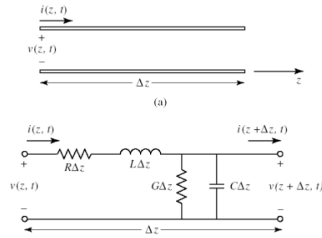
Microwave Techniques

Transmission Lines

Lumped Element Model for a Transmission Line



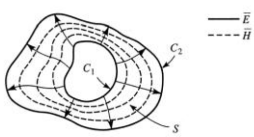
$$V_{12} = \Phi_1 - \Phi_2 = \int_1^2 \bar{E} \cdot d\bar{\ell},$$

$$I = \oint_C \bar{H} \cdot d\bar{\ell}$$

R = series resistance per unit length, for both conductors, in Ω/m.
L = series inductance per unit length, for both conductors, in H/m.
G = shunt conductance per unit length, in S/m.
C = shunt capacitance per unit length, in F/m.

Field Analysis of Transmission Lines

Transmission Line Parameters



Field lines on an arbitrary TEM transmission line.

$$W_m = \frac{\mu}{4}\int_S \bar{H}\cdot\bar{H}^* ds = L|I_o|^2/4$$

$$L = \frac{\mu}{|I_o|^2}\int_S \bar{H}\cdot\bar{H}^* ds \text{ H/m}$$

$$W_e = \frac{\epsilon}{4}\int_S \bar{E}\cdot\bar{E}^* ds = C|V_o|^2/4$$

$$C = \frac{\epsilon}{|V_o|^2}\int_S \bar{E}\cdot\bar{E}^* ds \text{ F/m.}$$

$$R = \frac{R_s}{|I_o|^2}\int_{C_1+C_2} \bar{H}\cdot\bar{H}^* dl \ \Omega\text{/m.}$$

$$G = \frac{\omega\epsilon''}{|V_o|^2}\int_S \bar{E}\cdot\bar{E}^* ds \text{ S/m.}$$

$R_s = 1/\sigma\delta_s$ is the surface resistance of the conductors

ϵ'' is the imaginary part of the complex permittivity

Transmission Line Parameters of a Coaxial Line

$$\bar{E} = \frac{V_o\hat{\rho}}{\rho \ln b/a} e^{-\gamma z},$$

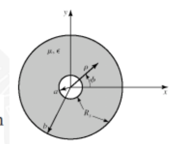
$$\bar{H} = \frac{I_o\hat{\phi}}{2\pi\rho} e^{-\gamma z},$$

$$L = \frac{\mu}{(2\pi)^2}\int_{\phi=0}^{2\pi}\int_{\rho=a}^{b}\frac{1}{\rho^2}\rho d\rho d\phi = \frac{\mu}{2\pi}\ln b/a \text{ H/m}$$

$$C = \frac{\epsilon'}{(\ln b/a)^2}\int_{\phi=0}^{2\pi}\int_{\rho=a}^{b}\frac{1}{\rho^2}\rho d\rho d\phi = \frac{2\pi\epsilon'}{\ln b/a} \text{ F/m}$$

$$R = \frac{R_s}{(2\pi)^2}\left\{\int_{\phi=0}^{2\pi}\frac{1}{a^2}a d\phi + \int_{\phi=0}^{2\pi}\frac{1}{b^2}b d\phi\right\} = \frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right) \Omega\text{/m}$$

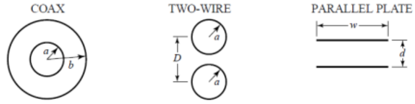
$$G = \frac{\omega\epsilon''}{(\ln b/a)^2}\int_{\phi=0}^{2\pi}\int_{\rho=a}^{b}\frac{1}{\rho^2}\rho d\rho d\phi = \frac{2\pi\omega\epsilon''}{\ln b/a} \text{ S/m}$$



Geometry of a coaxial line with surface resistance R_s on the inner and outer conductors.

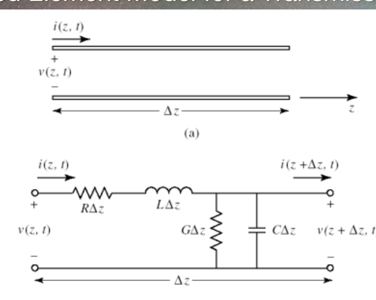
Transmission Line Parameters of Some Common Lines

TABLE 2.1 Transmission Line Parameters for Some Common Lines

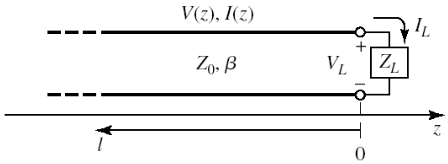


	COAX	TWO-WIRE	PARALLEL PLATE
L	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	$\frac{\mu}{\pi}\cosh^{-1}\left(\frac{D}{2a}\right)$	$\frac{\mu d}{w}$
C	$\frac{2\pi\epsilon'}{\ln b/a}$	$\frac{\pi\epsilon'}{\cosh^{-1}(D/2a)}$	$\frac{\epsilon' w}{d}$
R	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{R_s}{\pi a}$	$\frac{2R_s}{w}$
G	$\frac{2\pi\omega\epsilon''}{\ln b/a}$	$\frac{\pi\omega\epsilon''}{\cosh^{-1}(D/2a)}$	$\frac{\omega\epsilon'' w}{d}$

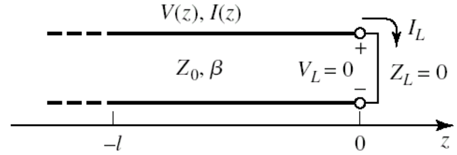
Lumped Element Model for a Transmission Line



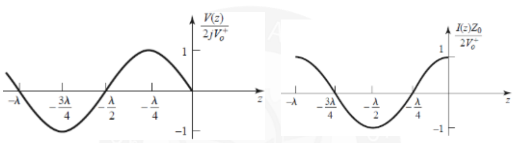
Terminated Lossless Transmission Lines

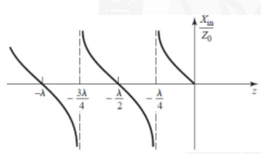


Short-Circuit Termination



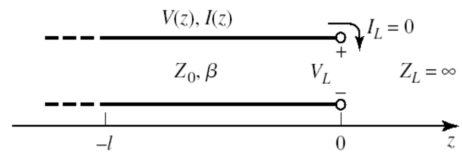
Short-Circuit Termination



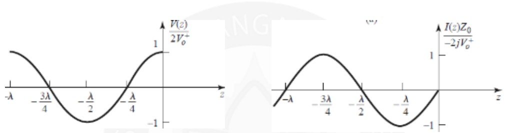


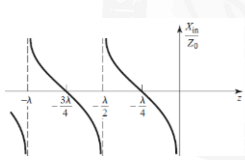
(a) Voltage, (b) current, and (c) impedance (R_{in} = 0 or ∞) variation along a short-circuited transmission line.

Open-Circuit Termination



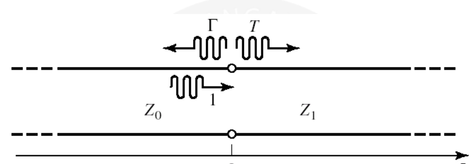
Open-Circuit Termination





(a) Voltage, (b) current, and (c) impedance (R_{in} = 0 or ∞) variation along a open-circuited transmission line.

Junction of Two Transmission Lines

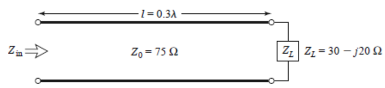


Examples

2.1 A 75 Ω coaxial line has a current $i(t, z) = 1.8\cos(3.77 \times 10^9\, t - 18.13z)$ mA. Determine (a) the frequency, (b) the phase velocity, (c) the wavelength, (d) the relative permittivity of the line, (e) the phasor form of the current, and (f) the time domain voltage on the line.

2.2 A transmission line has the following per-unit-length parameters: $L = 0.5\ \mu$H/m, $C = 200$ pF/m, $R = 4.0\ \Omega$/m, and $G = 0.02$ S/m. Calculate the propagation constant and characteristic impedance of this line at 800 MHz. If the line is 30 cm long, what is the attenuation in dB? Recalculate these quantities in the absence of loss ($R = G = 0$).

2.8 A lossless transmission line of electrical length $\ell = 0.3\lambda$ is terminated with a complex load impedance as shown in the accompanying figure. Find the reflection coefficient at the load, the SWR on the line, the reflection coefficient at the input of the line, and the input impedance to the line.



Smith Chart

$$\Gamma = \frac{z_L - 1}{z_L + 1} = |\Gamma| e^{j\theta} \qquad z_L = \frac{1 + |\Gamma| e^{j\theta}}{1 - |\Gamma| e^{j\theta}}$$

$$r_L + j x_L = \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i}$$

$$r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2},$$

$$x_L = \frac{2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}.$$

$$\left(\Gamma_r - \frac{r_L}{1 + r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1 + r_L}\right)^2,$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2,$$

Smith Chart

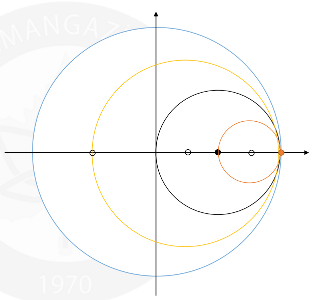
$r_L = 0$

$r_L = 1$

$r_L = 2$

$r_L = 1/3$

$r_L = \infty$



Smith Chart

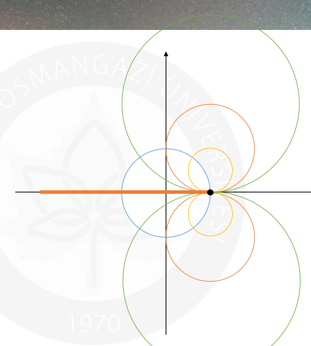
$x_L = \pm 1$

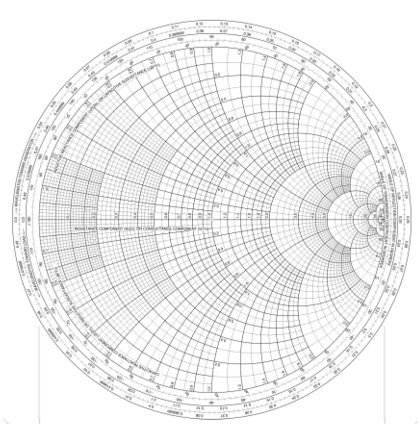
$x_L = \pm 2$

$x_L = \pm 0.5$

$x_L = 0$

$x_L = \infty$





Examples

EXAMPLE 2.2
A load impedance of $40 + j70\ \Omega$ terminates a 100 Ω transmission line that is 0.3λ long. Find the reflection coefficient at the load, the reflection coefficient at the input to the line, the input impedance, the standing wave ratio on the line, and the return loss.

EXAMPLE 2.3
A load of $Z_L = 100 + j50\ \Omega$ terminates a 50 Ω line. What are the load admittance and input admittance if the line is 0.15λ long?