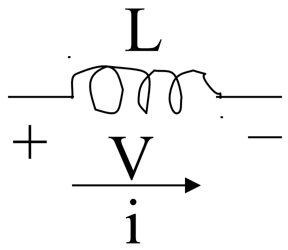
INDUCTANCE, CAPACITANCE AND MUTUAL INDUCTANCE

OSMAN PARLAKTUNA
OSMANGAZİ UNIVERSITY
ESKİŞEHİR, TURKEY
www.ogu.edu.tr/~oparlak

THE INDUCTOR

- An inductor is a electrical element that opposes any change in electrical current.
- It is composed of a coil of wire wound around a supporting core.
- The source of the magnetic field is current. If the current is varying with time, the magnetic field is varying with time. A time-varying magnetic field induces a voltage in any conductor linked by the magnetic field.

INDUCTANCE



Inductance is the circuit parameter used to describe an inductor. Inductance is symbolized by the letter L, is measured in henrys (H).

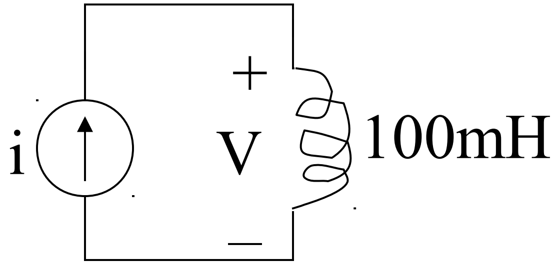
The voltage-current relationship for an inductor is given as

$$V = L\frac{di}{dt}$$

a) If the current is constant, the voltage across the ideal inductor is zero. Thus the inductor behaves as short circuit in the presence of dc current.

b) Current cannot change instantaneously in an inductor. An instantaneous change in current would require an infinite voltage, and infinite voltages are not possible.

EXAMPLE



$$i = \begin{cases} 0 & t < 0 \\ 10te^{-5t}A & t > 0 \end{cases}$$

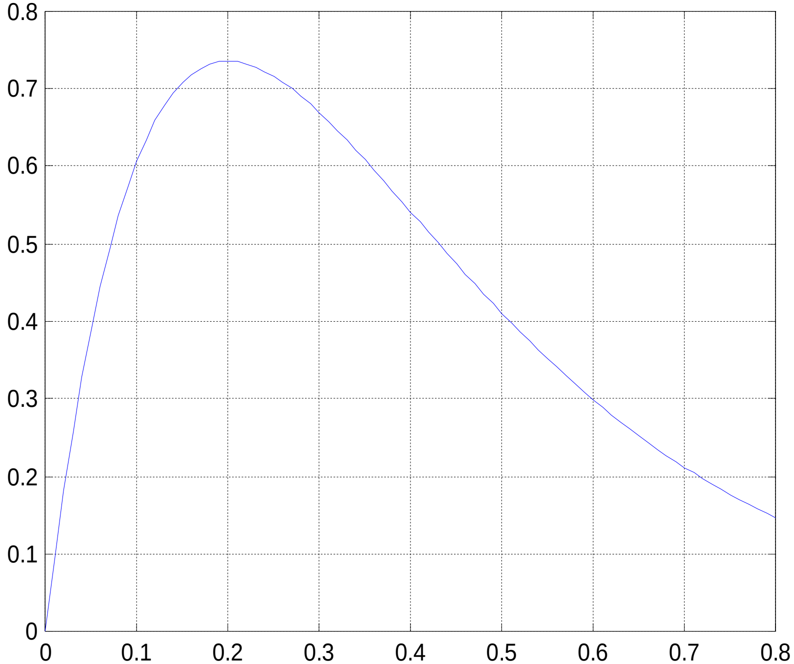
Find V. At what instant of time is the current maximum?

$$V = L\frac{di}{dt} = \begin{cases} 0 & t < 0 \\ (0.1)10e^{-5t}(1-5t) & t > 0 \end{cases}$$

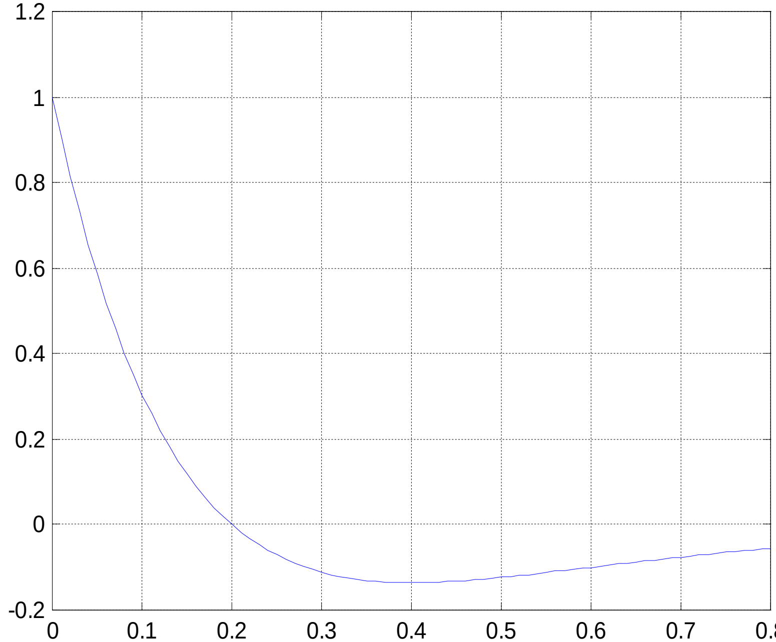
$$\frac{di}{dt} = 10e^{-5t}(1-5t)A/s$$

$$\frac{di}{dt} = 0 \quad \text{when} \quad t = 0.2s$$

$$i\Big|_{t=0.2s} = 10(0.2)e^{-5(0.2)} = 0.736A$$



Current



Voltage

Current in terms of Voltage

$$V = L\frac{di}{dt} \Rightarrow Vdt = Ldi$$

$$L\int_{i(t_0)}^{i(t)} dx = \int_{t_0}^{t} Vd\tau$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} Vd\tau + i(t_0)$$

Power and Energy

$$p = Vi = Li\frac{di}{dt}$$

$$p = V\left[\frac{1}{L}\int_{t_0}^{t} Vd\tau + i(t_0)\right]$$

Since power is the time rate of expending energy

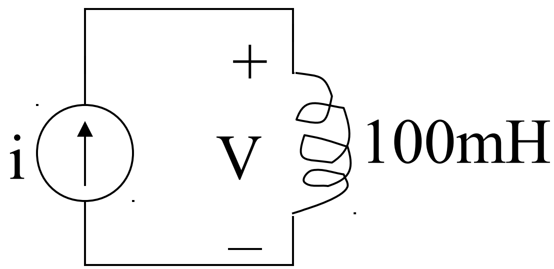
$$p = \frac{d\omega}{dt} = Li\frac{di}{dt}$$

$$d\omega = Lidi$$

$$\omega = \frac{1}{2}Li^2$$

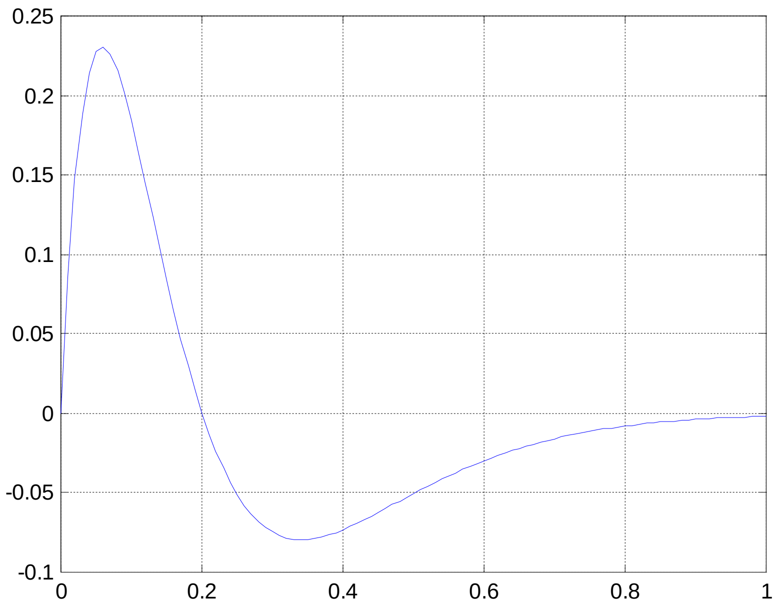
Unit of energy is Joules (J).

EXAMPLE

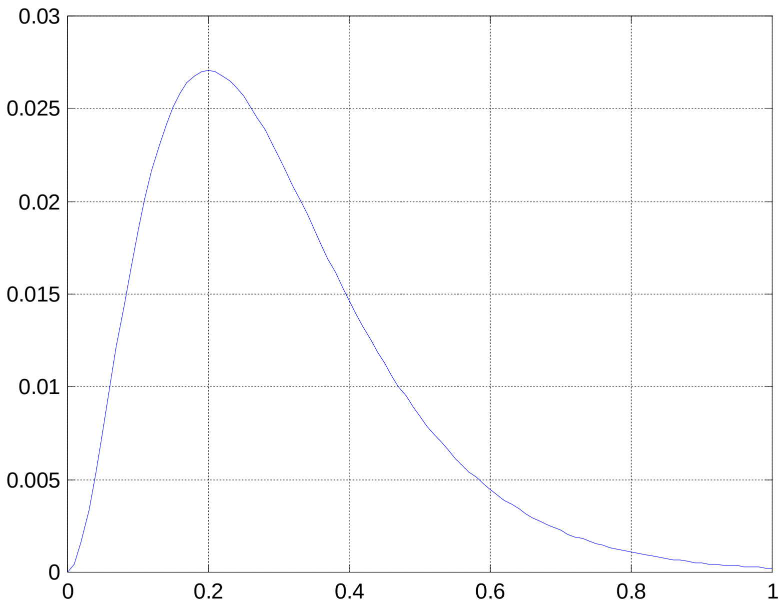


$$i = \begin{cases} 0 & t < 0 \\ 10te^{-5t} A & t > 0 \end{cases}$$

a)Plot the power and energy in the inductor.
b)In what time interval is energy being stored in the inductor?
c) In what time interval is energy being extracted from the inductor?
d)What is the maximum energy stored in the inductor?



Power



Energy

An increasing energy curve indicates that energy is being stored. Thus energy is being stored in the time interval 0-0.2s. Note that this corresponds to the interval when $p>0$.

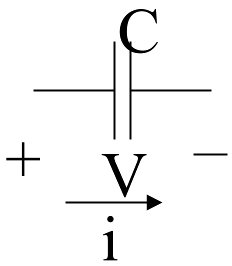
A decreasing energy curve indicates that energy is being extracted. Thus energy is being extracted in the time interval 0.2s-∞. Note that this corresponds to the interval when $p<0$.

Energy is maximum when current is at maximum. It occurs at t=0.2s. At t=0.2s $w_{max}=27.07J$

THE CAPACITOR

- A capacitor is an electrical component that consists of two conductors separated by an insulator or dielectric material.
- The source of the electric field is voltage. If the voltage is varying with time, the electric field is varying with time.
- A time-varying electric field produces a displacement current in the space occupied by the field.

CAPACITANCE



The circuit parameter of capacitance is represented by the letter C, is measured in Farads (F).

The current is proportional to the rate of the voltage across the capacitor.

$$i = C\frac{dV}{dt}$$

a) Voltage across a capacitor cannot change instantaneously. Such a change requires infinite current, a physical impossibility.

b) If the voltage across the capacitor is constant, the capacitor current is zero. Thus a capacitor acts as an open circuit in the presence of a dc voltage.

Voltage, power and energy

$$idt = CdV \Rightarrow \int_{V(t_0)}^{V(t)} dx = \frac{1}{C}\int_{t_0}^{t} id\tau$$

$$V(t) = \frac{1}{C}\int_{t_0}^{t} id\tau + V(t_0)$$

$$p = Vi = CV\frac{dV}{dt} = i\left[\frac{1}{C}\int_{t_0}^{t} id\tau + V(t_0)\right]$$

$$d\omega = CVdV \Rightarrow \omega = \frac{1}{2}CV^2$$

EXAMPLE

Voltage across the terminals of a 5μF capacitor is defined as

$$v(t) = \begin{cases} 0 & t < 0 \\ 4t \text{ V} & 0 \le t < 1 \\ 4e^{-(t-1)} \text{ V} & 1 \le 1 \le \infty \end{cases}$$

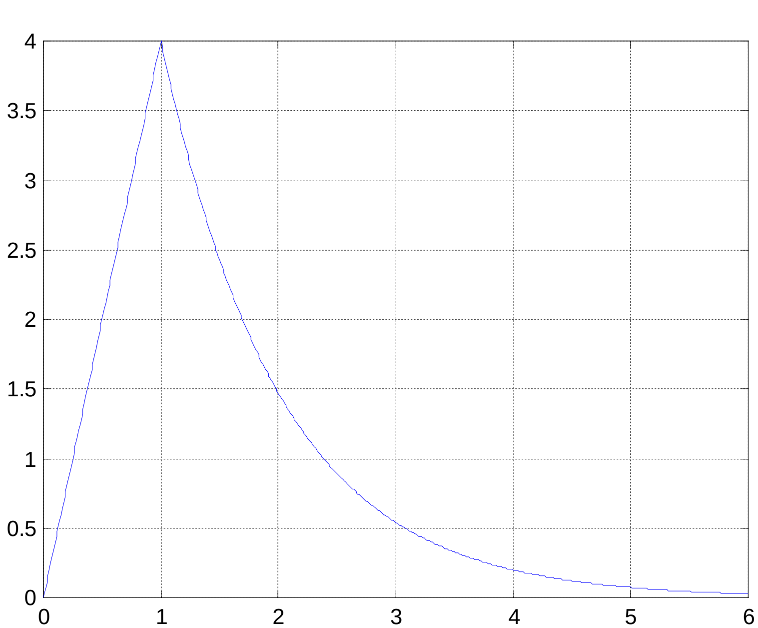
a) Derive the expressions for the current, power, and energy. Sketch the voltage, current, power, and energy plots.
b) Specify the time interval of time when energy is being stored in the capacitor.
c) Specify the time interval of time when energy is being delivered by the capacitor.

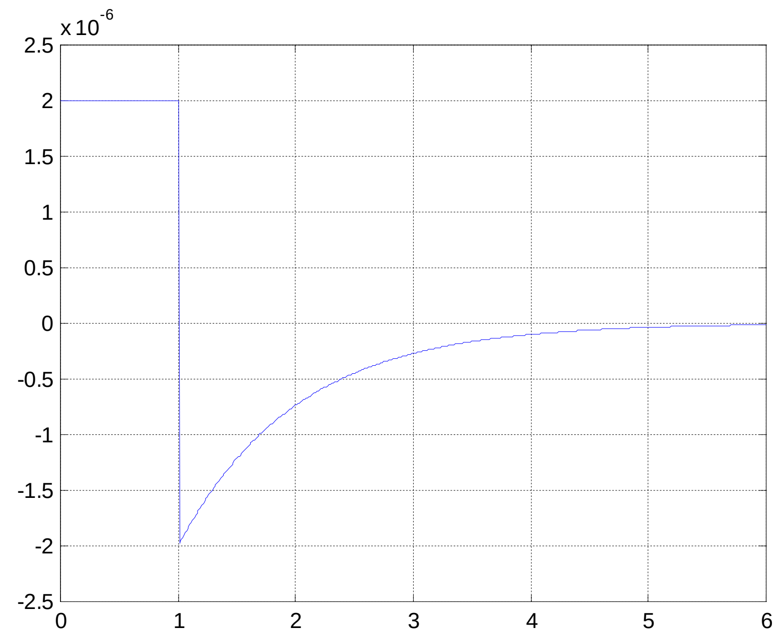
a)

$$i = C\frac{dV}{dt} = \begin{cases} 0 & t < 0 \\ 2\mu A & 0 \le t < 1 \\ -2e^{-(t-1)}\mu A & t \ge 1 \end{cases}$$

$$p = Vi = \begin{cases} 0 & t < 0 \\ (4t)2 = 8t\ \mu V & 0 \le t < 1 \\ 4e^{-(t-1)}(-2e^{-(t-1)}) = -8e^{-2(t-1)} & t \ge 1 \end{cases}$$

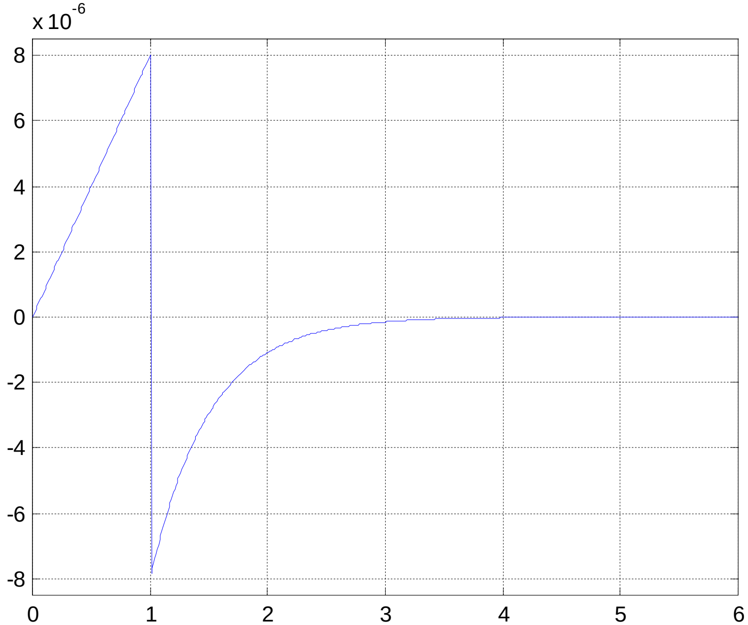
$$\omega = \frac{1}{2}CV^2 = \begin{cases} 0 & t < 0 \\ \frac{1}{2}(0.5)16t^2 = 4t^2\ \mu J & 0 \le t < 1 \\ \frac{1}{2}(0.5)16e^{-2(t-1)} = 4e^{-2(t-1)}\mu J & t \ge 1 \end{cases}$$

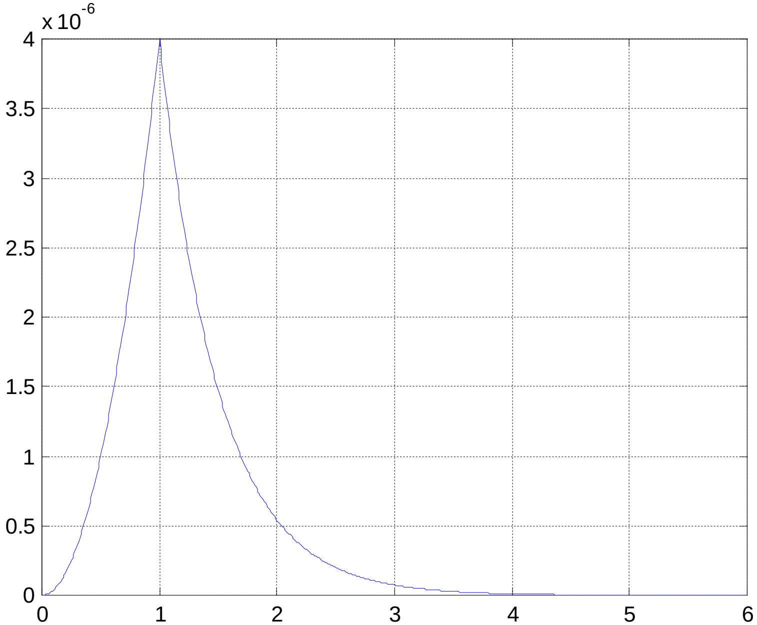




Voltage

Current



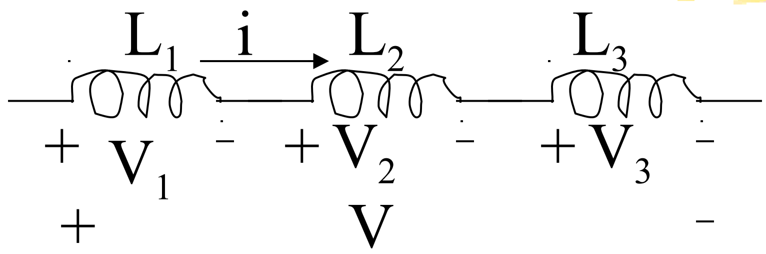


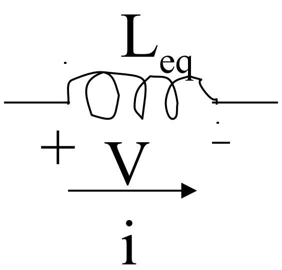
Power

Energy

Energy is being stored in the capacitor whenever the power is positive ($t<1$). Energy is being delivered by the capacitor whenever the power is negative ($t>1$).

Inductors in Series



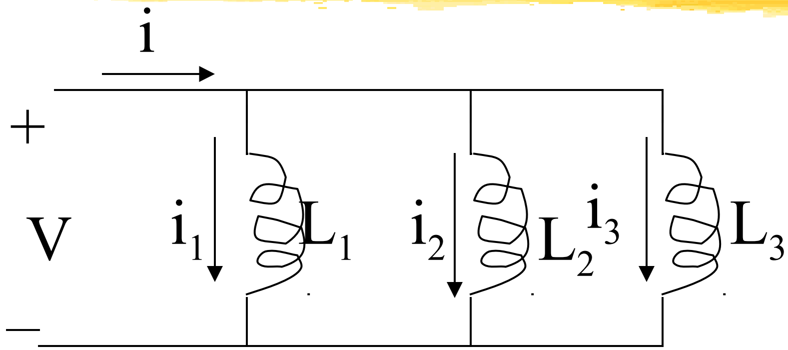


$$V_1 = L_1 \frac{di}{dt} \qquad V_2 = L_2 \frac{di}{dt} \qquad V_3 = L_3 \frac{di}{dt}$$

$$V = V_1 + V_2 + V_3 = (L_1 + L_2 + L_3)\frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3$$

Inductors in Parallel



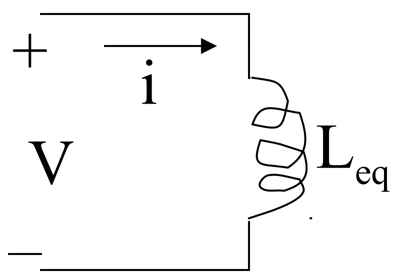
$$i_1 = \frac{1}{L_1}\int_{t_0}^{t} V d\tau + i_1(t_0)$$

$$i_2 = \frac{1}{L_2}\int_{t_0}^{t} V d\tau + i_2(t_0)$$

$$i_3 = \frac{1}{L_3}\int_{t_0}^{t} V d\tau + i_3(t_0)$$

$$i = i_1 + i_2 + i_3$$

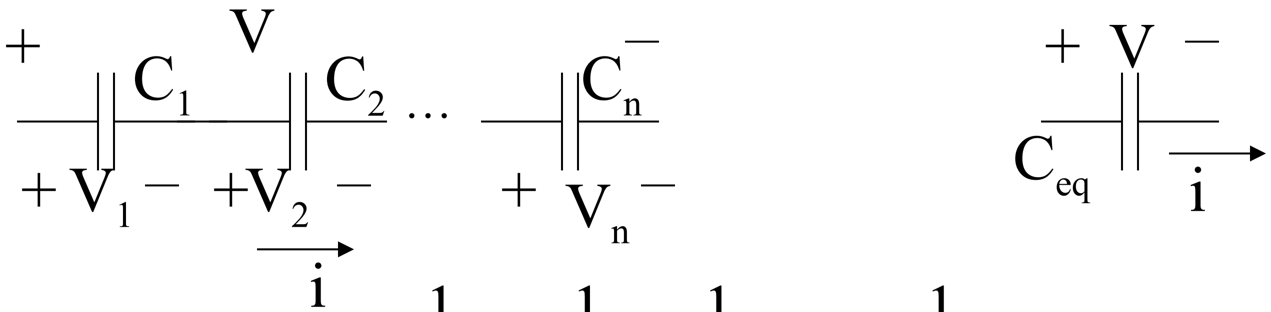
$$i = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_{t_0}^{t} V d\tau + i_1(t_0) + i_2(t_0) + i_3(t_0)$$



$$i = \frac{1}{L_{eq}}\int_{t_0}^{t} V d\tau + i(t_0) \qquad \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$i(t_0) = i_1(t_0) + i_2(t_0) + i_3(t_0)$$

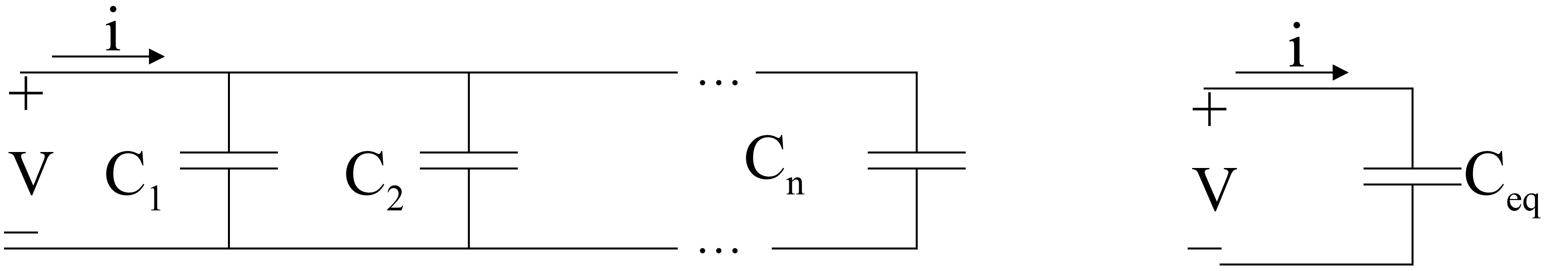
Capacitors in Series



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

$$V(t_0) = V_1(t_0) + V_2(t_0) + \cdots + V_n(t_0)$$

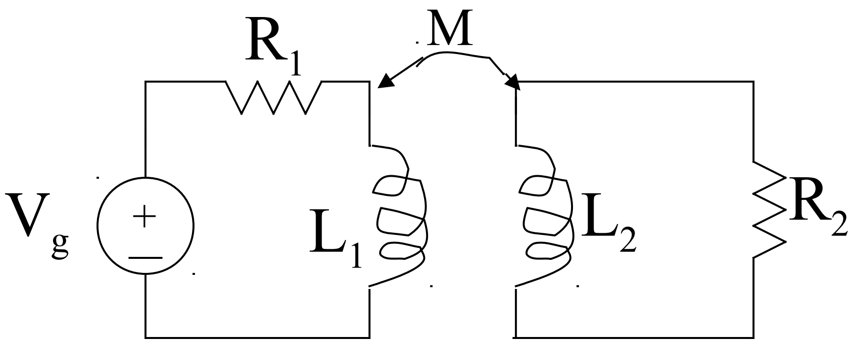
Capacitors in Parallel



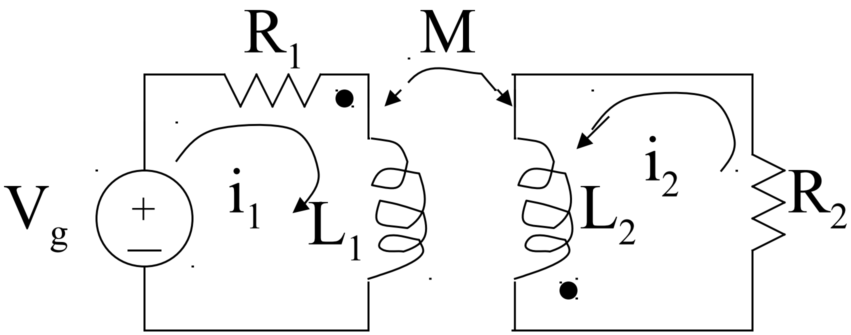
$$C_{eq} = C_1 + C_2 + \cdots + C_n$$

MUTUAL INDUCTANCE

When the two coils are placed in a magnetic field, current flowing through one coil will produce a voltage across the other. This phenomena is called magnetic coupling. The voltage induced in the second coil is related to a parameter known as mutual inductance (M).



DOT CONVENTION



Because of the mutual inductance M, there will be two voltages across each coil. i) Self-induced voltage (product of the self-inductance and the first derivative of the current in the coil).

ii) The mutually induced voltage (product of the mutual inductance and the first derivative of the current in the other coil).

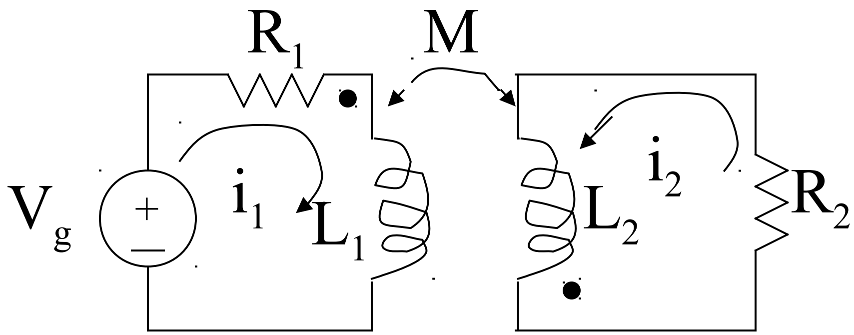
Using passive sign convention, we can easily determine the polarity of the self-induced voltages. But what about the polarity of the mutually induced voltage?

The polarity of the mutually induced voltage depends on the way coils wound in relation to the reference direction of coil currents. To determine the polarity of the mutually induced voltage, we will use the **dot convention.**

i) When the positive current enters the dotted terminal of a coil, the reference polarity of the voltage that induces in the other coil is positive at its dotted terminal.

ii) When the positive current leaves the dotted terminal of a coil, the reference polarity of the voltage that induces in the other coil is negative at its dotted terminal.

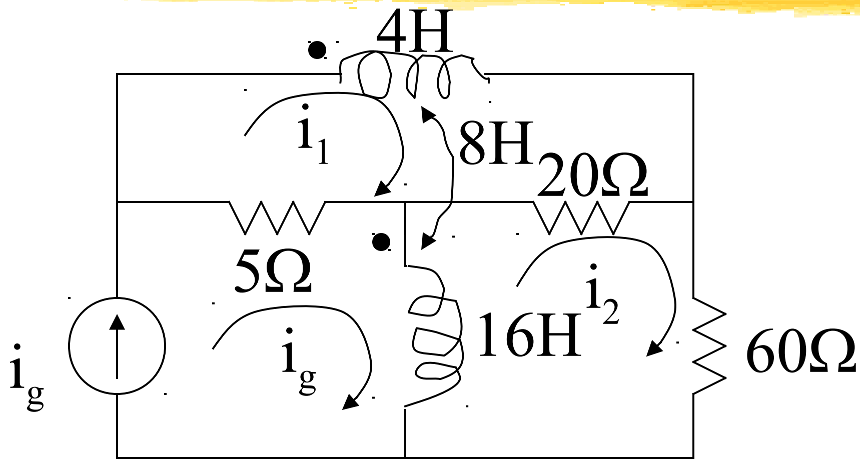
EXAMPLE



$$R_1 i_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = V_g$$

$$R_2 i_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} = 0$$

EXAMPLE



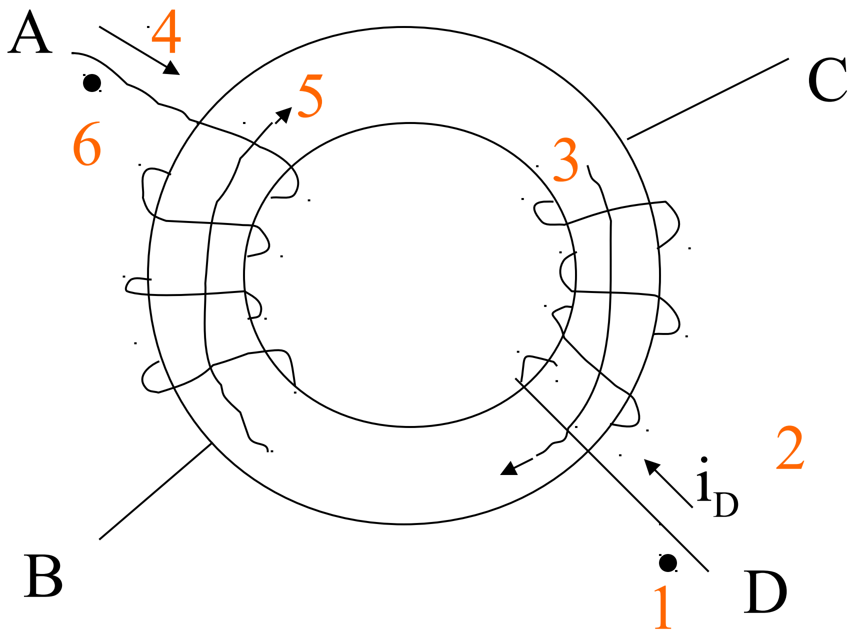
Write a set of mesh-current equations in terms of i_1 and i_2

$$4\frac{di_1}{dt} + 8\frac{d(i_g - i_2)}{dt} + 20(i_1 - i_2) + 5(i_1 - i_g) = 0$$

$$20(i_2 - i_1) + 60i_2 + 16\frac{d(i_2 - i_g)}{dt} - 8\frac{di_1}{dt} = 0$$

The Procedure for Determining Dot Markings

If we know the physical arrangement of the two coils of each winding in a magnetically coupled circuit, the following six steps will determine a set of dot markings



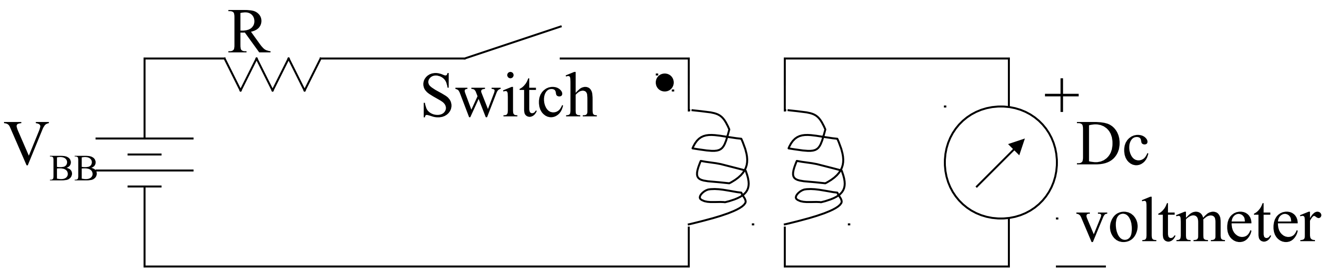
1) Arbitrarily select one terminal say D of one coil and mark with a dot.
2) Assign a current into the dotted terminal and label it i_D
3) Use the right-hand-rule to determine the direction of the magnetizing field established by i_D.

4) Arbitrarily pick one terminal of the second coil, say terminal A and assign a current into this terminal as i_A

5) Use the right-hand-rule to determine the direction of the flux established by i_A

6) Compare the directions of the two fluxes established by each current. If the fluxes have the same direction, place a dot on the terminal of the second coil where the test current i_A enters. If the fluxes have different directions, place a dot on the terminal of the second coil where the test current leaves.

If we don’t know how the coils are wound on the core, we may use an experimental procedure as follows:



When the switch is closed, the voltmeter deflection is observed. If the momentary deflection is upscale, the coil terminal connected to the positive terminal of the voltmeter receives the polarity mark. If the deflection is downscale, the coil terminal connected to the negative terminal of the voltmeter receives the polarity mark.