SIMPLE RESISTIVE CIRCUITS

OSMAN PARLAKTUNA

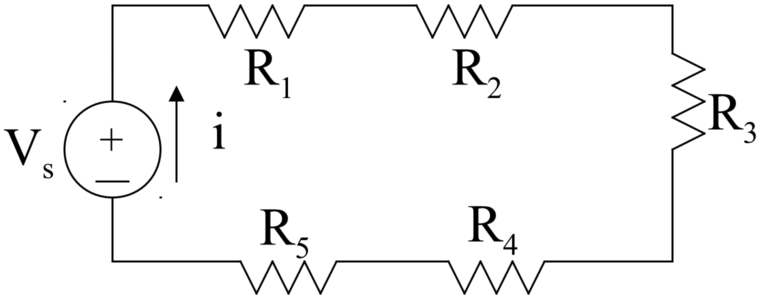
Osmangazi University

Eskişehir, TURKEY

www.ogu.edu.tr/~oparlak

RESISTORS IN SERIES

Elements are connected in **series** when they carry the same current.



Writing the KVL equation

$-V_s+R_1i+R_2i+R_3i+R_4i+R_5i=0$

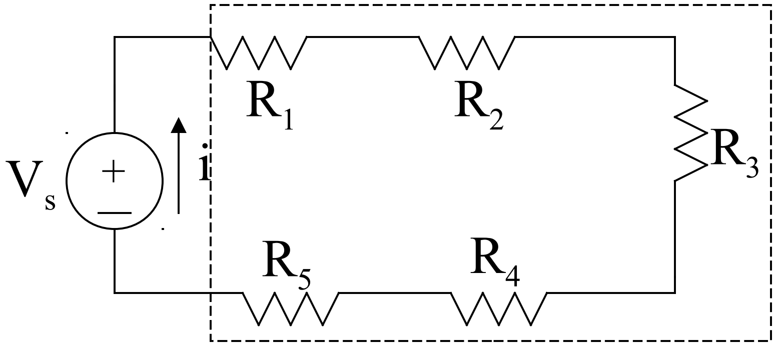
$V_s=i(R_1+R_2+R_3+R_4+R_5)$

Sum of the five resistors can be replaced by a single resistor as $R_{eq}= R_1+R_2+R_3+R_4+R_5$ and $V_s=iR_{eq}$. R_{eq} is the equivalent resistance of series resistor. In genera,l if k resistors are connected in series, then

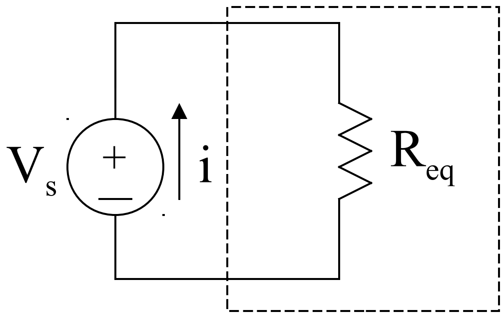
$$R_{eq} = \sum_{i=1}^{k} R_i = R_1 + R_2 + \cdots + R_k$$

EQUIVALENT CIRCUITS

Two circuits are said to be equivalent if they have the same voltage and current values at the corresponding terminals.

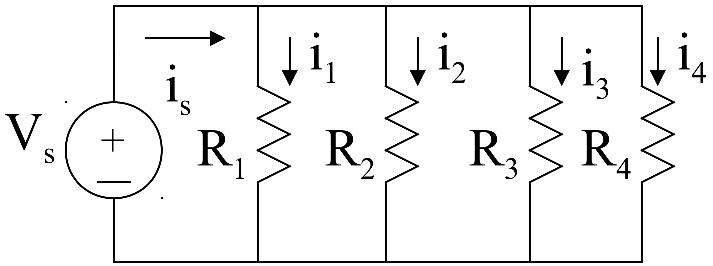


Replacing five resistors by their equivalent R_{eq}



These two circuits are equivalent because they have the same voltage current relationship at their input terminals.

RESISTORS IN PARALLEL



Elements are said to be in **parallel** when they have the same voltage across their terminals.

The KCL equation is : $i_s = i_1 + i_2 + i_3 + i_4$

$$i_1 = \frac{V_s}{R_1}, \quad i_2 = \frac{V_s}{R_2}, \quad i_3 = \frac{V_s}{R_3}, \quad i_4 = \frac{V_s}{R_4}$$

$$i_s = V_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)$$

$$\frac{i_s}{V_s} = \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

For k resistors connected in parallel, the equivalent resistance is

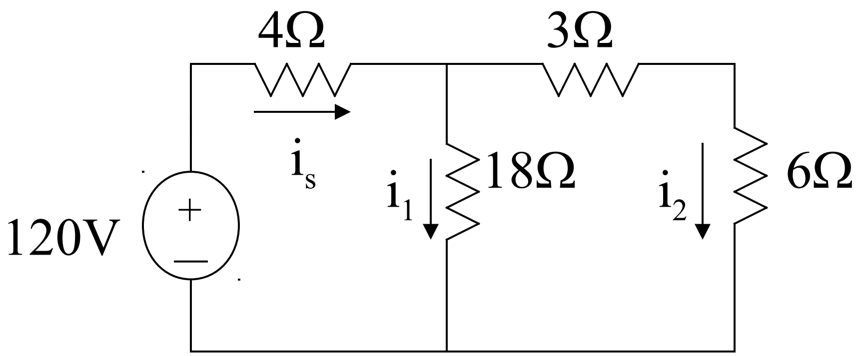
$$\frac{1}{R_{eq}} = \sum_{i=1}^{k} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_k}$$

For 2 resistors in parallel, we may use the following simple equation

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

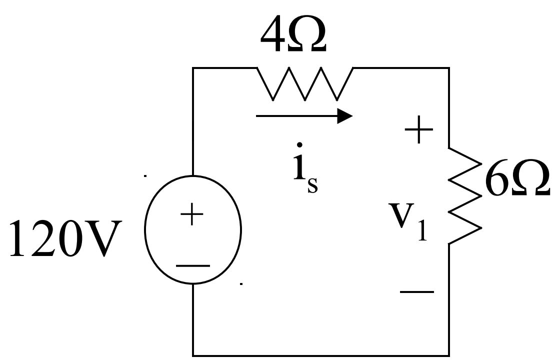
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

EXAMPLE



Find i_s, i_1, and i_2 in the circuit.

Finding the equivalent resistance of the three resistor in the right side.



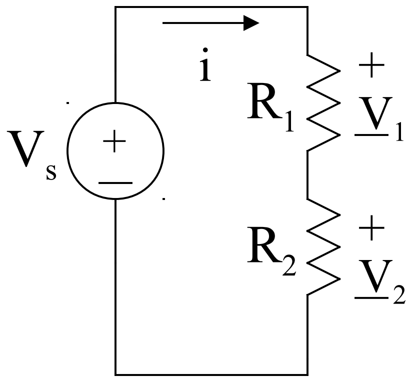
$$R_{eq} = (3+6)||(18) = \frac{(9)(18)}{9+18} = 6\Omega$$

$$i_s = \frac{120}{4+6} = 12A \Rightarrow v_1 = (12)6 = 72V$$

$$i_1 = \frac{v_1}{18} = \frac{72}{18} = 4A, \quad i_2 = \frac{v_1}{9} = \frac{72}{9} = 8A$$

THE VOLTAGE-DIVIDER CIRCUIT

Some times, in electrical circuits, developing more than one voltage level from a single supply is necessary. One way of doing this is by using a **voltage-divider circuit**.

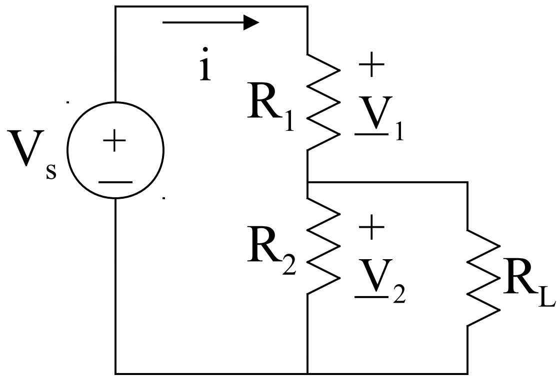


$$v_s = iR_1 + iR_2 \Rightarrow i = \frac{v_s}{R_1 + R_2}$$

$$v_1 = iR_1 = \frac{R_1}{R_1 + R_2} v_s$$

$$v_2 = iR_2 = \frac{R_2}{R_1 + R_2} v_s$$

LOADING



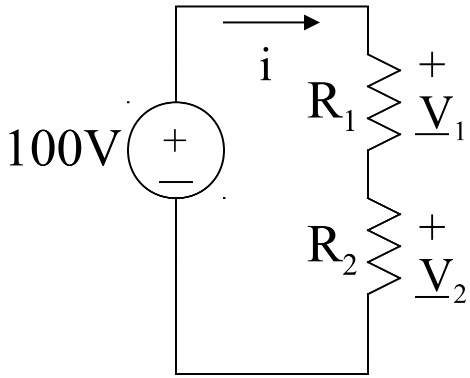
A **load** in any circuit consists of one or more circuit elements that draw power from the circuit. R_L acts as a load on the voltage-divider circuit. With the R_L, V_2 becomes

$$V_2 = \frac{R_{eq}}{R_1 + R_{eq}} V_s \qquad R_{eq} = \frac{R_2 R_L}{R_2 + R_L}$$

$$V_2 = \frac{R_2}{R_1[1 + (R_2 / R_L)] + R_2} v_s$$

The voltage V_2 with the load is smaller than the voltage V_2 of the unloaded circuit.

EXAMPLE



R_1=25 KΩ, R_2=100 KΩ. If the resistors have a tolerance of ±10%, find the minimum and maximum value of V_2

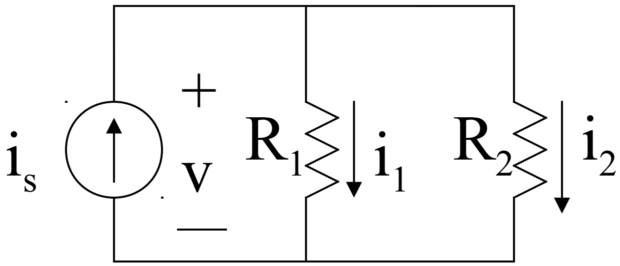
$$V_2 = \frac{R_2}{R_1 + R_2} 100$$

The maximum value of V_2 occurs when R_2 is 10% high and R_1 is 10% low, and the minimum value of V_2 occurs when R_2 is 10% low and R_1 is 10% high.

$$V_2(\max) = \frac{110}{22.5 + 110} 100 = 83.02V$$

$$V_2(\min) = \frac{90}{27.5 + 90} 100 = 76.60V$$

THE CURRENT-DIVIDER CIRCUIT

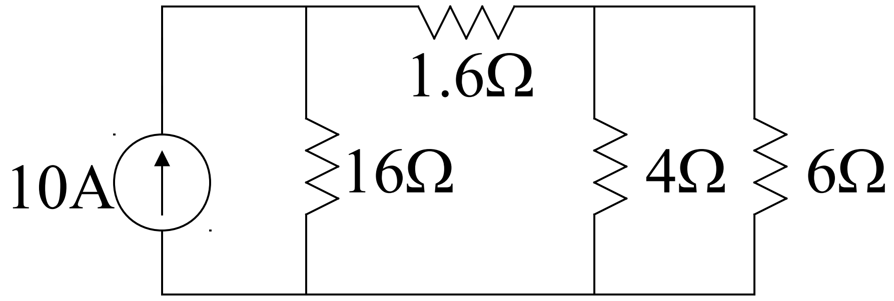


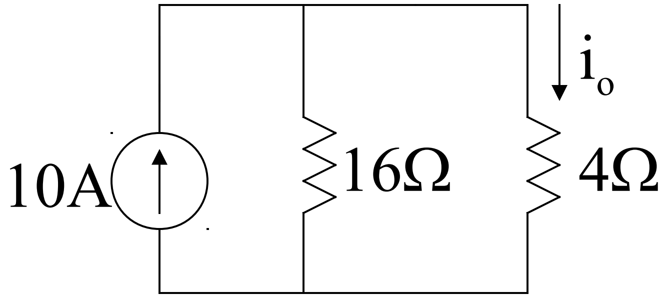
$$v = i_1 R_1 = i_2 R_2 = \frac{R_1 R_2}{R_1 + R_2} i_s$$

$$i_1 = \frac{R_2}{R_1 + R_2} i_s \quad i_2 = \frac{R_1}{R_1 + R_2} i_s$$

The current divides between two resistors in parallel such that the current in one resistor equals the current entering the parallel pair multiplied by the other resistance and divided by the sum of the resistors.

EXAMPLE





Find the power dissipated in the 6Ω resistor.

$$R_{eq1} = (4||6) + 1.6 = 4\Omega$$

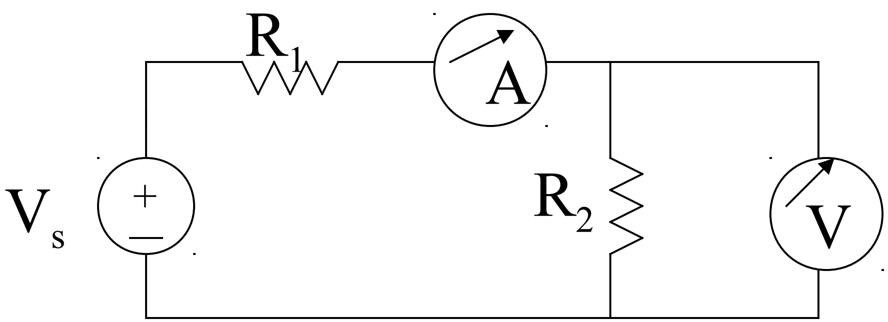
$$i_o = \frac{16}{4+16}10 = 8A$$

$$i_{6\Omega} = \frac{4}{4+6}8 = 3.2A$$

$$p_{6\Omega} = (3.2)^2(6) = 61.44W$$

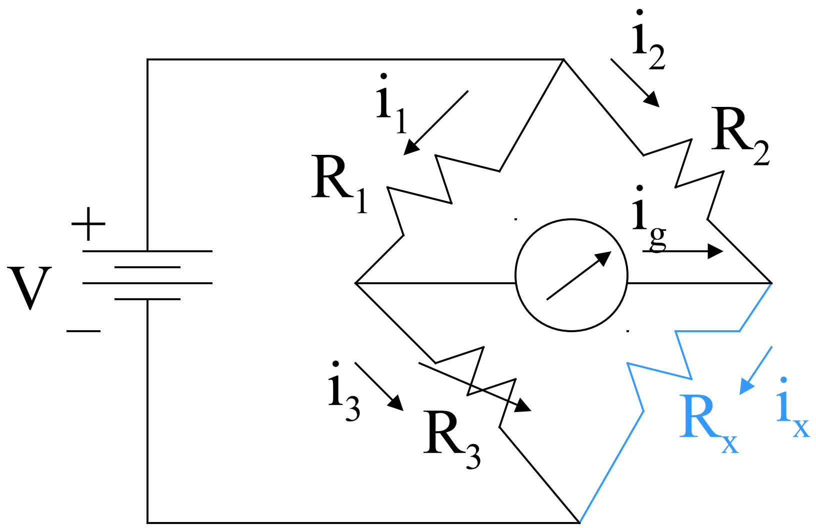
MEASURING VOLTAGE AND CURRENT

An **ammeter** is an instrument designed to measure current; it is placed in series with the circuit element whose current is being measured. A voltmeter is an instrument designed to measure voltage; it is placed in parallel with the element whose voltage is being measured.



An ideal ammeter has an equivalent resistance of 0Ω, and an ideal voltmeter has an infinite equivalent resistance.

THE WHEATSTONE BRIDGE

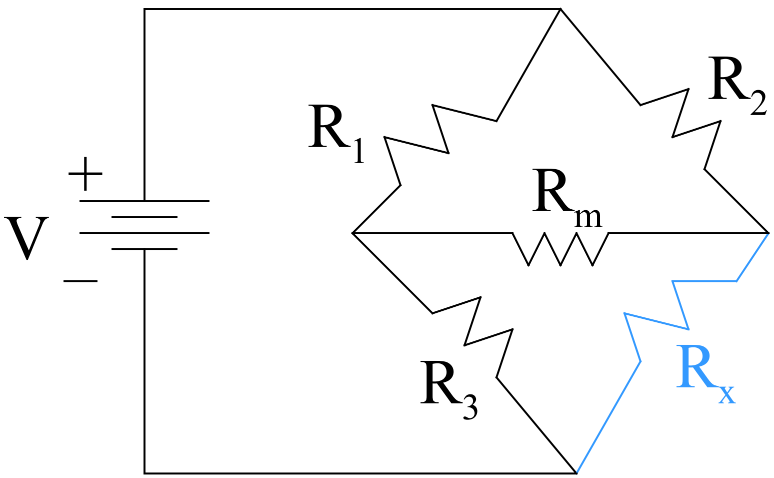


The Wheatstone bridge circuit is used to measure resistances in the range of 1Ω to 1MΩ.

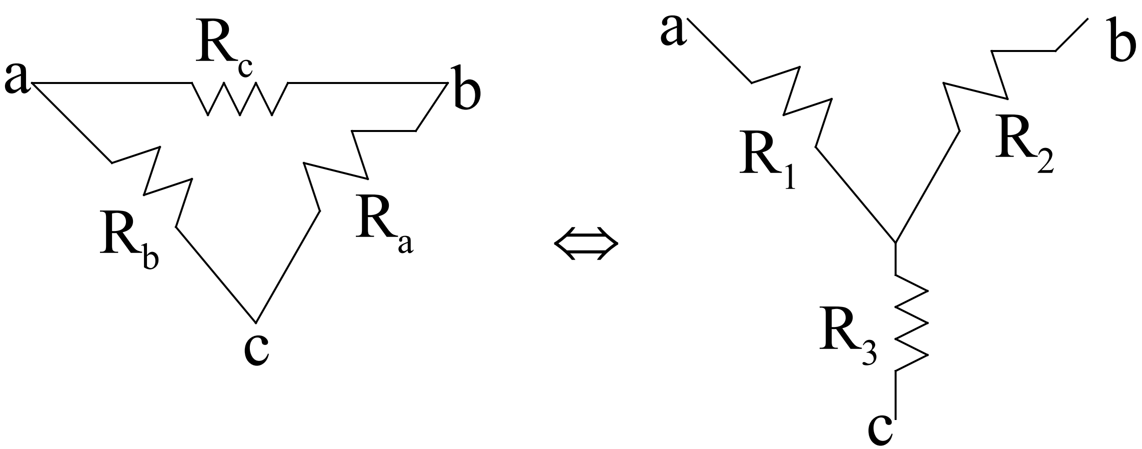
To find the value of R_x, R_3 is adjusted until i_g=0. Then, $i_1=i_3$ and $i_2=i_x$ $i_3R_3=i_xR_x$ and $i_1R_1=i_2R_2$ $i_1R_3=i_2R_x$

$$\frac{R_3}{R_1} = \frac{R_x}{R_2} \Rightarrow R_x = \frac{R_2}{R_1} R_3$$

DELTA-TO-WYE (Δ TO Y) EQUIVALENT CIRCUITS



This circuit cannot be reduced to a single resistance circuit across the terminals of the battery is restricted to simple series or parallel resistance procedures. This circuit can be reduced to a single resistance using delta-to-wye transformation.



$$R_{ab} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} = R_1 + R_2$$

$$R_{bc} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} = R_2 + R_3$$

$$R_{ca} = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} = R_1 + R_3$$

$$\Rightarrow$$

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

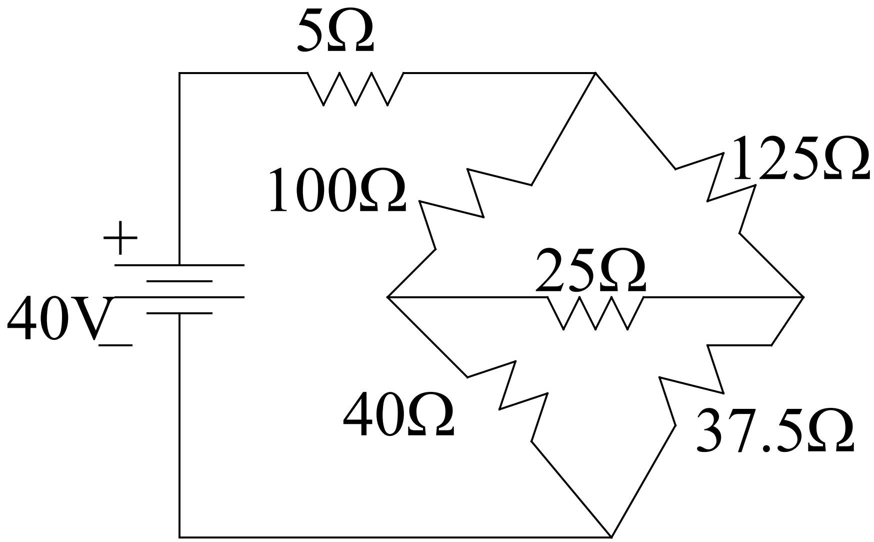
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_a = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

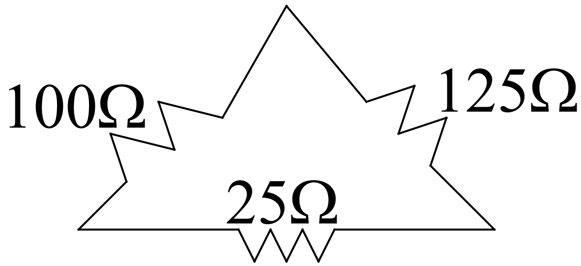
$$R_c = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

EXAMPLE



Find the power supplied by the 40V source.

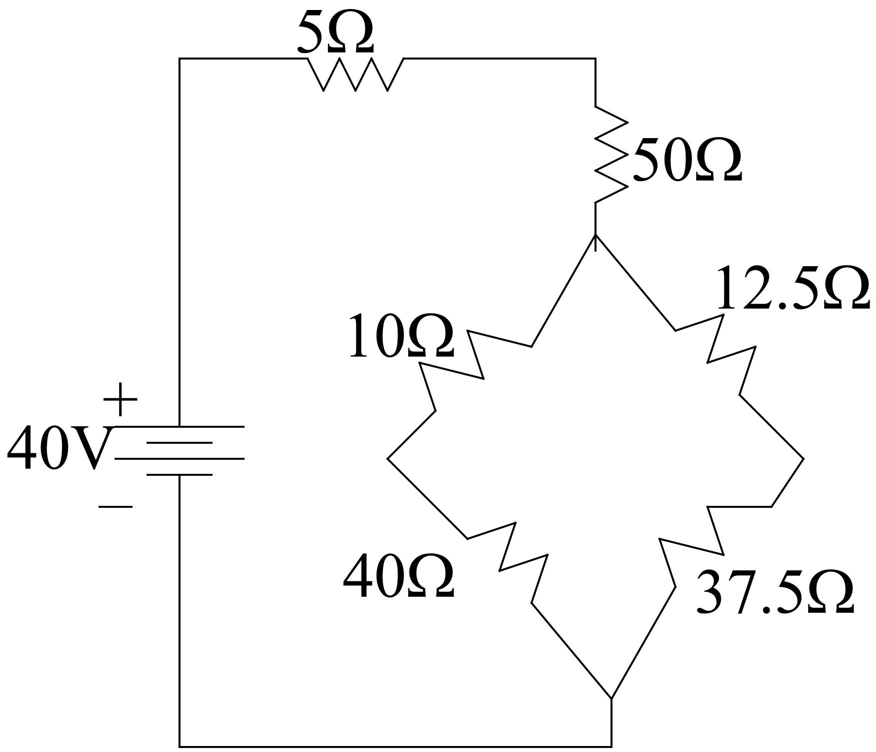
Replacing the upper Δ by its Y equivalent



$$R_1 = \frac{(100)125}{100+125+25} = 50\Omega$$

$$R_2 = \frac{(25)125}{100+125+25} = 12.5\Omega$$

$$R_3 = \frac{(100)25}{100+125+25} = 10\Omega$$



$$R_{eq} = 5 + 50 + ((10 + 40)||(12.5 + 37.5))$$

$$R_{eq} = 55 + \frac{(50)(50)}{50 + 50} = 80\Omega$$

$$i = \frac{40}{80} = 0.5A$$

$$p = 40(0.5) = 20W$$