Appendix A Wavelength in a Waveguide

APPENDIX A WAVELENGTH IN A WAVEGUIDE

For radio-frequency energy to travel along a waveguide, it must be possible for the associated electric and magnetic fields to exist inside the guide. One of the conditions that must be met is that there can be no electric field acting along a conducting surface. How is this possible with a transverse electric ware?

Imagine two plane waves in free space, having the same frequency (and therefore wavelength), and travelling at an angle to one another.

Fig A1

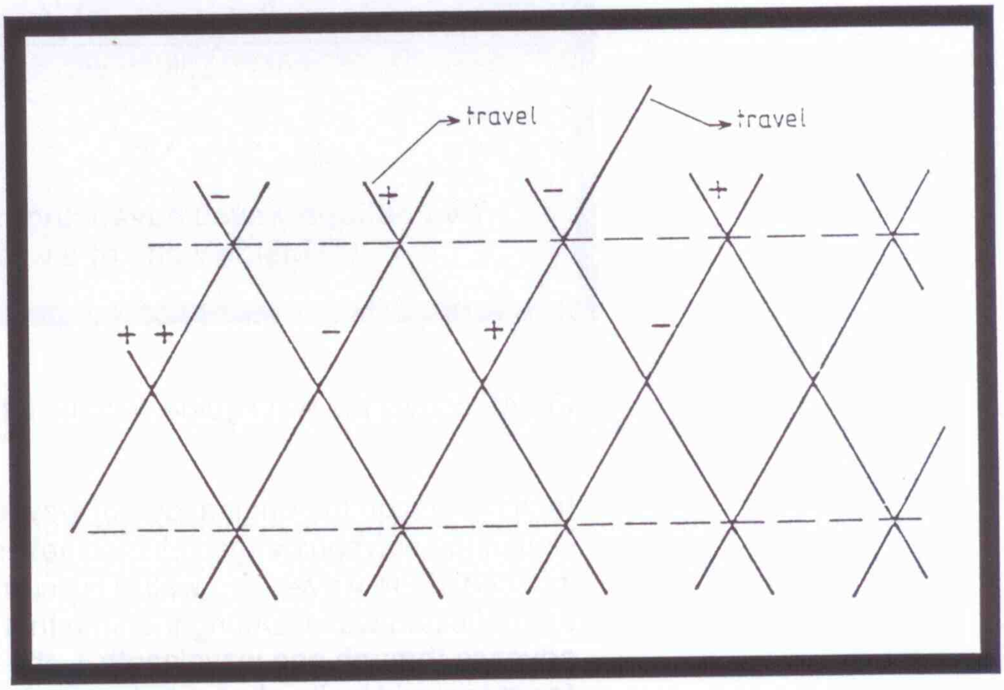

Fig A1 shows the waves as a series of lines representing the places where maximum positive and negative electric field strength are obtained (at a given instant).

Also shown in fig A1 is a pair of broken lines at which the maximum positive value of one of the waves is cancelled out by the maximum negative value of the other wave. These broken lines are therefore places where the electric field strength is zero.

It is possible therefore to place conductors at the site of the broken lines without altering the wave pattern. These can be the walls of a waveguide. We have thus shown a possible pattern of waves which could exist in a waveguide. (Clearly, however, the pattern will not be generated by the process we have imagined).

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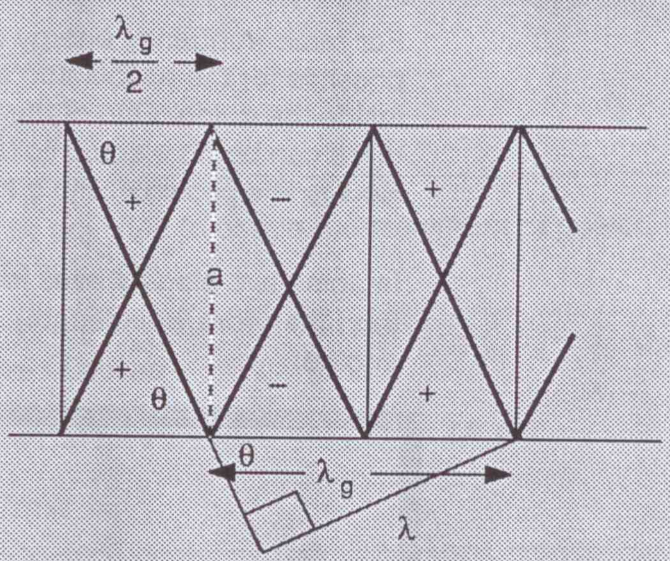
Fig A2



Two oblique waves advancing by reflections at the inside walls of a waveguide

Fig A2 shows the waveguide, with its internal field pattern.

In this situation the original pair of waves, meeting the conducting wall of the waveguide, has electric field reduced to zero by current induced in the waveguide wall. This current causes the generation of a reflected wave. During the time that the two oblique waves advance through one wavelength λ, the combined wave appears (as measured by its phase) to have advanced through the greater distance λ_g. From geometrical considerations, all the angles marked θ are equal. By equating values of $\tan\theta$, or otherwise, it is easy to show that:

$$\frac{\lambda_g}{\lambda} = \frac{1}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}} \qquad \text{A. 1}$$

It will be evident that λg is always greater than I.

What has been described leads to a waveguide propagating the wave in $TE_{1,0}$ mode.

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Cutoff frequency

As the free-space wavelength λ is increased, the angle between the guide's direction and that of each oblique wave increases. Eventually, at the cutoff frequency, the denominator of equation A.1 becomes zero (and therefore the calculated λ_g becomes infinite). The 'oblique' waves are now travelling perpendicular to the waveguide direction. No signal is therefore propagated along the waveguide at all.

In-guide wavelength in terms of cutoff frequency

It can be shown that, for all modes,

$$\frac{\lambda_g}{\lambda} = \frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \qquad \text{A.2}$$

where f is the signal frequency

f_c is the waveguide's cut–off frequency

the wavelength in the guide is therefore always longer than the free-space wavelength, by a factor which increases as the cutoff frequency is approached.