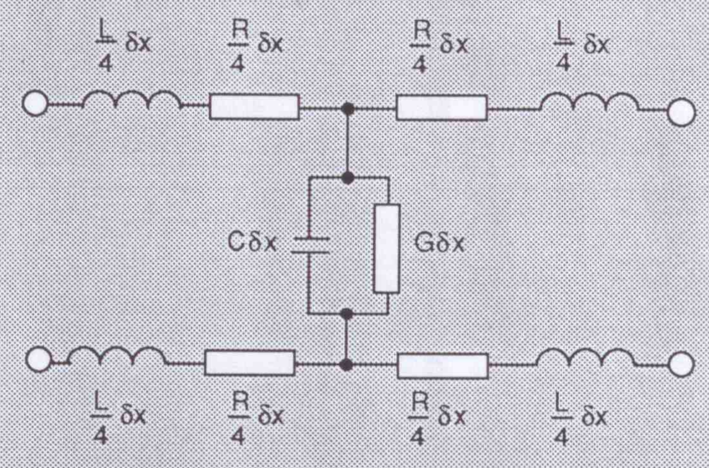
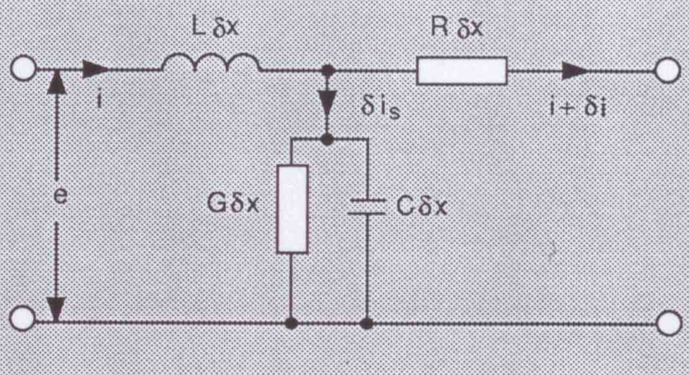
Fig 1



Typical Microwave Circuit

Fig 2



Simplified Microwave Circuit (Ref: fig 1)

Chapter 2 Microwave Theory

CHAPTER 2 MICROWAVE THEORY

This chapter briefly discusses transmission line theory

A microwave signal is simply an electromagnetic wave whose wavelength is small compared with ordinary radio waves. Like any radio wave, microwaves can be sent through space, or guided by conductors. Any extended system of conductors intended to guide a signal between two separated points is called a *transmission line*.

The MWT530 Microwave Trainer is mainly concerned with a particular form of transmission line, known as a *waveguide*. This is rather like a pipe in appearance, and it is rather difficult to give easy definitions of 'voltage' and 'current' in it. Its behaviour however resembles that of an ordinary line, such as a pair of wires. It is therefore instructive to consider how such an ordinary line behaves.

If we consider a short length δx of the transmission line, we can represent it by four types of component:

1 Series inductance, $L\delta x$

2 Series resistance, $R\delta x$

3 Shunt capacitance, $C\delta x$

4 Shunt conductance, $G\delta x$, where,

L, R, C, G are values of inductance etc per unit length.

The line can be represented by the circuit, fig 1. The various elements are shown distributed along the line, to show that the line is isotropic (transmits equally well in both directions). Fig 2 shows a simpler representation, which is equally valid if δx is small enough.

Suppose that a voltage 'e' is applied between the conductors in fig 2. A shunt current δi_s will flow between the conductors. But if the line current changes by δi,

$$(i + \delta i) - i + \delta i_s = 0$$

$$\delta i = -\delta i_s$$

Consequently the current along the conductors will decrease, by the same amount. The shunt current will be the sum of the currents due to capacitance and due to conductance:

$$\therefore \delta i = -\delta_s = -\left(eG + C\frac{\partial e}{\partial t}\right)\delta x$$

Dividing by δx and allowing δx to tend to zero,

$$\frac{\delta i}{\delta x} = -\left(eG + C\frac{\partial e}{\partial t}\right) \qquad 1.1$$

The current flowing through the inductance and resistance of the length δx of line will produce a voltage drop –δe:

$$\delta e = -\left(iR + L\frac{\partial i}{\partial t}\right)\delta x, \quad \text{so that:}$$

$$\frac{\delta e}{\delta x} = -\left(iR + L\frac{\partial i}{\partial t}\right) \qquad 1.2$$

Equations 1.1 and 1.2 are generally known as the 'line equations' which form the basis of an analysis of line behaviour. Such an analysis shows that R and G cause the signal to be weakened as it travels along the line. In useful lines however, the signal is weakened as little as possible, so that R and G are made small. The line behaviour can then be approximated quite well by putting R and G equal to zero, which greatly simplifies the analysis. The equations then reduce to:

$$\frac{\delta i}{\delta x} = \frac{-C\partial e}{\partial t} \quad \text{and} \qquad 1.3$$

$$\frac{\delta e}{\delta x} = \frac{-L\partial i}{\partial t} \qquad 1.4$$

It can be shown* that these equations represent a wave of current and of voltage, travelling with velocity $\frac{1}{\sqrt{LC}}$. The voltage is equal to the current multiplied by $\sqrt{LC}$.

* See for example Johnson "Transmission Lines and Networks" McGraw Hill, 1950.

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Equation 1.3 specifies the ratio between the voltage and the current at any point in the line (assuming that a wave is travelling in either one direction but not the other).

Equation 1.4 describes a wave, of waveform f(x), travelling to the right (minus sign) or to the left (plus sign) with velocity v.

Solutions of these equations are:

$$e = iZ_0, \quad \text{and:} \qquad 1.5$$

$$e = f(x \pm vt) \qquad 1.6$$

where $Z_0 = \sqrt{\frac{L}{C}}$ and is called the *characteristic impedance* and:

$$v = \frac{1}{\sqrt{LC}}$$

This can be seen by using 1.5 to substitute for e in 1.3, giving:

$$\frac{\delta i}{\delta x} = -\sqrt{LC}\,\frac{\partial i}{\partial t}$$