CIRCUIT ELEMENTS

OSMAN PARLAKTUNA
Osmangazi University
Eskişehir, TURKEY
www.ogu.edu.tr/~oparlak

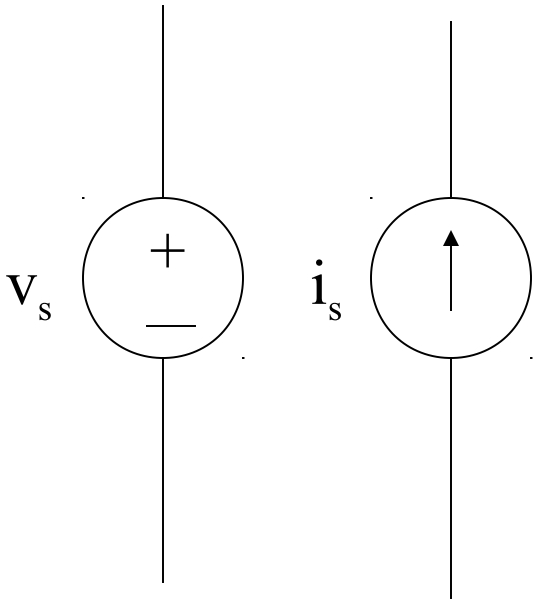
INDEPENDENT IDEAL VOLTAGE AND CURRENT SOURCES

An electrical source is a device that is capable of converting non-electric energy to electric energy and vice versa.

An **independent source** establishes a voltage or current in a circuit without relying on voltages or currents elsewhere in the circuit.

An **ideal voltage source** is a circuit elements that maintains a prescribed voltage across its terminals regardless of the current flowing through it.

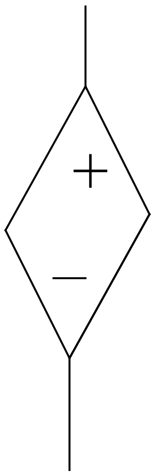
An **ideal current source** is a circuit element that maintains a prescribed current through its terminals regardless of the voltage across those terminals.



DEPENDENT IDEAL VOLTAGE AND CURRENT SOURCES

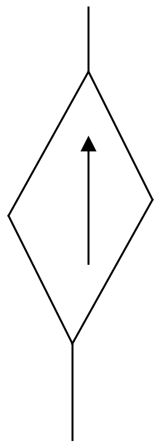
A **dependent** source establishes a voltage or current whose value depends on the value of a voltage or current elsewhere in the circuit.

$v_s = \mu v_x$



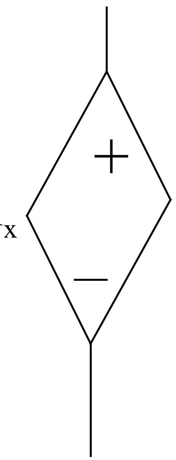
Voltage controlled voltage source
VCVS

$i_s = \alpha v_x$



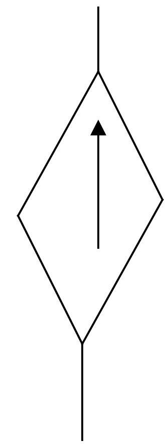
Voltage controlled current source
VCCS

$v_s = \rho i_x$



Current controlled voltage source
CCVS

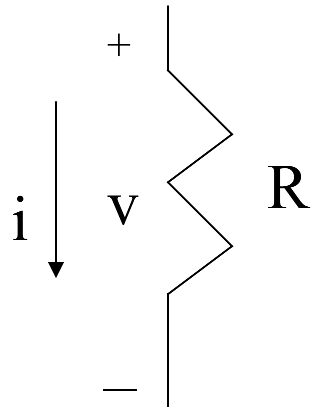
$i_s = \beta i_x$



Current controlled current source
CCCS

ELECTRICAL RESISTANCE (OHM'S LAW)

Resistance is the capacity of materials to impede the flow of current. The circuit element used to model this behavior is the resistor. For purposes of circuit analysis, we must reference the current in the resistor to the terminal voltage. We use **positive sign convention**: the direction of the voltage drop is in the direction of the current.



Ohm's Law: Voltage across a resistor is proportional to the current flowing through the resistor. The proportionality constant is the resistance.

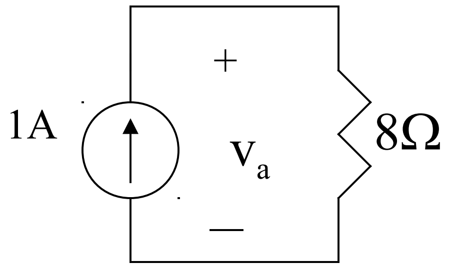
$$v = iR$$

The unit for resistance is ohms (Ω)

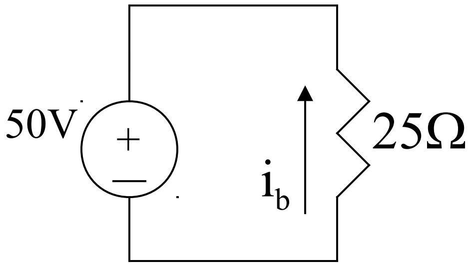
POWER OF A RESISTOR

Power at the terminals of a resistor is the product of the voltage and current.

$$p = vi = i^2 R = \frac{v^2}{R}$$



$$v_a = (1)(8) = 8V \qquad p = \frac{8^2}{8} = (1)^2 8 = 8W$$



$$i_b = \frac{-50}{25} = -2A, \qquad p = \frac{(50)^2}{25} = (-2)^2 25 = 100W$$

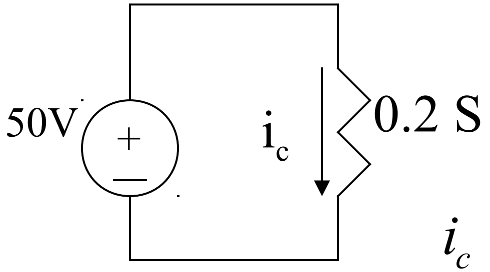
CONDUCTANCE

The reciprocal of a resistance is referred to as **conductance** and it is measured in in mhos or siemens (S).

$$G = \frac{1}{R}$$

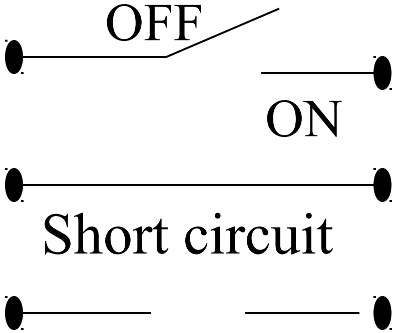
For example an 8 Ω resistor has a conductance of 0.125 S.

Power in terms of conductance is: $p=i^2/G=v^2G$.



$$i_c = (50)(0.2) = 10A \qquad p = (50)^2(0.2) = \frac{(10)^2}{0.2} = 500W$$

SHORT CIRCUIT AND OPEN CIRCUIT



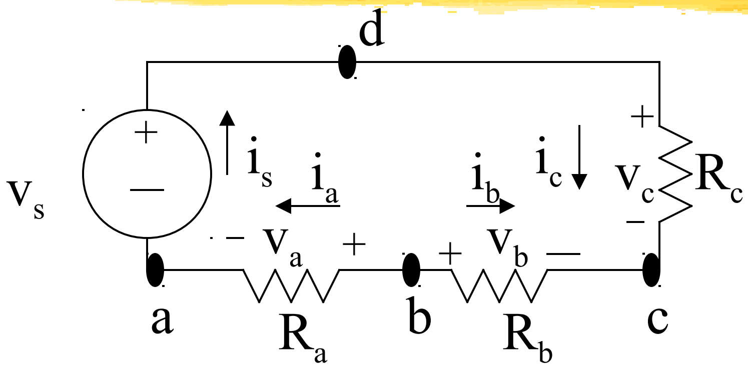
Open circuit

A switch is a two-state device. It is either ON or OFF. An ideal switch offers no resistance to the current when it is in the ON state, but it offers infinite resistance to the current in the OFF state.

These two states represent the limiting values of a resistor. The ON state corresponds to a resistor with a value of zero resistance. This case is called the **short circuit**. In a short circuit, $R=0$ and $v=0$, but there may be a finite current flow through the circuit.

The OFF state corresponds to a resistor with a numerical value of infinity. This state is called the **open circuit**. In an open circuit, $R=\infty$, $i=0$, but there may be finite voltage across the circuit.

KIRCHOFF'S LAWS



Consider the given circuit. We have placed dots in this circuit. Those dots are the start and end points of an individual circuit element.

A **node** is a point where two or more circuit elements meet. It is necessary to identify the nodes in a circuit in order to use the Kirchoff's current law. There are 7 unknowns in this circuit: i_s, i_a, i_b, i_c, v_a, v_b, and v_c. To find these unknowns we need 7 independent equations. From Ohm's law we can write 3 equations:

$$v_a = i_a R_a \qquad v_b = i_b R_b \qquad v_c = i_c R_c$$

KIRCHOFF'S CURRENT LAW

The algebraic sum of all currents at any node in a circuit equals zero. To use KCL, an algebraic sign corresponding to a reference direction must be assigned to every current at the node. For example, assigning a positive sign to entering currents to a node requires a negative sign assignment to leaving currents (or vice versa). In the above circuit, assuming that currents leaving a node have positive sign yields

node a $i_s - i_a = 0$

node b $i_a - i_b = 0$

node c $-i_b - i_c = 0$

node d $i_c - i_s = 0$

Now, there are 7 equations to find 7 unknowns.

KIRCHOFF'S VOLTAGE LAW

Closed path or **loop**: Starting at an arbitrary node, trace a closed path in a circuit through circuit elements and return to the original node without passing through each element and node more than once.

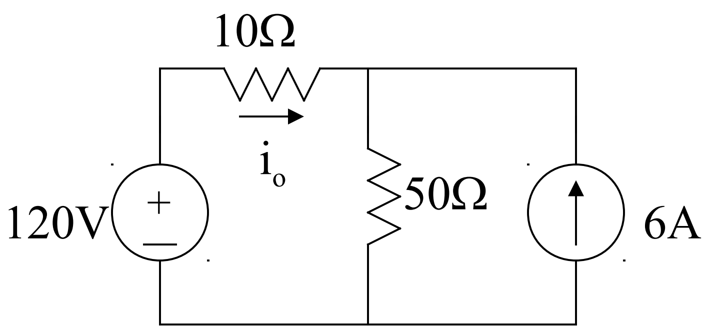
Kirchoff's Voltage Law (KVL) The algebraic sum of all the voltages around any closed path in a circuit equals zero.

To use KVL, we must assign an algebraic sign to each voltage in the loop. As we trace a closed path, a voltage will appear either as a rise or a drop in the tracing direction. Assigning a positive sign to a voltage rise requires assigning a negative sign to a voltage drop.

Starting at node d and tracing clockwise yields:

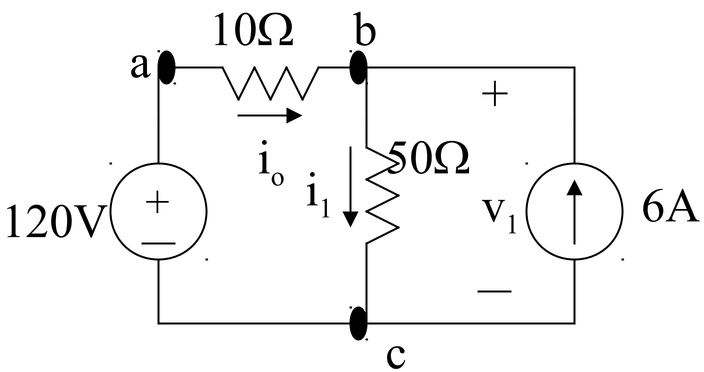
$$v_c - v_b + v_a - v_s = 0$$

EXAMPLE



a) Use Kirchoff's Laws and Ohm's Law to find i_o in the circuit

b) Test the solution by verifying that the total power generated equal the total power dissipated.



Using KCL at node b by assigning positive sign to the leaving currents

$$i_i - i_o - 6 = 0$$

Summing the voltages around the closed path cabc

$$-120 + 10i_o + 50i_1 = 0$$

Solving the two equations yields: i_o=-3A, i_1=3A

b) $p_{50\Omega}= (3)^2 50=450$ W

$p_{10\Omega}= (-3)^2 10=90$ W

$p_{120V}=-120i_o=-120(-3)=360$ W

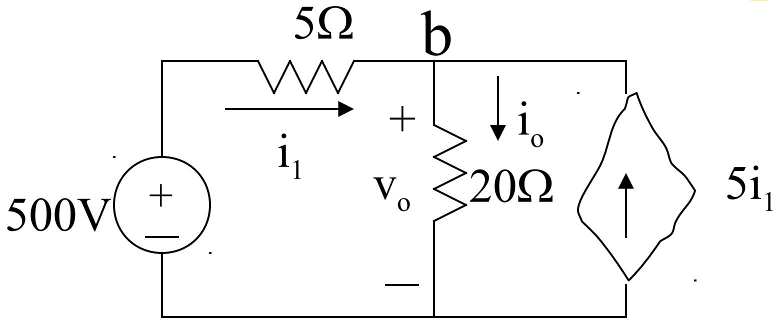
$p_{6A}=-v_1(6)=-50i_1(6)=-50(3)(6)=-900$ W

$p_{con}=450+90+360=900$ W

$p_{del}=900$ W

$p_{con}=p_{del}$

EXAMPLE



Writing KVL for the left loop

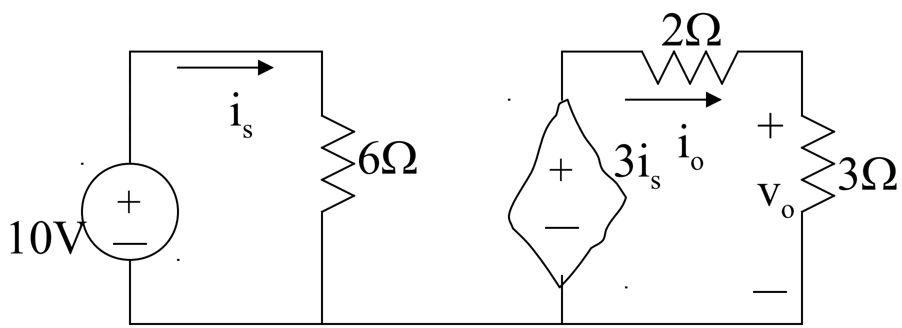
$$500=5i_1+20i_o$$

KCL equation at node b $i_o=i_1+5i_1=6i_1$

Solving the equations yields: $i_1=4$ A, $i_o=24$ A

$v_o=20i_o=480$ V

EXAMPLE



Writing KVL for both loops

$10=6i_s$

$3i_s=2i_o+3i_o$

$i_s=1.67$ A, $i_o=1$ A

$v_o=3i_o=3$ V

$p_{10V}=10(-1.67)=-16.7$ W,

$p_{3is}=(3i_s)(-i_o)=(5)(-1)=-5$ W

$p_{6\Omega}=(1.67)^2(6)=16.7$ W

$p_{2\Omega}=(1)^2(2)=2$ W

$p_{3\Omega}=(1)^2(3)=3$ W

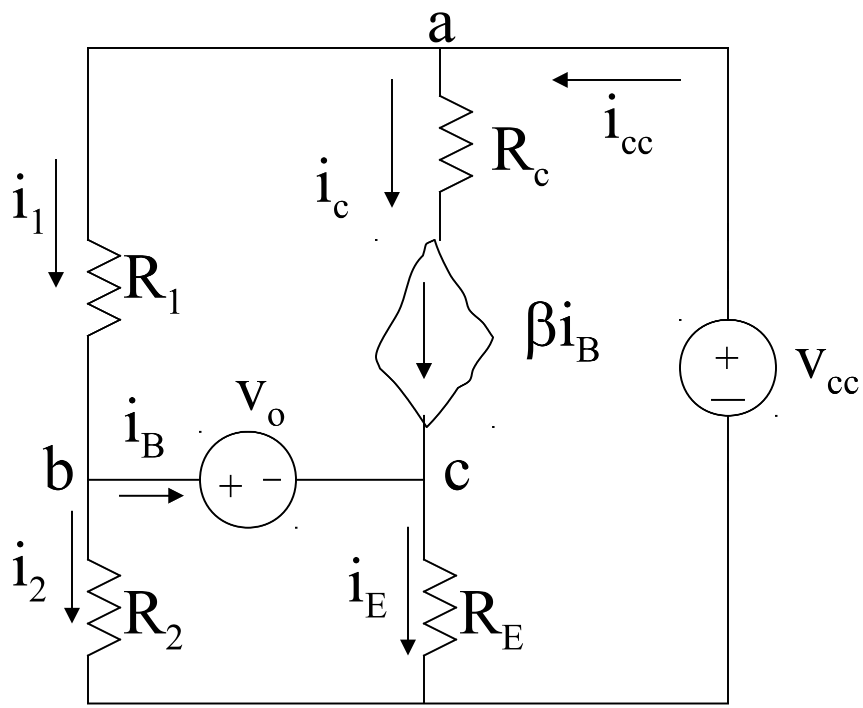
$p_{del}=p_{10V}+p_{3is}=16.7+5=21.7$ W

$p_{con}=p_{6\Omega}+p_{2\Omega}+p_{3\Omega}$

$=16.7+2+3=21.7$ W

$p_{del}=p_{con}$

EXAMPLE



Writing the KCL equations at nodes a, b, and c.

$i_1+i_c-i_{cc}=0$

$i_B+i_2-i_1=0$

$i_E-i_B-i_c=0$

and $i_c=\beta i_B$

Writing the KVL equations for the lower left loop and the outer loop

$$v_o+i_ER_E-i_2R_2=0$$

$$-i_1R_1+v_{cc}-i_2R_2=0$$

In order to write i_B in terms of the known circuit variables, eliminate other currents from the equations by substitutions. Then,

$$i_B = \frac{(v_{cc}R_2)/(R_1+R_2)-v_o}{(R_1R_2)/(R_1+R_2)+(1+\beta)R_E}$$