TECHNIQUES OF CIRCUIT ANALYSIS

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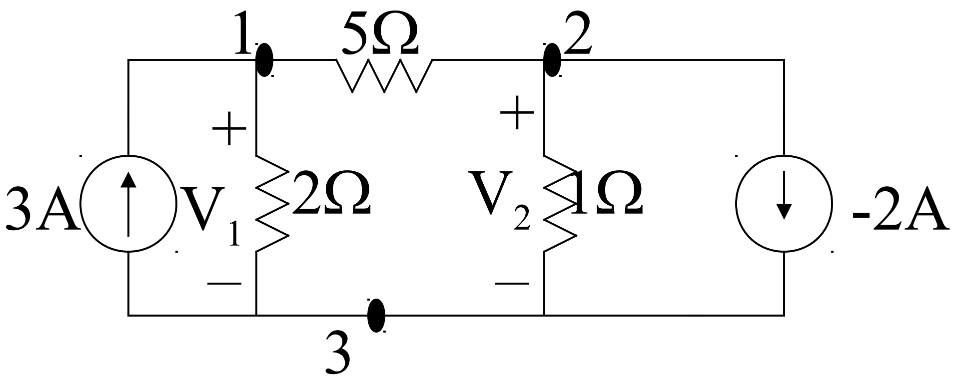
THE NODE-VOLTAGE METHOD



Consider the circuit with 3 nodes. Select one of the nodes as the reference node and assign voltages to two remaining nodes as V_1 and V_2.

Since the number of elements connected to node 3 is the greatest, choosing this node as the reference node will simplify the equations. Now apply KCL to nodes 1 and 2

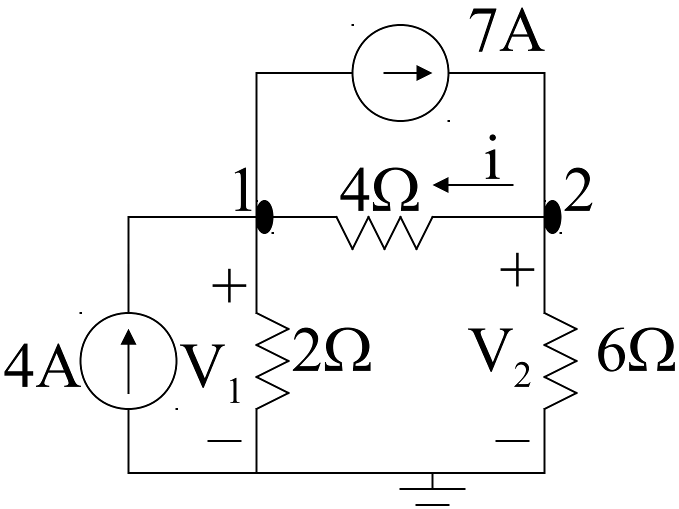
Assuming the entering currents to be negative and leaving currents to be positive

$$-3+\frac{V_1-0}{2}+\frac{V_1-V_2}{5}=0 \Rightarrow 0.5V_1+0.2(V_1-V_2)=3$$

$$\frac{V_2-0}{1}+\frac{V_2-V_1}{5}+(-2)=0 \Rightarrow -0.2V_1+1.2V_2=2$$

$$V_1=5V \qquad V_2=2.5V$$

EXAMPLE



Find i using nodal analysis.

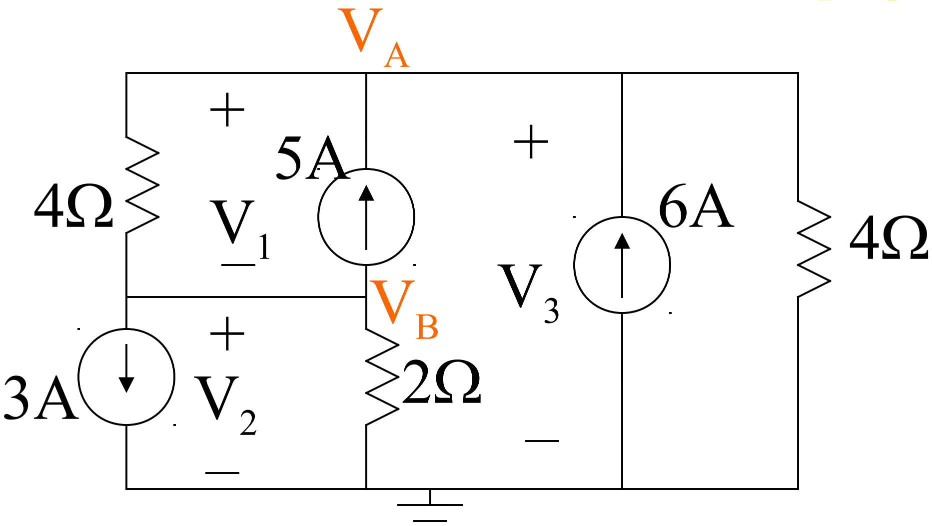
$$-4+\frac{V_1}{2}+\frac{V_1-V_2}{4}+7=0 \Rightarrow 3V_1-V_2=-12$$

$$\frac{V_2-V_1}{4}+\frac{V_2}{6}-7=0 \Rightarrow -3V_1+5V_2=84$$

$$V_1=2V \qquad V_2=18V$$

$$i=\frac{V_2-V_1}{4}=\frac{18-2}{4}=4A$$

EXAMPLE



Using nodal analysis, find V_1, V_2, and V_3.

There are 3 nodes in the circuit. Choosing the bottom node as the reference, assign voltages to nodes as V_A and V_B.

$$\frac{V_A - V_B}{4} - 5 - 6 + \frac{V_A}{4} = 0$$

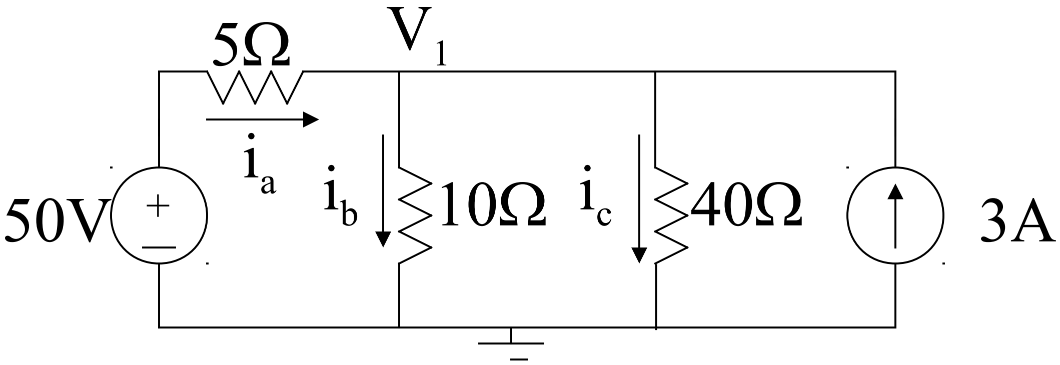
$$\frac{V_B - V_A}{4} + 3 + \frac{V_B}{2} + 5 = 0$$

$$V_A = 20V \qquad V_B = -4V$$

From the figure $V_2=V_B=-4V$ and $V_3=V_A=20V$. As you see they are node voltages. But, since V_1 is not defined with respect to the reference node, it is not a node voltage. It is the voltage across 4Ω resistor and 5A current source. It can be calculated as

$$V_A = V_1 + V_2 \Rightarrow 20 = V_1 - 4$$

$$V_1 = 24V$$



$$\frac{V_1 - 50}{5} + \frac{V_1}{10} + \frac{V_1}{40} - 3 = 0$$

$$V_1 = 40V$$

$$i_a = \frac{50-40}{5} = 2A$$

$$i_b = \frac{40}{10} = 4A$$

$$i_c = \frac{40}{40} = 1A$$

$$p_{50v} = -50 i_a = -100W (\text{delivering})$$

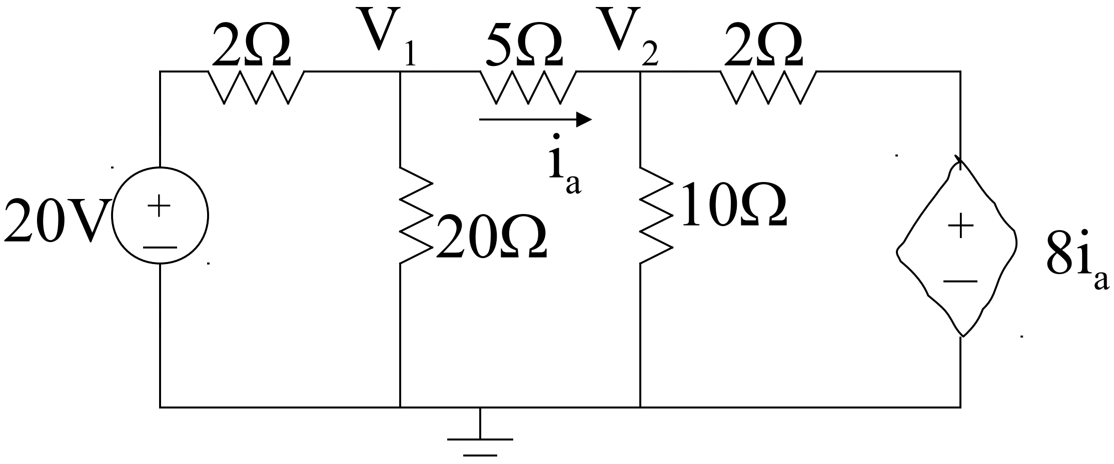
$$p_{3A} = -3v_1 = -120W (\text{delivering})$$

$$p_5 = (2)^2 5 = 20W (\text{absorbing})$$

$$p_{10} = (4)^2 10 = 160W (\text{absorbing})$$

$$p_{40} = (1)^2 40 = 40W (\text{absorbing})$$

EXAMPLE



$$\frac{V_1 - 20}{2} + \frac{V_1}{20} + \frac{V_1 - V_2}{5} = 0$$

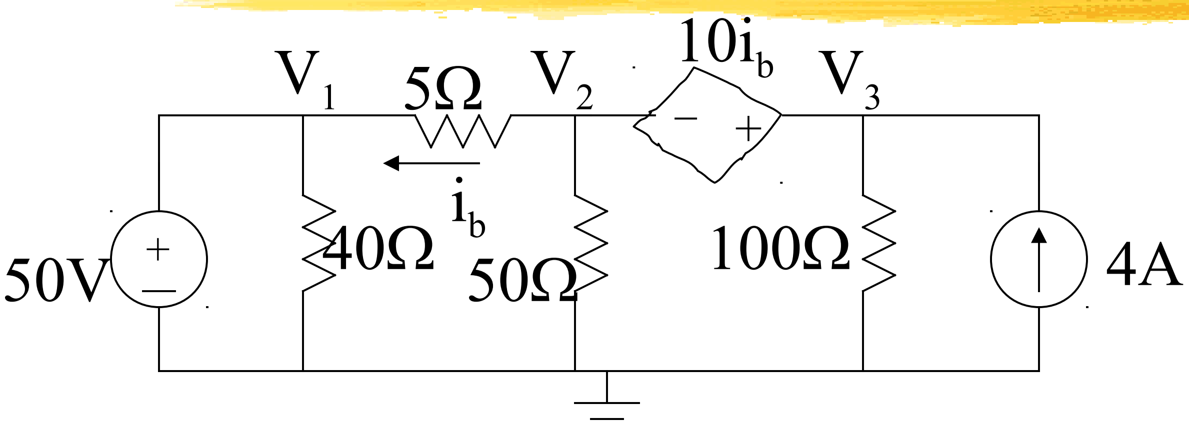
$$\frac{V_2 - V_1}{5} + \frac{V_2}{10} + \frac{V_2 - 8i_a}{2} = 0$$

$$i_a = \frac{V_1 - V_2}{5}$$

$V_1 = 16V \quad V_2 = 10V \quad i_a = 1.2A$

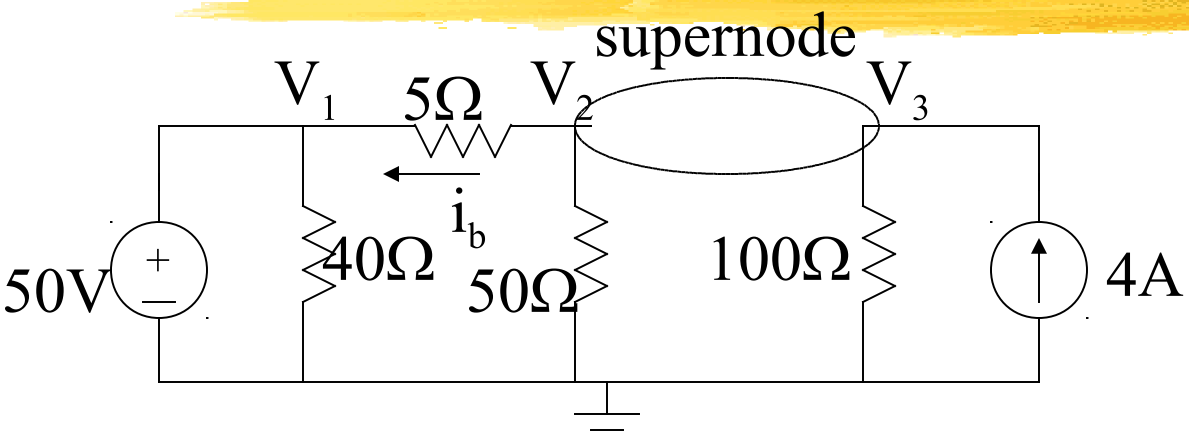
$p_5 = (1.2)^2 5 = 7.2W$

SUPERNODE



$V_1 = 50V$

To write the KCL equation at node 2 or node 3 we cannot express current through the dependent source in terms of V_2 or V_3. To do so, we consider nodes 2 and 3 to be a single node (replacing the voltage source by a short circuit) . This node is called a **supernode.** KCL must hold for the supernode



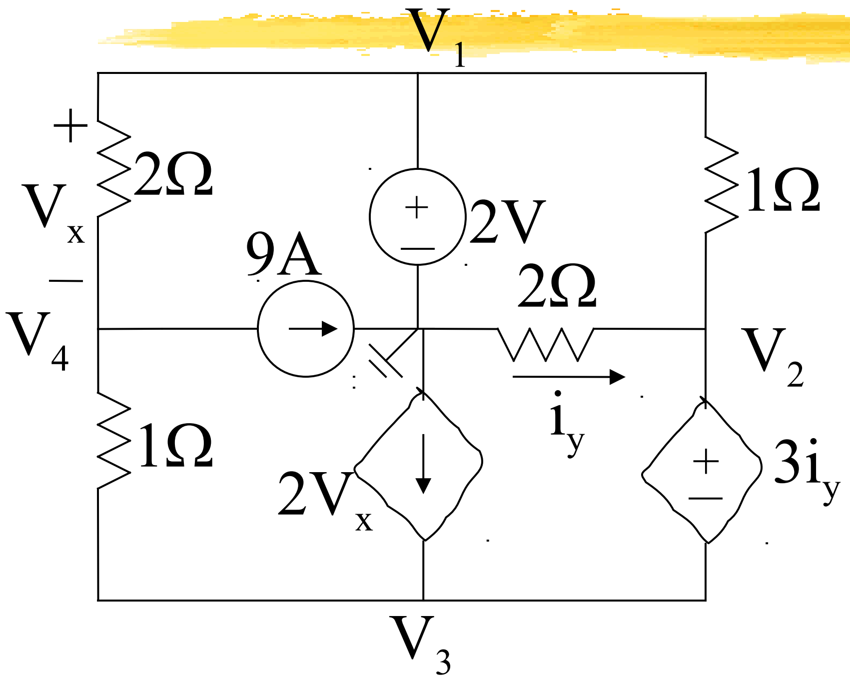
$$\frac{V_2 - V_1}{5} + \frac{V_2}{50} + \frac{V_3}{100} - 4 = 0$$

$$V_3 - V_2 = 10 i_b$$

$$i_b = \frac{V_2 - 50}{5}$$

$$V_2 = 60V \qquad V_3 = 80V \qquad i_b = 2A$$

EXAMPLE



$$V_1 = 2V \qquad V_2 = -2i_y$$

$$V_3 = -5i_y \qquad V_4 = 2 - V_x$$

$$-\frac{V_x}{2} + 9 + \frac{V_4 - V_3}{1} = 0$$

$$\frac{V_2 - V_1}{1} - i_y - 2V_x + \frac{V_3 - V_4}{1} = 0$$

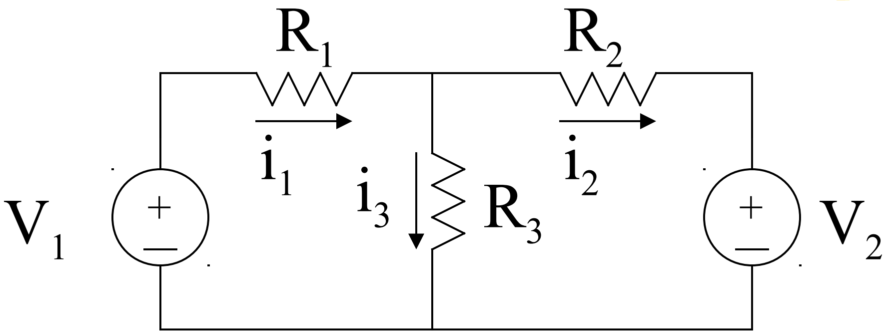
$$V_x = V_1 - V_4 \qquad i_y = -\frac{V_2}{2}$$

$$-3V_x + 10i_y = -22$$

$$-V_x - 8i_y = 4$$

$$V_x = 4V \qquad i_y = -1A$$

MESH-CURRENT METHOD



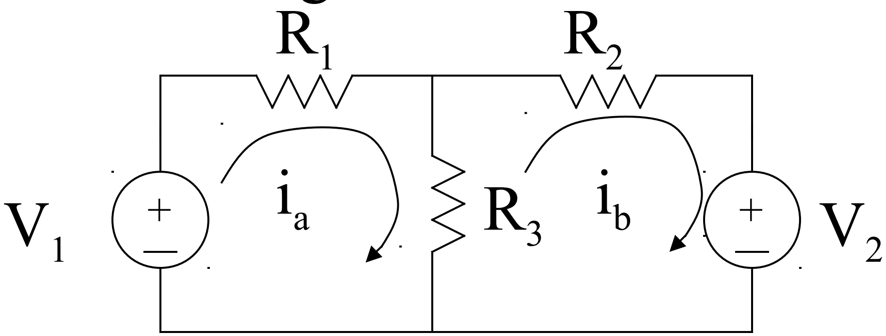
$$i_1 = i_2 + i_3$$
$$V_1 = i_1 R_1 + i_3 R_3$$
$$-V_2 = i_2 R_2 - i_3 R_3$$

Solving i_3 from the first equation and substituting into the other equations results

$$V_1 = i_1(R_1 + R_3) - i_2 R_3$$
$$-V_2 = -i_1 R_3 + i_2(R_2 + R_3)$$

Two equations can be solved to find i_1 and i_2.

i_1, i_2, and i_3 in the example are branch currents. Same problem can be solved by mesh currents. A mesh current is the current that exists only in the perimeter of a mesh. Consider the following circuit



i_a and i_b are mesh currents with arbitrary directions.

$$V_1 = i_a R_1 + (i_a - i_b) R_3$$

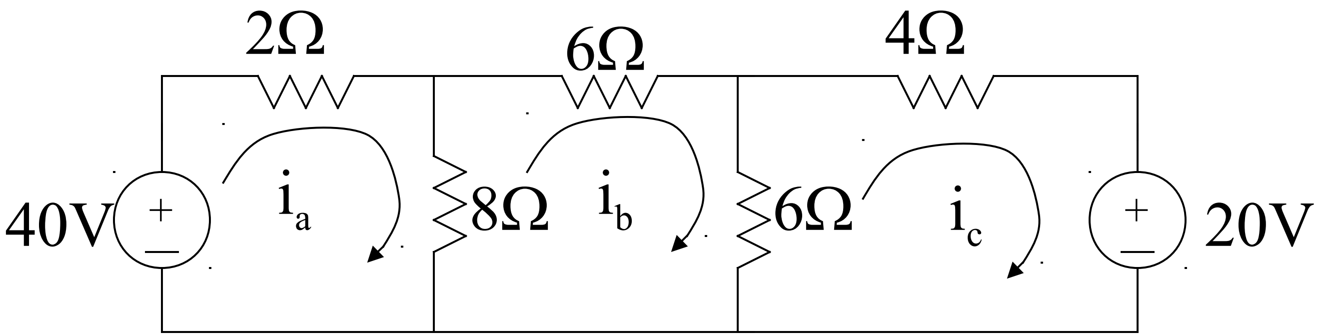
$$-V_2 = (i_b - i_a) R_3 + i_b R_2$$

$$V_1 = i_a(R_1 + R_3) - i_b R_3$$
$$-V_2 = -i_a R_3 + i_b(R_2 + R_3)$$

These equations are identical with the previous equations when the mesh currents i_a and i_b replace the branch currents i_1 and i_2. Note that the branch currents can be expressed in terms of the mesh currents as

$$i_1 = i_a \qquad i_2 = i_b \qquad i_3 = i_a - i_b$$

EXAMPLE

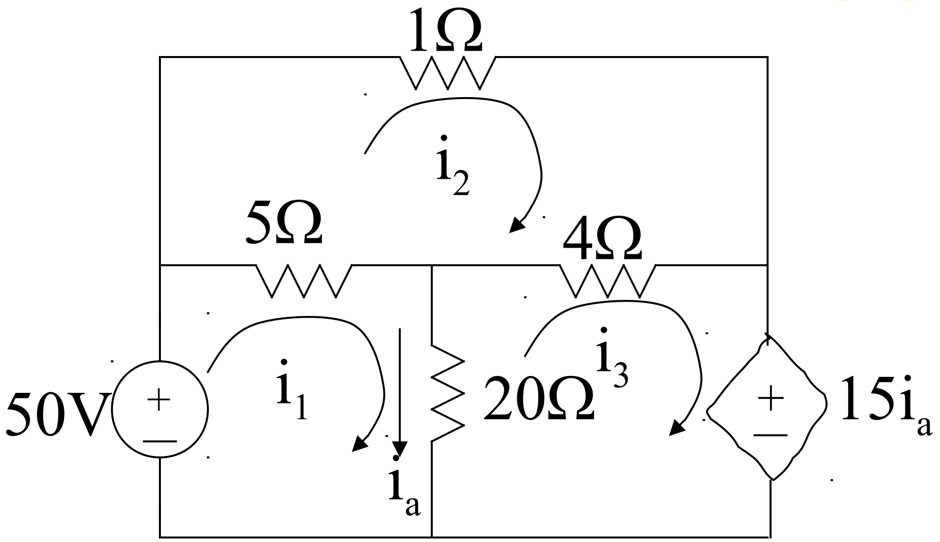


Use the mesh-current method to determine the power supplied by each source.

$$10i_a - 8i_b = 40$$
$$-8i_a + 20i_b - 6i_c = 0$$
$$-6i_b + 10i_c = -20$$

$$i_a = 5.6A \quad i_b = 2A \quad i_c = -0.8A$$
$$p_{40V} = -40i_a = -224W$$
$$p_{20V} = 20i_c = -16W$$

EXAMPLE



Use the mesh-current method to find the power dissipated in the 4Ω resistor.

$$25i_1 - 5i_2 - 20i_3 = 50$$

$$-5i_1 + 10i_2 - 4i_3 = 0$$

$$-20i_1 - 4i_2 + 24i_3 = -15i_a$$

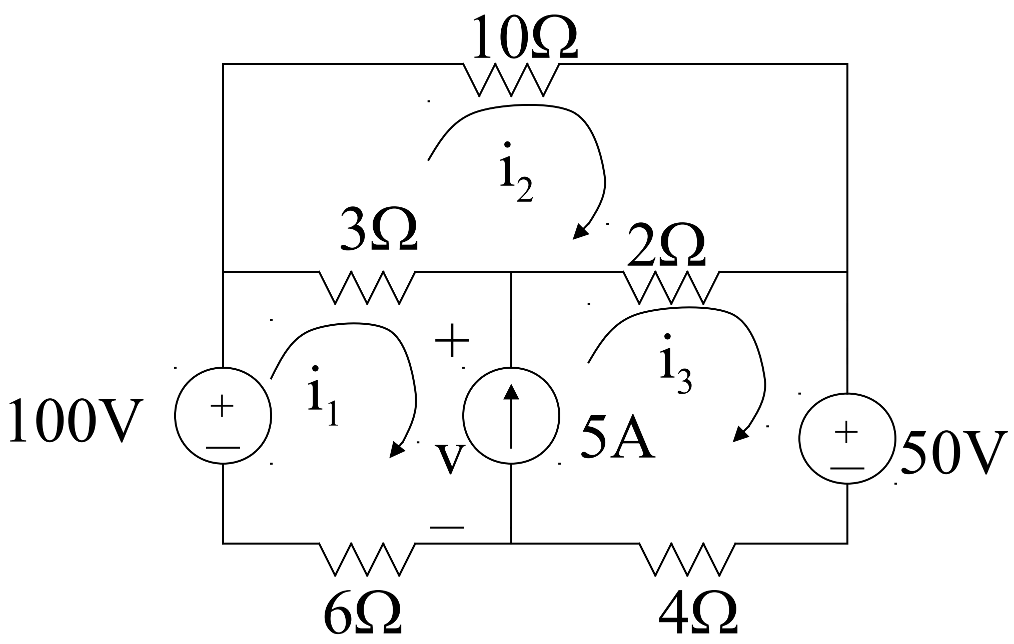
$$i_a = i_1 - i_3$$

$$i_2 = 26A \qquad i_3 = 28A$$

$$p_{4\Omega} = (i_3 - i_2)^2 4 = (2)^2 4 = 16W$$

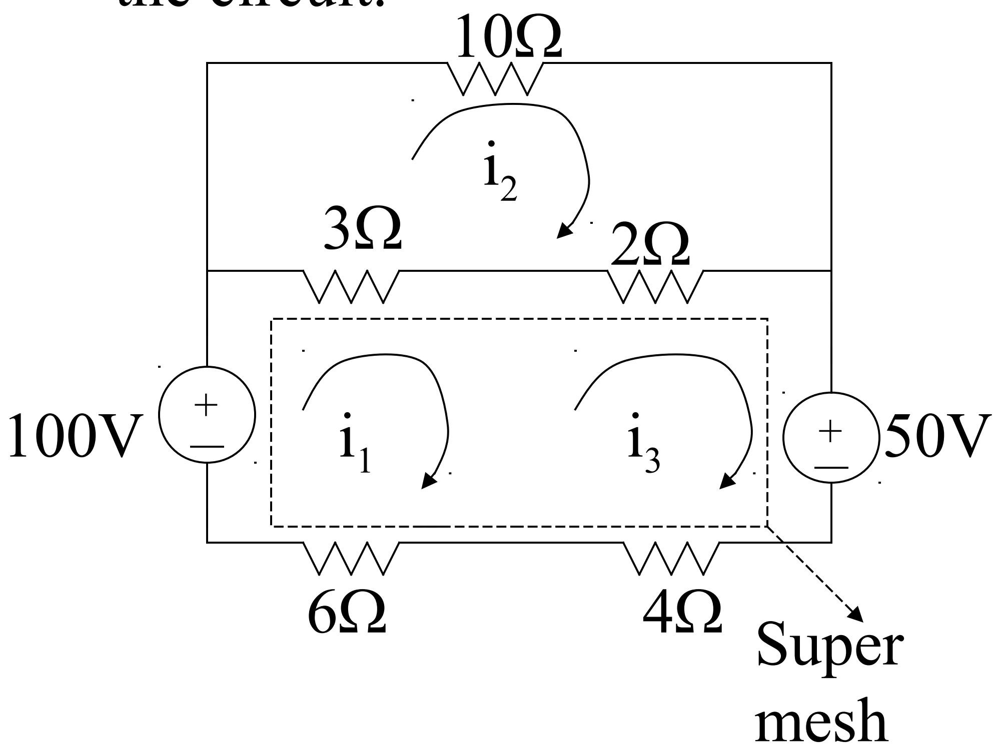
SUPERMESH

When a branch includes a current source, the mesh-current method requires some modifications. Consider the following circuit



When we attempt to write the KVL equation for left or right mesh containing the 5A current source, we cannot write the voltage drop in terms of i_2 and i_3. This difficulty can be solved using the supermesh concept.

A supermesh is created by removing the current source from the circuit.



For the upper mesh

$$-3i_1 + 15i_2 - 2i_3 = 0$$

For the supermesh

$$3(i_1 - i_2) + 2(i_3 - i_2) + 50$$

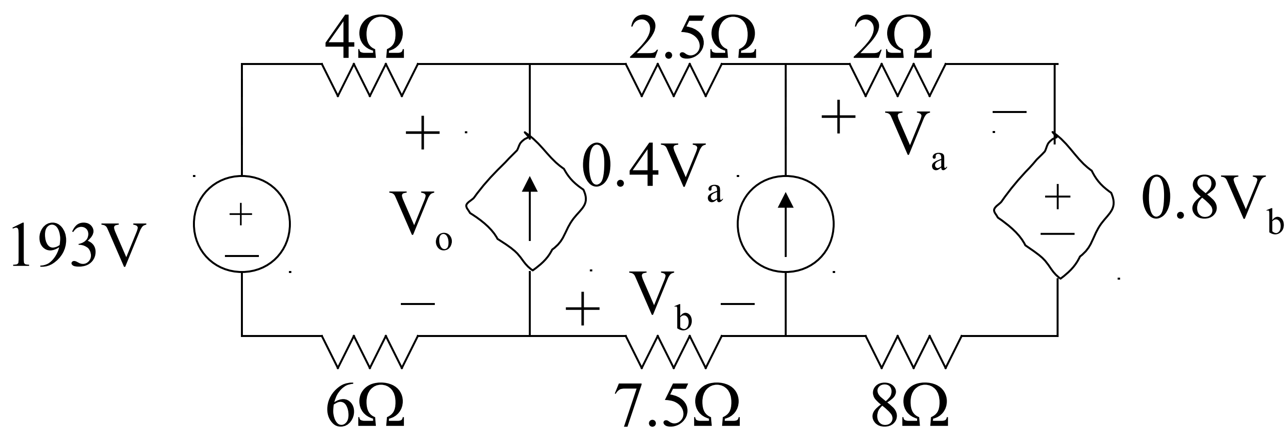
$$+ 4i_3 + 6i_1 - 100 = 0$$

$$9i_1 - 5i_2 + 6i_3 = 50$$

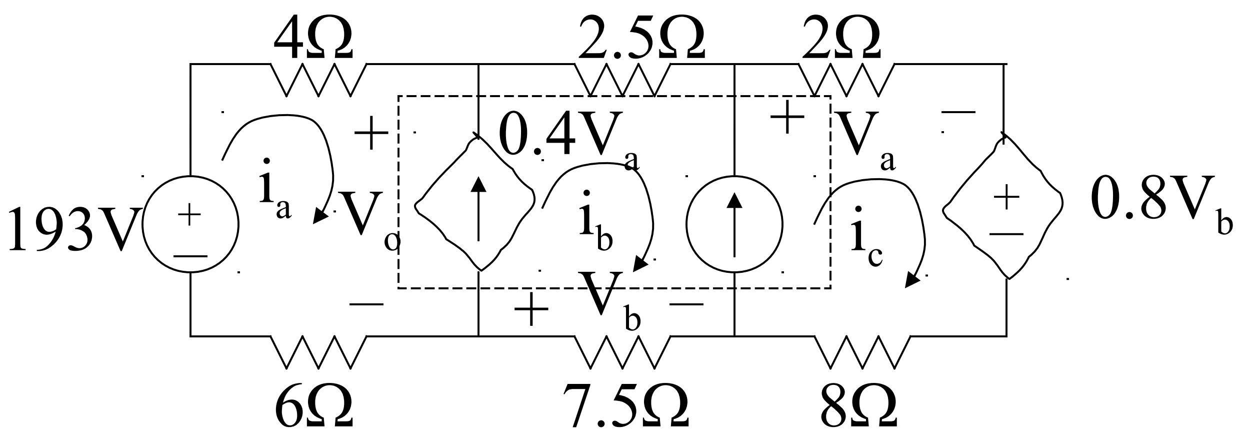
$$i_3 - i_1 = 5$$

$$i_1 = 1.75A \quad i_2 = 1.25A \quad i_3 = 6.75A$$

EXAMPLE



Using both the node-voltage and mesh-current methods, find the voltage V_o in the circuit.



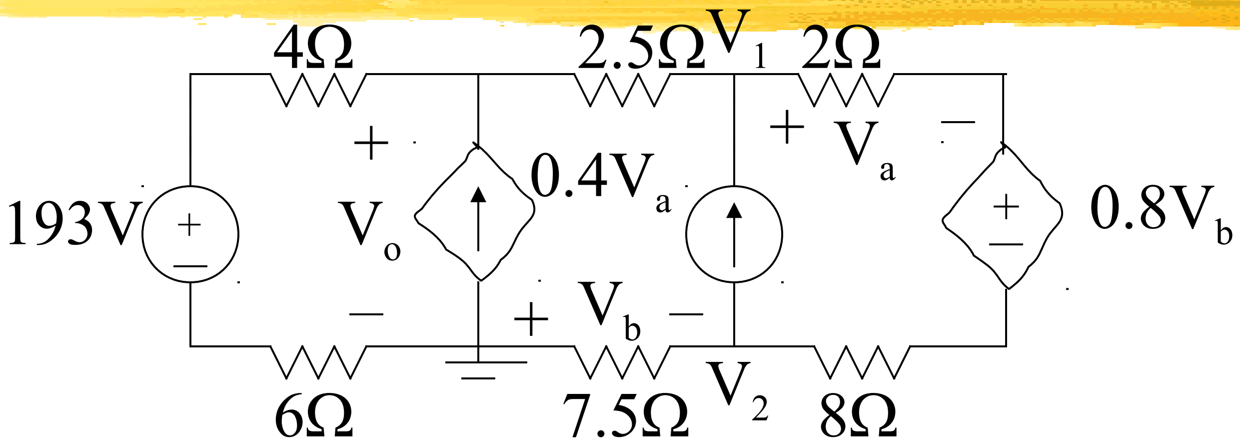
$$193 = 10i_a + 10i_b + 10i_c + 0.8V_b$$

$$i_b - i_a = 0.4V_a = 0.8i_c$$

$$V_b = -7.5i_b \qquad i_c - i_b = 0.5$$

$$160 = 80i_a \Rightarrow i_a = 2A$$

$$V_0 = 193 - 20 = 173V$$



$$\frac{V_o - 193}{10} - 0.4V_a + \frac{V_o - V_1}{2.5} = 0$$

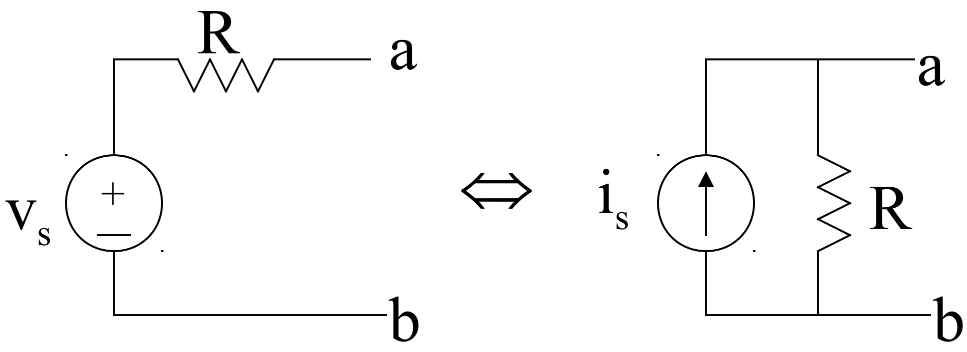
$$\frac{V_1 - V_o}{2.5} - 0.5 + \frac{V_1 - (V_2 + 0.8V_b)}{10} = 0$$

$$\frac{V_2}{7.5} + 0.5 + \frac{V_2 + 0.8V_b - V_1}{10} = 0$$

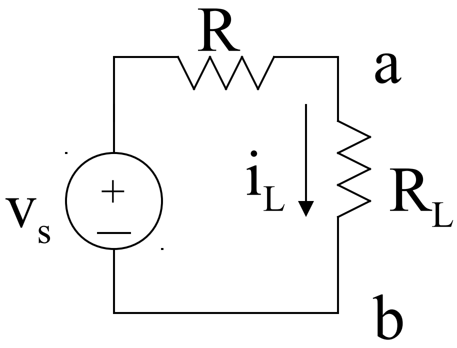
$$V_b = -V_2 \qquad V_a = \left[\frac{V_1 - (V_2 + 0.8V_b)}{10}\right]2$$

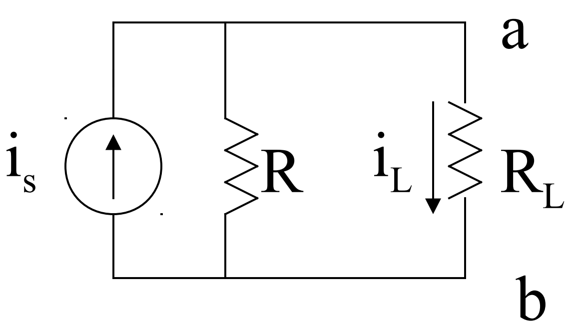
SOURCE TRANSFORMATIONS

A **source transformation** allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor or vice versa. Following figure shows the source transformation.



We need to find the relationship between v_s and i_s that guarantees the two circuits to be equivalent with respect to nodes a and b.



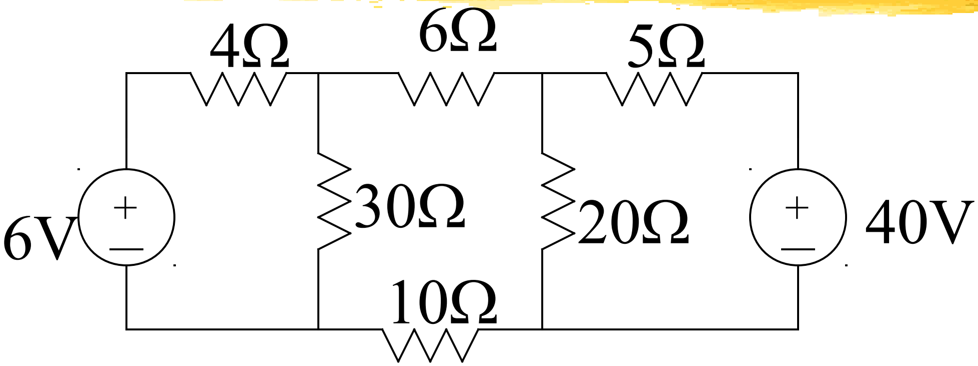


$$i_L = \frac{v_s}{R + R_L}$$

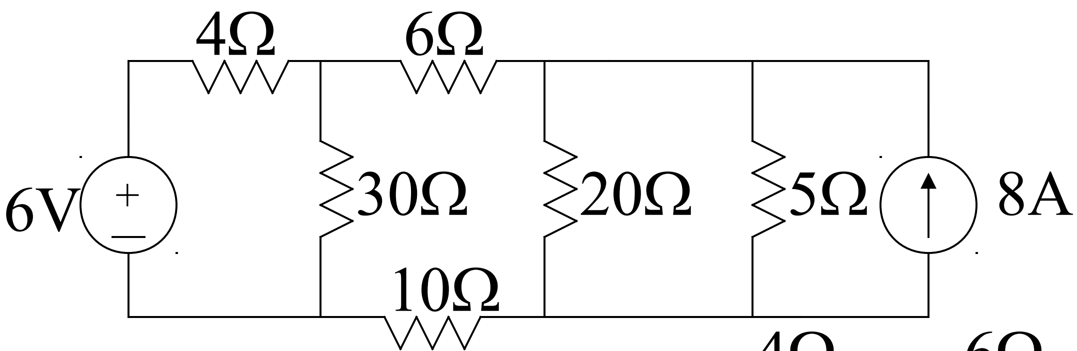
$$i_L = \frac{R}{R + R_L} i_s$$

From the equations it is seen that $i_s = \dfrac{V_s}{R}$

EXAMPLE

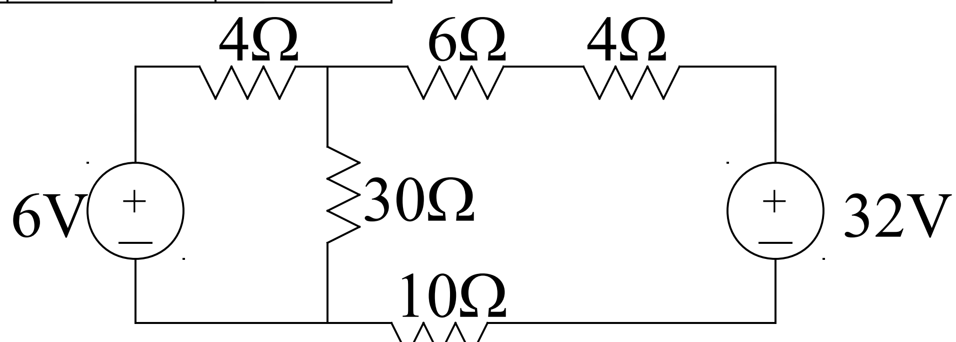


Using successive source transformations, find the delivered (absorbed) by the 6V source.

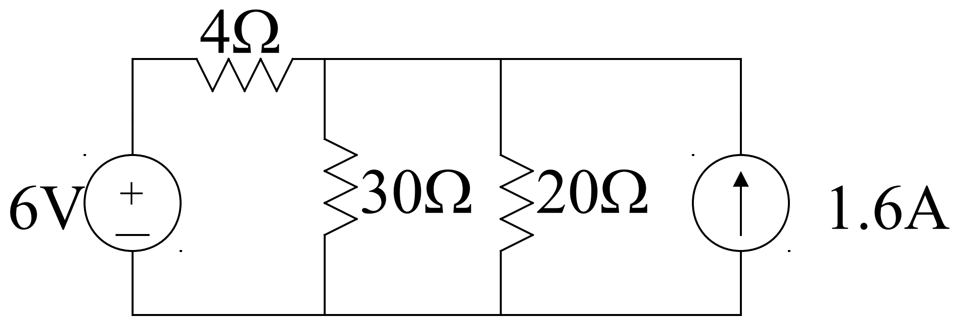


$20||5 = 4\Omega$

$4(8) = 32V$

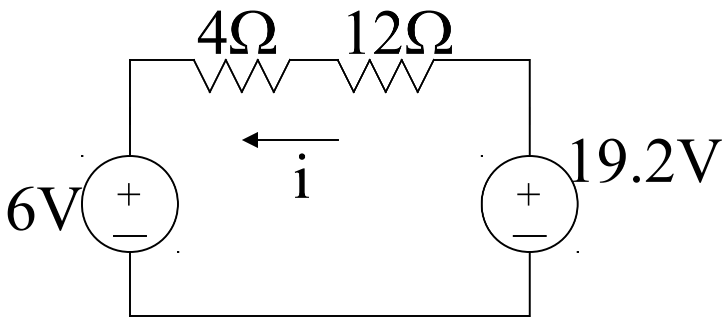


4+6+10=20Ω   32/20=1.6A



30||20=12Ω

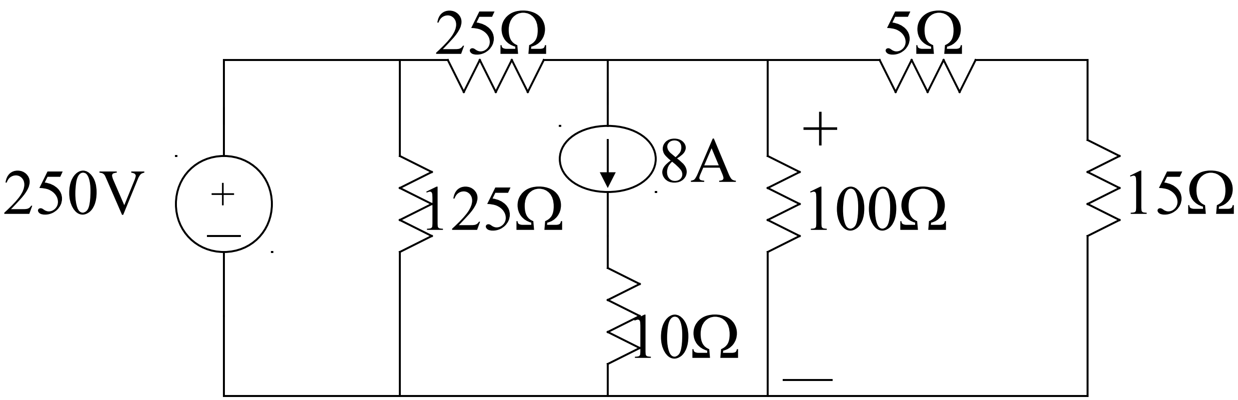
12(1.6)=19.2V



$$i = \frac{(19.2 - 6)}{16} = 0.825\,A$$

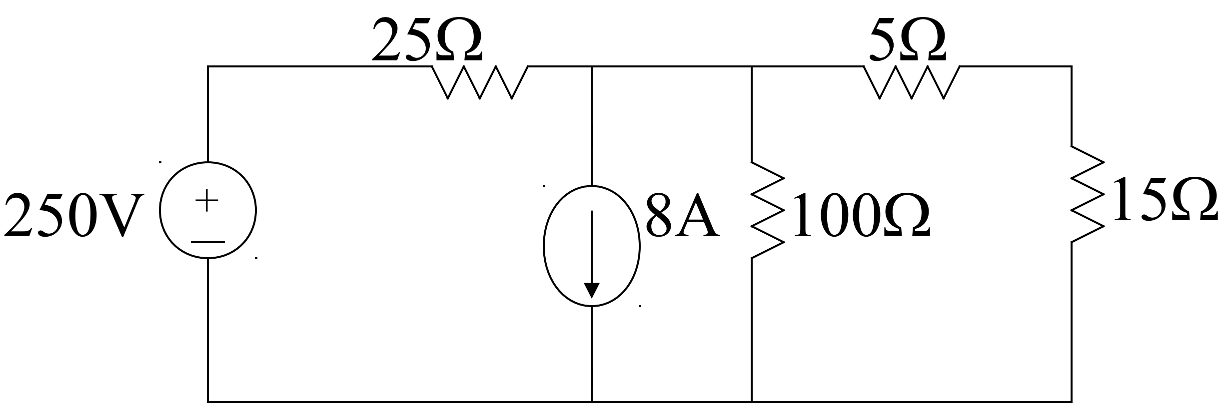
$$p_{6V} = (0.825)(6) = 4.95W \text{ (absorbing)}$$

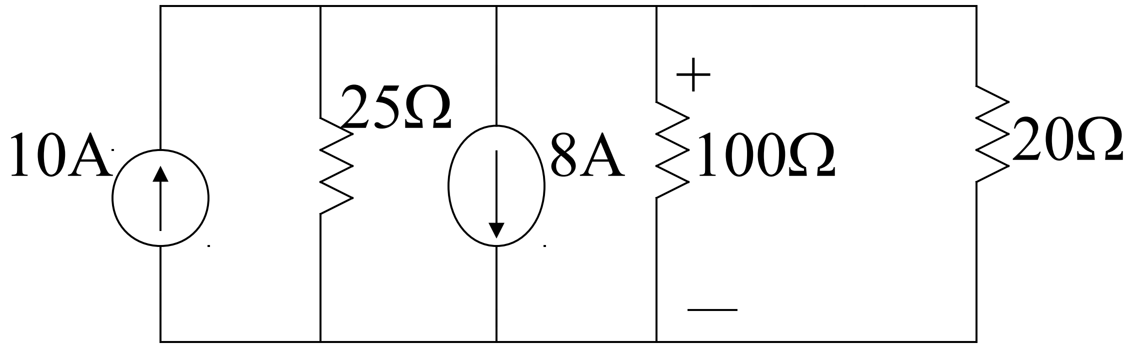
EXAMPLE



Using the source transformations, find the voltage across 100Ω resistor.

Find the power developed by 250V voltage source in the circuit.





25||100||20=10Ω

10-8=2A

V_{100}=2(10)=20V

Current supplied by the 250V source is

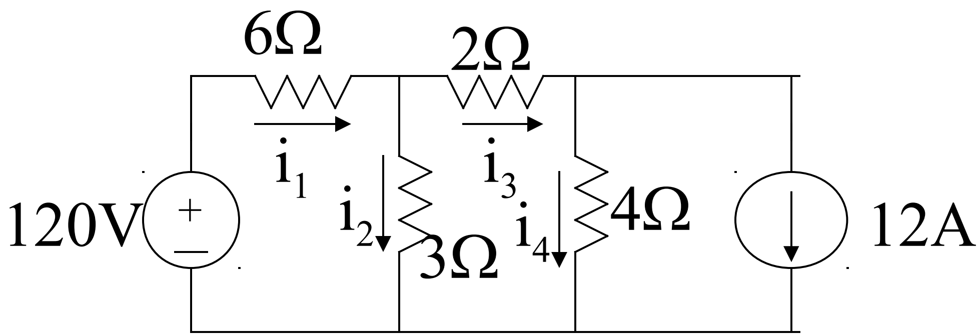
$$i_s = \frac{250}{125} + \frac{250-20}{25} = 11.2A$$

$$p_{250V} = (250)(11.2) = 2800W$$

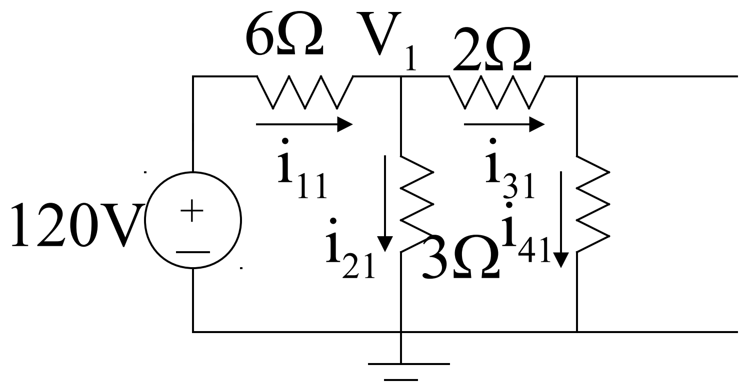
SUPERPOSITION

- A linear system obeys the principle of **superposition**, which states that when a linear circuit is driven by more than one independent source of energy, the total response is the sum of the individual responses. An individual response is the result of an independent source acting alone.

EXAMPLE



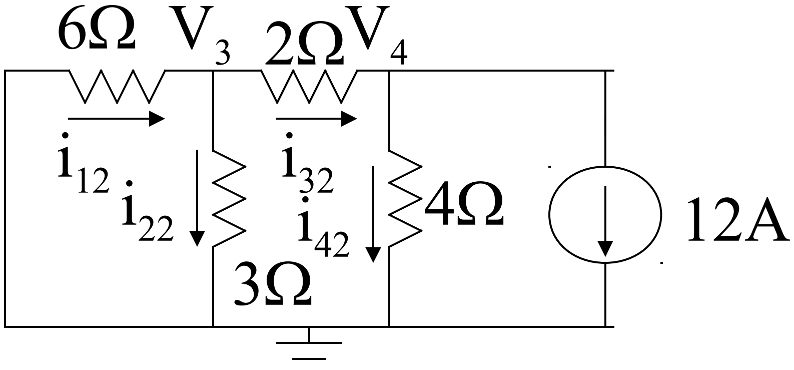
Use superposition to find the branch currents in the circuit.



$$\frac{V_1 - 120}{6} + \frac{V_1}{3} + \frac{V_1}{2+4} = 0 \Rightarrow V_1 = 30V$$

$$i_{11} = \frac{120 - 30}{6} = 15A \qquad i_{21} = \frac{30}{3} = 10A$$

$$i_{31} = i_{41} = \frac{30}{6} = 5A$$



$$\frac{V_3}{3}+\frac{V_3}{6}+\frac{V_3-V_4}{2}=0$$

$$\frac{V_4-V_3}{2}+\frac{V_4}{4}+12=0$$

$$V_3=-12V \qquad V_4=-24V$$

$$i_{12}=\frac{-V_3}{6}=\frac{12}{6}=2A \qquad i_{22}=\frac{V_3}{3}=-4A$$

$$i_{32}=\frac{V_3-V_4}{2}=\frac{-12+24}{2}=6A$$

$$i_{42}=\frac{V_4}{4}=-6A$$

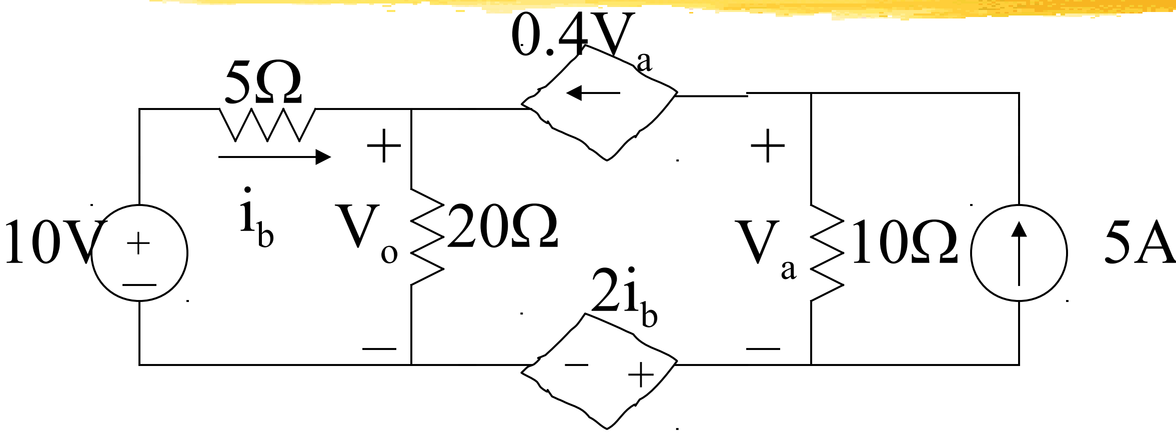
$$i_1=i_{11}+i_{12}=5+12=17A$$

$$i_2=i_{21}+i_{22}=10-4=6A$$

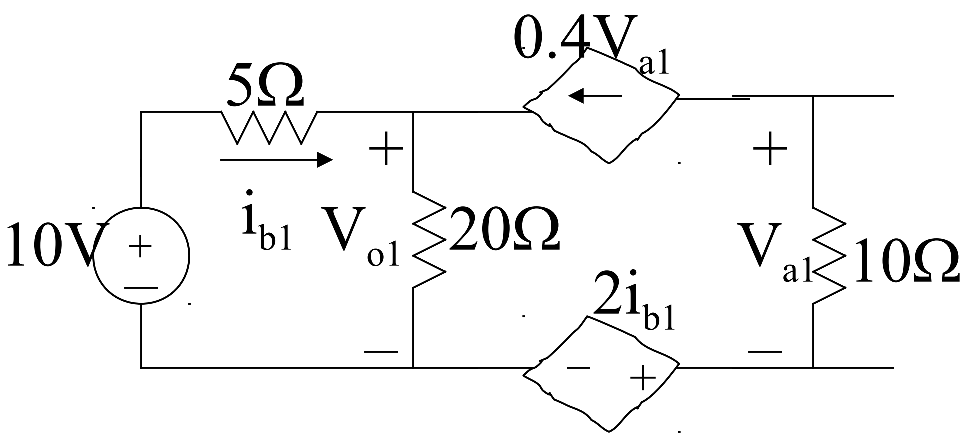
$$i_3=i_{31}+i_{32}=5+6=11A$$

$$i_4=i_{41}+i_{42}=5-6=-1A$$

EXAMPLE

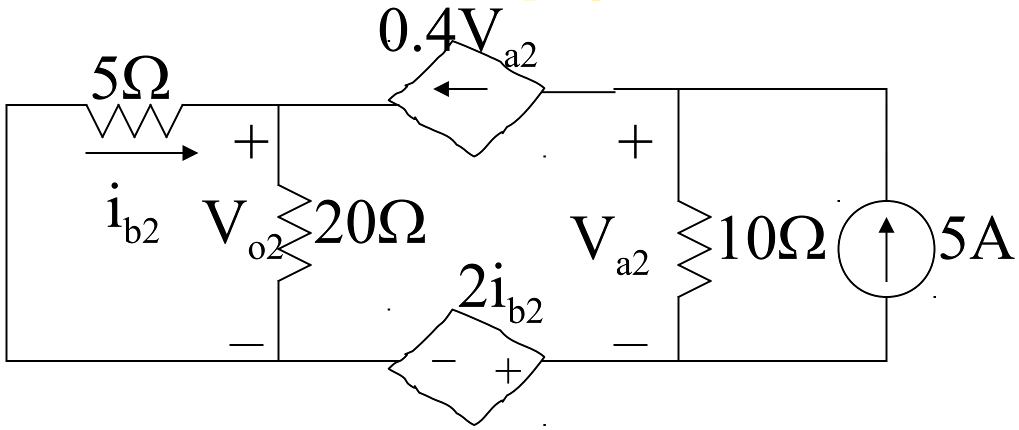


Use the principle of superposition to find V_o in the circuit.



Since V_{a1}=-(0.4V_{a1})10=-4V_{a1}, V_{a1}=0. Then the branch containing two dependent sources is open.

$$V_{o1} = \frac{20}{20+5}10 = 8V$$



$$0.4V_{a2} + \frac{V_{a2}}{10} - 5 = 0 \Rightarrow V_{a2} = 10V$$

$$\frac{V_{o2}}{20} + \frac{V_{o2}}{5} - 0.4V_{a2} = 0 \Rightarrow V_{o2} = 16V$$

$$V_o = V_{o1} + V_{o2} = 8 + 16 = 24V$$