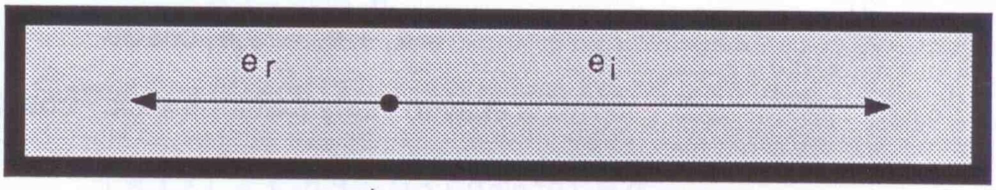
Appendix B Analysis of Double-Minimum Method for VSWR

APPENDIX B ANALYSIS OF DOUBLE-MINIMUM METHOD FOR VSWR

An incident wave of strength e_i and a reflected wave of strength e_r produce a standing wave whose maximum value is e_{max} and whose minimum value is e_{min}. For a point where the minimum signal (e_{min}) is obtained, fig B1 shows the phasors for e_i and e_r to be in antiphase.

Fig B1

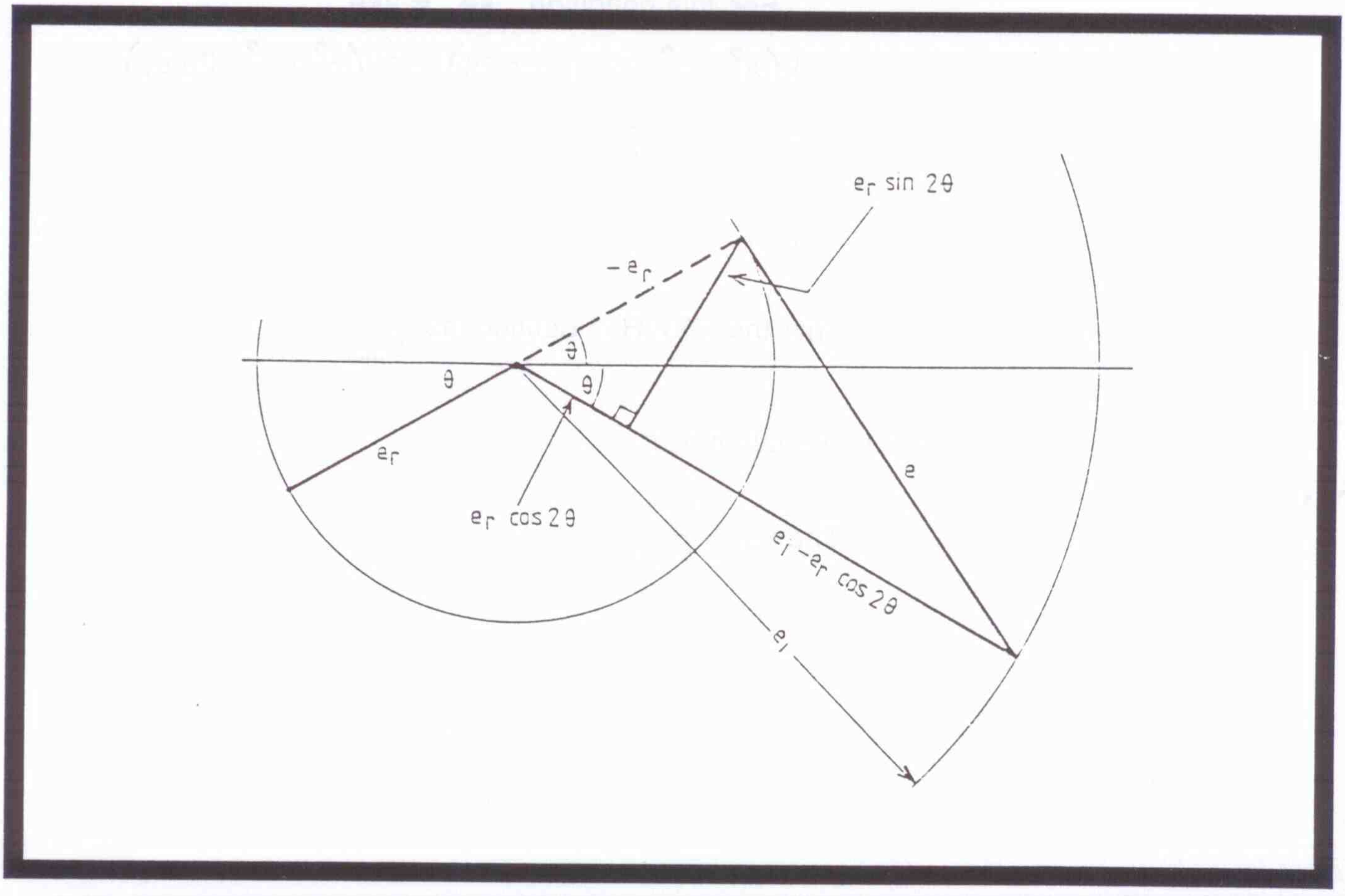


On moving a distance $\frac{d}{\lambda_g}$ from the minimum point, where λ_g is the wavelength within the guide, the phasor of the incident wave will be retarded $\frac{2\pi d}{\lambda_g}$ radians, and that of the reflected wave will be advanced by the same amount.

For convenience let $\theta = \frac{2\pi d}{\lambda_g}$

Then fig B2 shows the displaced phasors (whose tips move around circles, since the magnitudes of incident and reflected waves are supposed constant).

Fig B2



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The resultant e is most easily calculated as the difference between the larger phasor ei and the negative of the smaller, $-e_r$.

Thus from fig B2, with Pythagoras' Theorem,

$$e^2 = e_r^2 \sin^2 2\theta + \left(e_i^2 - e_i e_r \cos 2\theta + e_r^2 \cos 2\theta\right)$$
$$= e_r^2 + e_i^2 - 2e_i e_r \cos 2\theta$$

It is easy to verify that for $\theta = n\pi$, $\cos 2\theta = 1$

and $e^2 = (e_i - e_r)^2 = e_{min}{}^2$

while for $\theta = n\pi + \frac{\pi}{2}$, $\cos 2\theta = -1$

so that $e^2 = (e_1 + e_r)^2 = e_{max}^2$

The double-minimum method finds the value of θ for which the detector current

$i = ke^2$ is doubled, i.e when $i = 2ke_{min}{}^2$.

For this condition $ke_0^2 = 2ke_{min}^2$

$$\text{or } k\left(e_r^2 + e_i^2 - 2e_i e_r \cos 2\theta\right) = 2k\left(e_i^2 + e_r^2 - 2e_i e_r\right)$$

$$\therefore e_r^2 + e_i^2 - 2e_i e_r (2 - \cos 2\theta) = 0$$

$$\therefore \left(\frac{e_i}{e_r}\right)^2 + 1 - 2\frac{e_i}{e_r}(2 - \cos 2\theta) = 0 \qquad (1)$$

Now the VSWR is defined as $\frac{e_{max}}{e_{min}}$.

Let us denote it by s

$$\text{Then } s = \frac{e_i + e_r}{e_i - e_r}$$

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$$e_i(s-1) = e_r(s+1)$$

$$\frac{e_i}{e_r} = \frac{s+1}{s-1} \qquad (2)$$

Also $\cos 2\theta = 1 - 2\sin^2\theta$

$$\therefore 2 - \cos 2\theta = 2 - (1 - 2\sin^2\theta)$$

$$= 1 + 2\sin^2\theta \qquad (3)$$

Substituting (2) and (3) into (1) gives:

$$\left(\frac{s+1}{s-1}\right)^2 + 1 - 2\frac{s+1}{s-1}(1 + 2\sin^2\theta) = 0$$

which reduces to:

$$s^2 = 1 + \frac{1}{\sin^2\theta}$$