Power Flow Equations and sample problems

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$$\bar{I}_k = \sum_{n=1}^{N} \bar{Y}_{kn}\bar{V}_n \qquad (7.4.4)$$

$$\bar{S}_k = P_k + jQ_k = \bar{V}_k \bar{I}_k^* \qquad (7.4.5)$$

$$P_k + jQ_k = \bar{V}_k \left[\sum_{n=1}^{N} \bar{Y}_{kn}\bar{V}_n\right]^* \qquad k = 1, 2, \ldots N \qquad (7.4.6)$$

With following notation,

$$\bar{V}_n = V_n e^{j\delta_n}$$

$$\bar{V}_k = V_k e^{j\delta_k} \qquad k, n = 1, 2, \ldots N$$

$$\bar{Y}_{kn} = Y_{kn} e^{j\theta_{kn}}$$

(7.4.6) becomes

$$P_k + jQ_k = V_k \sum_{n=1}^{N} Y_{kn} V_n e^{j(\delta_k - \delta_n - \theta_{kn})} \qquad (7.4.9)$$

Taking the real and imaginary parts of (7.4.9)

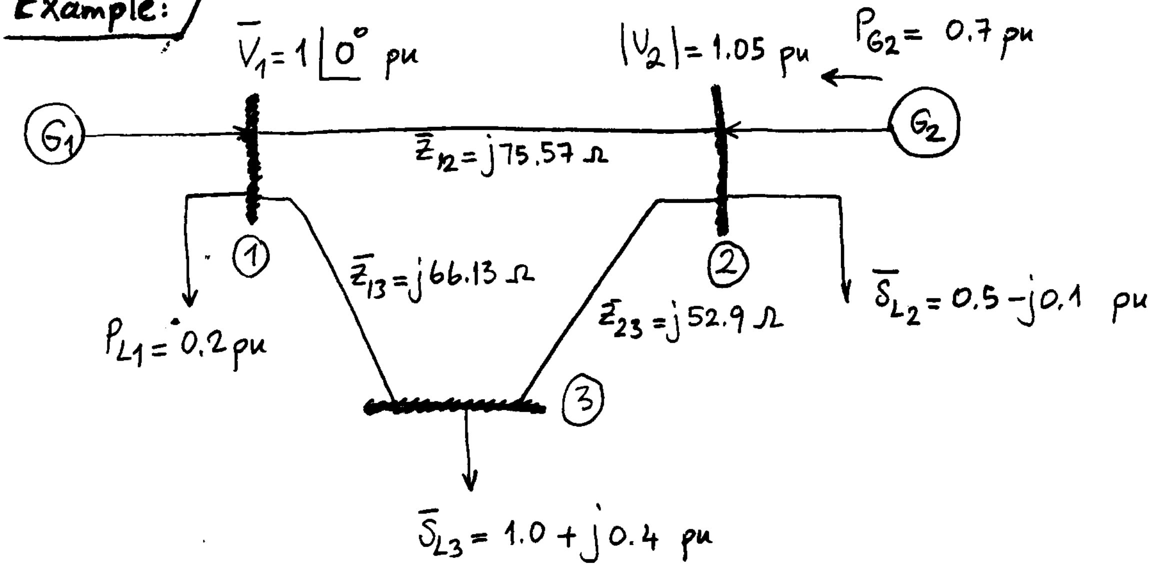
$$P_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn}) \qquad (7.4.10)$$

$$Q_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn}) \qquad (7.4.11)$$

Power-Flow Solution by Gauss-Seidel

$$\bar{V}_k(i+1) = \frac{1}{\bar{Y}_{kk}} \left[\frac{P_k - jQ_k}{V_k^*(i)} - \sum_{n=1}^{K-1} \bar{Y}_{kn}\bar{V}_n(i+1) - \sum_{n=k+1}^{N} \bar{Y}_{kn}\bar{V}_n(i)\right] \qquad (7.5.2)$$

Example:



BUS #	BUS TYPE	INPUT DATA	UNKNOWNS
1	swing bus	$V_1 = 1.0$, $\delta_1 = 0^\circ$, $P_{L1} = 0.2$	P_1, Q_1
2	Voltage controlled bus	$\lvert V_2\rvert = 1.05$, $Q_{L2} = -0.1$ $P_2 = P_{G2} - P_{L2} = 0.7 - 0.5 = 0.2$	δ_2, Q_2
3	Load bus	$P_3 = P_{G3} - P_{L3} = 0 - 1.0 = -1$ $Q_3 = Q_{G3} - Q_{L3} = 0 - 0.4 = -0.4$	V_3, δ_3

* $S_{base3\phi} = 100$ MVA and $V_{base} = 230\ kV_{LL}$

* Transmission lines are lossless lines

a)) Find the $\underline{Y}_{bus}$.

$$Z_{base} = \frac{V_{base\,LL}^2}{S_{base3\phi}} = \frac{(230)^2}{100} = 529\ \Omega$$

$$\bar{Z}_{12} = j\frac{75.57}{529} = j0.143 \text{ pu} \Rightarrow \bar{y}_{12} = \frac{1}{j0.143} = -j7.0 \text{ pu}$$

$$\bar{Z}_{13} = j\frac{66.13}{529} = j0.125 \text{ pu} \Rightarrow \bar{y}_{13} = \frac{1}{j0.125} = -j8 \text{ pu}$$

$$\bar{Z}_{23} = j\frac{52.9}{529} = j0.1 \text{ pu} \Rightarrow \bar{y}_{23} = \frac{1}{j0.1} = -j10 \text{ pu}$$

$$Y_{bus} = j\begin{bmatrix} -15 & 7 & 8 \\ 7 & -17 & 10 \\ 8 & 10 & -18 \end{bmatrix} \quad pu$$

b) $\delta_2(0) = \delta_3(0) = 0°$, $|V_3(0)| = 1.0$ pu

Using Gauss-Seidel method, find $\delta_2(1)$, $\delta_3(1)$, $|V_3(1)|$.

$Q_{G2min} = -0.5$ pu , $Q_{G2max} = 1.5$ pu

$$\bar{V}_k(i+1) = \frac{1}{\bar{Y}_{kk}}\left[\frac{P_k - jQ_k}{\bar{V}_k^*(i)} - \sum_{n=1}^{k-1} \bar{Y}_{kn}\bar{V}_n(i+1) - \sum_{n=k+1}^{N} \bar{Y}_{kn}\bar{V}_n(i)\right]$$

In this problem, $N = 3$.

Since the first bus (bus 1) is a swing bus, no iterations are requirek for bus 1.

So, for bus 2, $k=2$

$$\bar{V}_2(1) = \frac{1}{\bar{Y}_{22}}\left[\frac{P_2 - jQ_2}{\bar{V}_2^*(0)} - \bar{Y}_{21}\bar{V}_1 - \bar{Y}_{23}\bar{V}_3(0)\right] \qquad (1)$$

$$\bar{V}_3(1) = \frac{1}{\bar{Y}_{33}}\left[\frac{P_3 - jQ_3}{\bar{V}_3^*(0)} - \bar{Y}_{31}\bar{V}_1 - \bar{Y}_{32}\bar{V}_2(1)\right] \qquad (2)$$

where

$P_2 = 0.2$ pu

Q_2 is unknown and should be calalated.

$P_3 = -1$ pu, $Q_3 = -0.4$ pu

To find $\bar{V}_2(1)$, we should first find Q_2. So, use equation (7.4.11).

$$Q_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn}) \qquad 7.4.11$$

$$Q_2 = |V_2(0)| \sum_{n=1}^{3} |\bar{Y}_{2n}|\,|\bar{V}_n(0)| \sin(\delta_2(0) - \delta_n(0) - \theta_{2n})$$

$$Q_2 = 1.05\left[7 \times 1.0 \times \sin(-90^\circ) - 17 \times 1.05 \times \sin(-90^\circ) + 10 \times 1.0 \times \sin(-90^\circ)\right]$$

$$= 1.05\left[-7 + 17.85 - 10\right] = 1.05 \times 0.85 = 0.8925 \text{ pu}$$

$$= 0.8925 \text{ pu}$$

$$Q_2 = Q_{G2} - Q_{L2} = Q_{G2} - (-0.1) = Q_{G2} + 0.1 \Rightarrow Q_{G2} = Q_2 - 0.1$$

$$= 0.8925 - 0.1$$

$$Q_{G2} = 0.7925 \text{ pu}$$

$$-0.5 < 0.7925 < 1.5$$

The calculated value ($Q_{G2} = 0.7925$) does not exceed its limits. So, we can use $Q_2 = 0.8925$ pu in (1) to calculate $\bar{V}_2(1)$.

$$\bar{V}_2(1) = \frac{1}{-j17}\left[\frac{0.2 - j0.8925}{1.05} - j7 \times 1.0 - j10 \times 1.0\right]$$

$$= j\frac{1}{17}\left(0.190 - j0.85 - j17\right) = 1.05 + j0.011$$

$$\bar{V}_2(1) = 1.05006 \angle 0.6^\circ \text{ pu}$$

The magnitude of $\bar{V}_2(1)$ should be changed to 1.05, which is the input data for the voltage-contralled bus. Thus, we use equation (1) to compute only the angle $\delta_2(i)$ for the voltage-contralled buses.

Therefore,

$$\boxed{\begin{aligned} |V_2(1)| &= 1.05 \text{ pu} \\ \delta_2(1) &= 0.6^\circ \end{aligned}}$$

$$V_3(1) = \frac{1}{-j18}\left[\frac{-1 - j(-0.4)}{1.0} - j8 \times 1.0 - j10 \times 1.05\angle 0.6\right]$$

$$\bar{V}_3(1) = -\frac{1}{j18}\left[-1.0 + j0.4 - j8 \underbrace{-j10\left(1.049942 + j\,0.010995\right)}_{(-j10.4994 + 0.10995)}\right]$$

$$= -\frac{1}{j18}\left[-0.89005 - j\,18.0994\right] = 1.00552 - j\,0.04944 \quad pu$$

$$\bar{V}_3(1) = 1.00673 \angle -2.815^\circ \quad pu$$

$$\boxed{\begin{array}{c} |V_3(1)| = 1.00673 \; pu \\ \delta_3 = -2.815^\circ \end{array}}$$

Example 2

Use the same system given in example 1. The only change is: 2nd bus is changed from voltaged-controlled bus to a load bus. In addition, $Q_{G2} = 0.4$ pu is assumed.
Using the following initial values,

$\delta_2(0) = \delta_3(0) = 0^\circ$, $|V_2(0)| = |V_3(0)| = 1.0$ pu, calculate $\delta_2(1)$, $\delta_3(1)$, $|V_2(1)|$, and $|V_3(1)|$ using Newton-Raphson method.

$P_2 = 0.2$ pu $\qquad Q_2 = Q_{G2} - Q_{L2} = 0.4 - (-0.1) = 0.5$ pu

$P_3 = -1.0$ pu $\qquad Q_3 = -0.4$ pu

$$x = \begin{bmatrix} \bar{\delta} \\ \bar{V} \end{bmatrix} = \begin{bmatrix} \delta_2 \\ \vdots \\ \delta_N \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \qquad y = \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} P_2 \\ \vdots \\ P_N \\ Q_2 \\ \vdots \\ Q_N \end{bmatrix} \qquad f(x) = \begin{bmatrix} P(x) \\ Q(x) \end{bmatrix} = \begin{bmatrix} P_2(x) \\ \vdots \\ P_N(x) \\ Q_2(x) \\ \vdots \\ Q_N(x) \end{bmatrix}$$

The swing bus variables V_1 and δ_1 are omitted from above equations, since they are alreddy known.

$$y_k = P_k = P_k(x) = V_k \sum_{n=1}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

$$y_{k+N} = Q_k = Q_k(x) = V_k \sum_{n=1}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

$$k = 2, 3, \dots N$$

The Jacobian matrix has the form

$$J = \begin{array}{c} \begin{array}{cc} J1 \qquad\qquad\qquad & \qquad\qquad\qquad J2 \end{array} \\ \left[\begin{array}{ccc|ccc} \frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_2}{\partial \delta_N} & \frac{\partial P_2}{\partial |V_2|} & \cdots & \frac{\partial P_2}{\partial |V_N|} \\ \vdots & & & \vdots & & \\ \frac{\partial P_N}{\partial \delta_2} & \cdots & \frac{\partial P_N}{\partial \delta_N} & \frac{\partial P_N}{\partial |V_2|} & \cdots & \frac{\partial P_N}{\partial |V_N|} \\ \hline \frac{\partial Q_2}{\partial \delta_2} & \cdots & \frac{\partial Q_2}{\partial \delta_N} & \frac{\partial Q_2}{\partial |V_2|} & \cdots & \frac{\partial Q_2}{\partial |V_N|} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial Q_N}{\partial \delta_2} & \cdots & \frac{\partial Q_N}{\partial \delta_N} & \frac{\partial Q_N}{\partial |V_2|} & \cdots & \frac{\partial Q_N}{\partial |V_N|} \end{array}\right] \\ \begin{array}{cc} J3 \qquad\qquad\qquad & \qquad\qquad\qquad J4 \end{array} \end{array}$$

$$\underline{x}(0) = \begin{bmatrix} \delta_2(0) \\ \delta_3(0) \\ V_2(0) \\ V_3(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$ we start with these initial values

Step 1 : Compute $\Delta y(i) \rightarrow$ Power mismatches

Power mismatches

$$\Delta \underline{y}(i) = \begin{bmatrix} \Delta \underline{P}(i) \\ \Delta \underline{Q}(i) \end{bmatrix} = \begin{bmatrix} \underline{P} - \underline{P}[x(i)] \\ \underline{Q} - \underline{Q}[x(i)] \end{bmatrix}$$

$$\Delta \underline{y}(0) = \begin{bmatrix} \Delta P_2(0) \\ \Delta P_3(0) \\ \Delta Q_2(0) \\ \Delta Q_3(0) \end{bmatrix} = \begin{bmatrix} P_2 - P_2[\underline{x}(0)] \\ P_3 - P_3[\underline{x}(0)] \\ Q_2 - Q_2[\underline{x}(0)] \\ Q_3 - Q_3[\underline{x}(0)] \end{bmatrix}$$

where $P_2 = 0.2 \quad P_3 = -1.0 \quad Q_2 = 0.5 \quad Q_3 = -0.4$

In this problem, $N=3$

$$P_K(\underline{x}) = V_k \sum_{n=1}^{3} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

$$Q_K(\underline{x}) = V_k \sum_{n=1}^{3} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

$\Bigg\}\ k=2,3$

$$P_2(\underline{x}) = \{ V_2 Y_{21} V_1 \cos(\delta_2 - \delta_1 - \theta_{21}) + V_2 Y_{22} V_2 \cos(\delta_2 - \delta_2 - \theta_{22}) + V_2 Y_{23} V_3 \cos(\delta_2 - \delta_3 - \theta_{23}) \}$$

$$P_2[\underline{x}(0)] = 1 \times 7 \times 1 \cdot \cos(-90) + 1 \times (-17) \times 1 \cos(-90) + 1 \times 10 \times 1 \times \cos(-90^\circ)$$

$$= 0$$

$$P_3(\underline{x}) = V_3 Y_{31} V_1 \cos(\delta_3 - \delta_1 - \theta_{31}) + V_3 Y_{32} V_2 \cos(\delta_3 - \delta_2 - \theta_{32}) + V_3 Y_{33} V_3 \cos(\delta_3 - \delta_3 - \theta_{33})$$

$P_3[\underline{x}(0)] = 0$ – due to $\cos(-90)$ in each term

$$Q_2(\underline{x}) = V_2 Y_{21} V_1 \sin(\delta_2 - \delta_1 - \theta_{21}) + V_2 Y_{22} V_2 \sin(\delta_2 - \delta_2 - \theta_{22}) + V_2 Y_{23} V_3 \sin(\delta_2 - \delta_3 - \theta_{23})$$

$$Q_2[\underline{x}(0)] = 7 \sin(-90^\circ) - 17 \sin(-90) + 10 \sin(-90^\circ)$$

$$= -7 + 17 - 10 = 0$$

$$Q_3(\underline{x}) = V_3 Y_{31} V_1 \sin(\delta_3 - \delta_1 - \theta_{31}) + V_3 Y_{32} V_2 \sin(\delta_3 - \delta_2 - \theta_{32}) + V_3 Y_{33} V_3 \sin(\delta_3 - \delta_3 - \theta_{33})$$

$$\Delta \underline{y}(0) = \begin{bmatrix} 0.2 - 0 \\ -1.0 - 0 \\ 0.5 - 0 \\ -0.4 - 0 \end{bmatrix} = \begin{bmatrix} 0.2 \\ -1.0 \\ 0.5 \\ -0.4 \end{bmatrix}$$

$Q_3[\underline{x}(0)] = 0$

Step 2

Calculate the Jacobian matrix

$$\underline{J1} = \frac{\partial \underline{P}}{\partial \underline{\delta}} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} \end{bmatrix}$$

$$\left.\frac{\partial P_2}{\partial \delta_2}\right|_{\underline{X}(0)} = -\left[7\sin(-90) + 10\sin(-90)\right] = 17$$

$$\left.\frac{\partial P_2}{\partial \delta_3}\right|_{\underline{X}(0)} = 10\sin(-90^\circ) = -10$$

$$\left.\frac{\partial P_3}{\partial \delta_2}\right|_{\underline{X}(0)} = 10\sin(-90^\circ) = -10$$

$$\left.\frac{\partial P_3}{\partial \delta_3}\right|_{\underline{X}(0)} = 18 \qquad J1 = \begin{bmatrix} 17 & -10 \\ -10 & 18 \end{bmatrix}$$

$$J2 = \frac{\partial P}{\partial |V|} = \begin{bmatrix} \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_2|} \\ \frac{\partial P_2}{\partial |V_3|} & \frac{\partial P_3}{\partial |V_3|} \end{bmatrix} \qquad J2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J3 = \frac{\partial Q}{\partial \delta} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J4 = \frac{\partial Q}{\partial |V|} = \begin{bmatrix} 17 & -10 \\ -10 & 18 \end{bmatrix}$$

step 3

$$\left[\begin{array}{c|c} J1(i) & J2(i) \\ \hline J3(i) & J4(i) \end{array}\right] \begin{bmatrix} \Delta\delta(i) \\ \Delta V(i) \end{bmatrix} = \begin{bmatrix} \Delta P(i) \\ \Delta Q(i) \end{bmatrix}$$

Since in this problen $J2 = J3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\underbrace{\left(\tfrac{10}{17}\right) \rightarrow \overbrace{\begin{bmatrix} 17 & -10 \\ -10 & 18 \end{bmatrix}}^{J1} \begin{bmatrix} \Delta\delta_2(0) \\ \Delta\delta_3(0) \end{bmatrix} = \begin{bmatrix} 0.2 \\ -1.0 \end{bmatrix}} \qquad \underbrace{\left(\tfrac{10}{17}\right) \rightarrow \begin{bmatrix} 17 & -10 \\ -10 & 18 \end{bmatrix} \begin{bmatrix} \Delta V_2(0) \\ \Delta V_3(0) \end{bmatrix} = \begin{bmatrix} 0.5 \\ -0.4 \end{bmatrix}}$$

$$\begin{bmatrix} 17 & -10 \\ 0 & 12.1176 \end{bmatrix} \begin{bmatrix} \Delta\delta_2(0) \\ \Delta\delta_3(0) \end{bmatrix} = \begin{bmatrix} 0.2 \\ -0.8823 \end{bmatrix} \qquad \begin{bmatrix} 17 & -10 \\ 0 & 12.1176 \end{bmatrix} \begin{bmatrix} \Delta V_2(0) \\ \Delta V_3(0) \end{bmatrix} = \begin{bmatrix} 0.5 \\ -0.10588 \end{bmatrix}$$

After solving above equations

$$\Delta\delta_3(0) = \frac{-0.8823}{12.1176} = -0.07281$$

$$17\,\Delta\delta_2(0) = -10\times(-0.07281) = 0.2 \Rightarrow \Delta\delta_2(0) = -0.03106 \text{ pu}$$

$$\Delta V_3(0) = \frac{-0.10588}{12.1176} = -8.7433\times10^{-3}$$

$$17\Delta V_2(0) - 10\times(-8.7433\times10^{-3}) = 0.5 \Rightarrow \Delta V_2(0) = 0.024268 \text{ pu}$$

Step 4.

$$\delta_2(1) = \delta_2(0) + \Delta\delta_2(0) = -0.03106$$

$$\delta_3(1) = \delta_3(0) + \Delta\delta_3(0) = -0.07281$$

$$V_2(1) = V_2(0) + \Delta V_2(0) = 1.0 + 0.024268 = 1.024268 \text{ pu}$$

$$V_3(1) = V_3(0) + \Delta V_3(0) = 1.0 - 8.7433\times10^{-3} = 0.991256 \text{ pu}$$