} V_3 \sin(\delta_2 - \delta_3 - \theta_{23})$$

$$Q_2(1) = (1.05)(10)(1.0)\sin\left(-0.051632 - 0 - \frac{\pi}{2}\right) +$$

$$(1.05)^2(19.98)\sin\left(\frac{\pi}{2}\right) + (1.05)(10)(0.9638)\sin\left(-0.051632 - 0.16622 - \frac{\pi}{2}\right)$$

$= 1.66$ pu $= Q_{G2}$ since $Q_2 = Q_{G2} - Q_{L2}$ and $Q_{L2} = 0$

This value is within the limits.

$Q_{G2} = 1.66$ pu $< Q_{G_2 max} = 10$ pu

$Q_{G2} = 1.66$ pu $> Q_{G2 min} = -5$ pu

So, we can go to the next iteration assuming bus 2 is a voltage controlled bus.