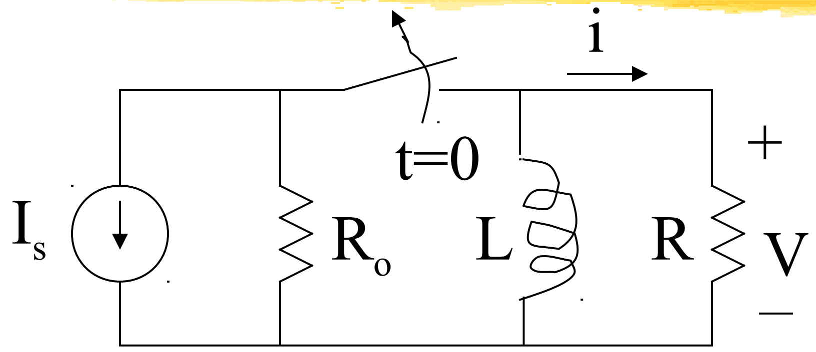
RESPONSE OF FIRST-ORDER RL AND RC CIRCUITS

OSMAN PARLAKTUNA
OSMANGAZİ UNIVERSITY
ESKİŞEHİR, TURKEY
www.ogu.edu.tr/~oparlak

NATURAL RESPONSE AND STEP RESPONSE

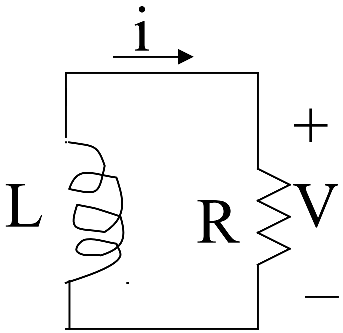
- When an inductor or a capacitor is abruptly disconnected from its dc source, the currents and voltages in this circuit are referred to as the **natural response**.
- The currents and voltages that arise from the sudden application of a dc voltage or current source are referred to as the **step response.**

THE NATURAL RESPONSE OF AN RL CIRCUIT



We assume that the current source is a dc source and the switch has been closed for a long time before it opens at t=0.

This means that all the currents and voltages have reached a constant value. Thus only dc currents can exist in the circuit just prior to the switch's being opened, and therefore the inductor appears as a short circuit. Then all the source current I_s appears in the inductive branch. We denote this by $i(0^-)=I_s$.



For t>0, the circuit reduces to the one shown in the figure.

$$L\frac{di}{dt}+Ri=0$$

This equation is a first-order ordinary constant coefficient differential equation.

$$\frac{di}{dt}dt=-\frac{R}{L}idt \Rightarrow \frac{di}{i}=-\frac{R}{L}dt$$

$$\int_{i(t_0)}^{i(t)}\frac{dx}{x}=-\frac{R}{L}\int_{t_0}^{t}dy \Rightarrow \ln\frac{i(t)}{i(0)}=-\frac{R}{L}t$$

$$i(t)=i(0)e^{-(R/L)t}$$

We know that the current through an inductor cannot change instantaneously. If we 0^- to denote the time just prior to switching, and 0^+ for the time immediately following switching, then $i(0^-)=i(0^+)=I_0$.

$$i(t) = I_0 e^{-(R/L)t} \qquad t \geq 0$$

$$V(t) = i(t)R = I_0 R\, e^{-(R/L)t} \qquad t \geq 0^+$$

$$V(0^-) = 0 \qquad V(0^+) = I_0 R$$

$$p = Vi = I_0^2 R\, e^{-2(R/L)t} \qquad t \geq 0^+$$

$$\omega = \frac{1}{2} L I_0^2 (1 - e^{-2(R/L)t}) \qquad t \geq 0$$

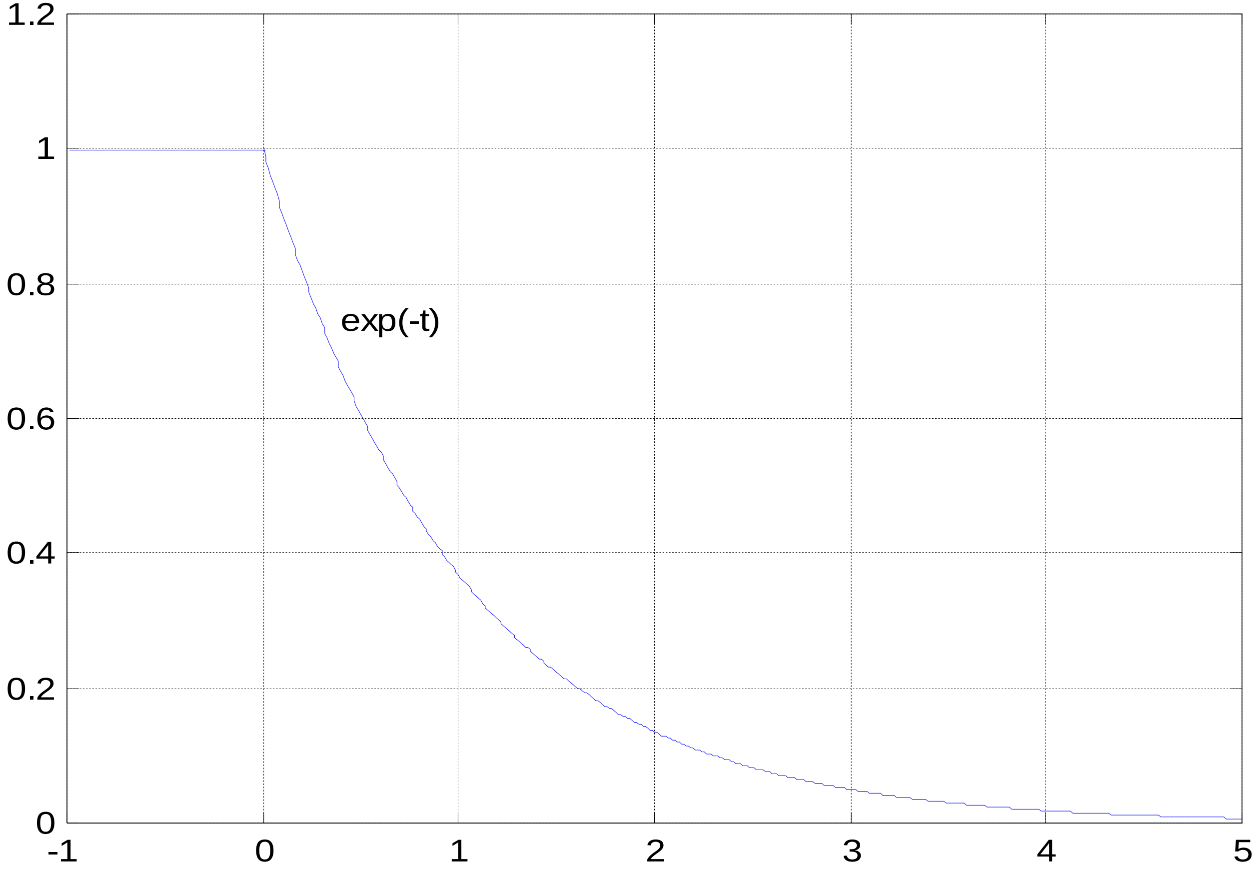
TIME CONSTANT

The coefficient of t in the $e^{-(R/L)t}$ term (R/L) determines the rate at which the current or voltage approaches zero. The reciprocal of this ratio is the **time constant** of the circuit.

$$\tau = \frac{L}{R}$$

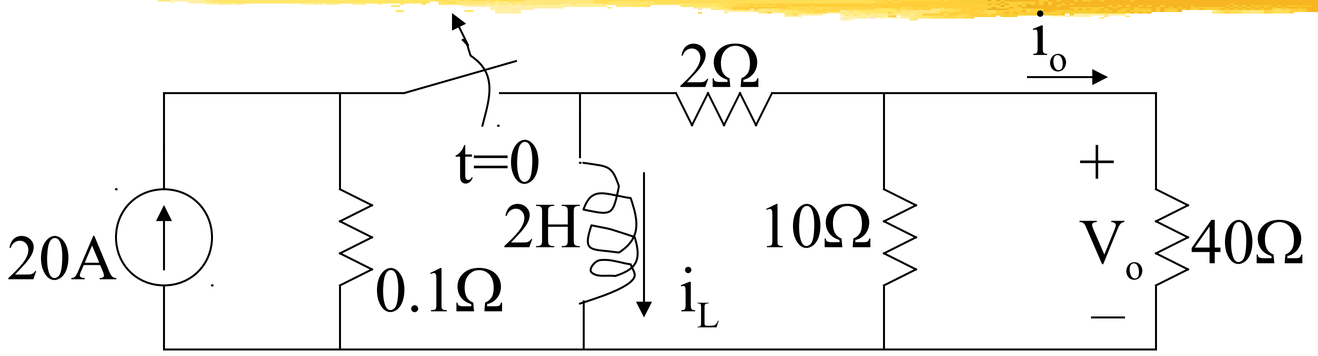
$$i(t) = I_0 e^{-t/\tau} \qquad t \geq 0$$

t	$e^{-t/\tau}$
τ	0.368
2τ	0.135
3τ	0.0497
4τ	0.0183
5τ	0.0067


1.2
1
0.8
0.6
0.4
0.2
0
-1
0
1
2
3
4
5
exp(-t)

Note that when the elapsed time exceeds five time constants, the current is less than 1% of its initial value. Thus after five time constants of switching, the currents and voltages have reached their final values. For first-order circuits with 1% accuracy, the phrase **a long time** implies that five or more time constants have elapsed. The response shown in the figure is referred to as the **transient response**. The response that exists a long time after switching has taken place is called the **steady-state response**.

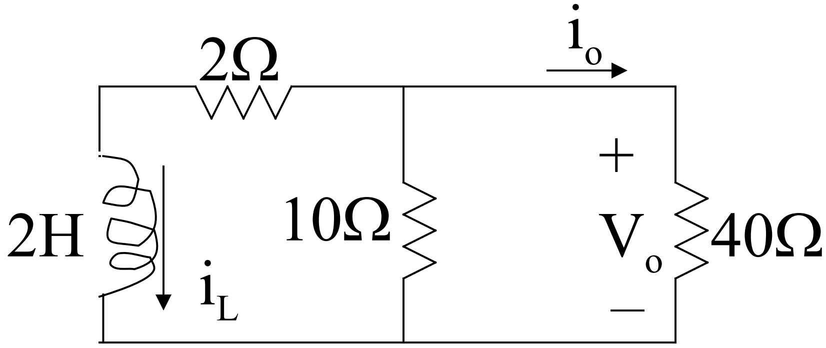
EXAMPLE



The switch has been in the circuit has been closed for a long time before it is opened at t=0. Find $i_L(t)$, $i_o(t)$, and $V_o(t)$ for t>0. Find the percentage of the total energy stored in the 2H inductor that is dissipated in the 10Ω resistor.

Since the switch has been closed for a long time prior to t=0, the circuit has reached to dc steady-state and inductor is short circuit. Then, $i_L(0^-)=20A$.

Since the current through inductor cannot change instantaneously, $i_L(0^+)=i_L(0^-)=20A$. Circuit for t>0 becomes



The equivalent circuit seen by the inductor

$$R_{eq} = 2 + (40||10) = 10\Omega$$

$$\tau = L / R_{eq} = 2/10 = 0.2s$$

$$i_L(t) = 20e^{-5t}A \quad \text{for} \;\; t \geq 0$$

$$i_o(t) = -i_L(t)\frac{10}{10+40}$$

$$i_o(t) = -4e^{-5t}A \quad \text{for} \;\; t \geq 0^+$$

$$V_o(t) = 40 i_o(t) = -160e^{-5t}V \quad \text{for} \;\; t \geq 0^+$$

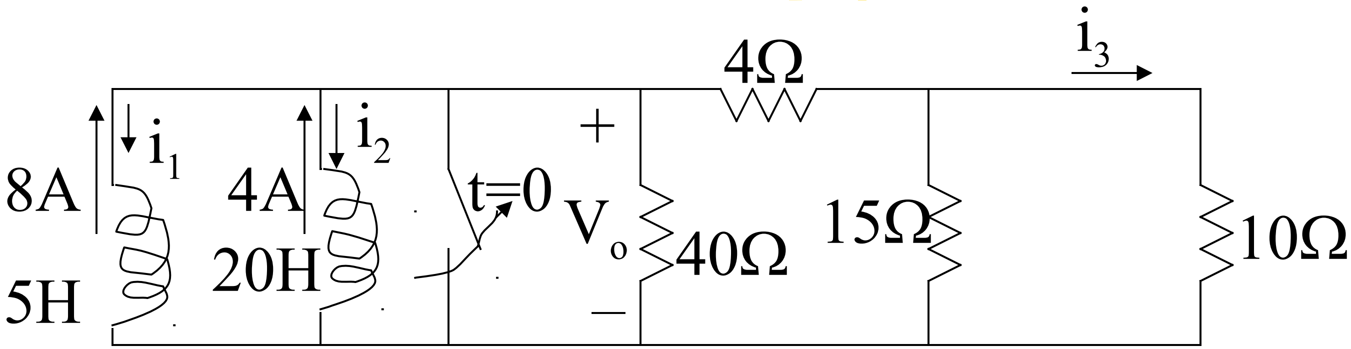
$$p_{10\Omega}(t) = \frac{V_o^2}{10} = 2560e^{-10t} W \quad \text{for} \;\; t \geq 0^+$$

$$\omega_{10\Omega}(t) = \int_0^{\infty} 2560e^{-10t} dt = 256J$$

$$\omega_L(0) = \frac{1}{2} Li^2(0) = \frac{1}{2}(2)(400) = 400J$$

$$\frac{256}{400}(100) = 64\%$$

EXAMPLE



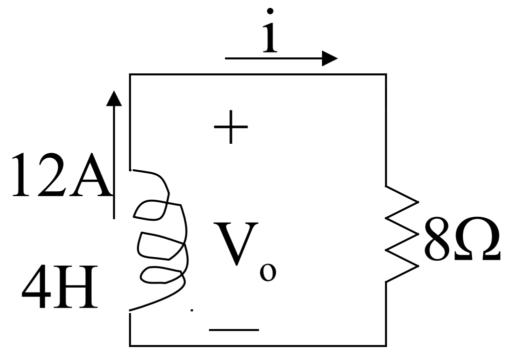
The initial currents in the inductors are shown in the figure. The switch is opened at t=0. a) Find i_1, i_2, and i_3 for t>0

b) Calculate the initial energy stored in the inductors.

c) Determine how much energy is stored in the inductors as $t \longrightarrow \infty$

d) Show that the total energy delivered to the resistive network equals the difference between the results of a and b)

Parallel inductors reduce to an equivalent of 4H with an initial current of 12A. The resistive network reduces to an equivalent resistance of 8Ω. Then, for t>0, circuit becomes



$$i(t) = 12e^{-t/\tau}\text{A} \quad t \geq 0 \quad \tau = \frac{L_{eq}}{R_{eq}} = \frac{4}{8} = 0.5\,\text{sec}$$

$$V_o(t) = 8i(t) = 96e^{-2t}\text{V} \qquad t \geq 0^+$$

$$i_1(t) = \frac{1}{5}\int_0^t 96e^{-2x}dx - 8$$

$$= 1.6 - 9.6e^{-2t}\text{A} \qquad t \geq 0$$

$$i_2(t) = \frac{1}{20}\int_0^t 96e^{-2x}dx - 4$$

$$= -1.6 - 2.4e^{-2t} \text{ A} \quad t \geq 0$$

$$i_3(t) = \frac{V_o(t)}{10}\frac{15}{25} = 5.76e^{-2t} A \quad t \geq 0^+$$

$$\omega(0) = \frac{1}{2}(5)(64) + \frac{1}{2}(20)(16) = 320J$$

As $t \to \infty,\ i_1 \to 1.6A$ and $i_2 \to -1.6A$

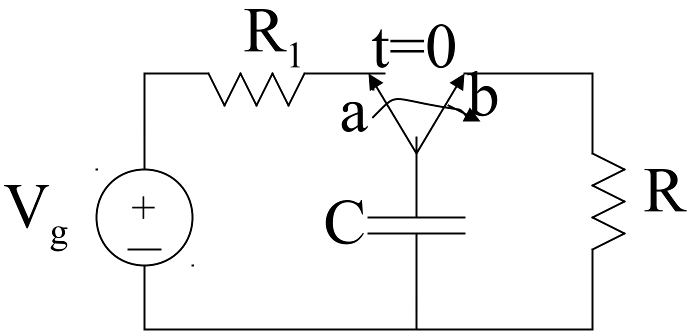
$$\omega(\infty) = \frac{1}{2}(5)(1.6)^2 + \frac{1}{2}(20)(-1.6)^2 = 32J$$

$$\omega_R = \int_0^\infty p\,dt = \int_0^\infty (8)(12e^{-2t})\,dt$$

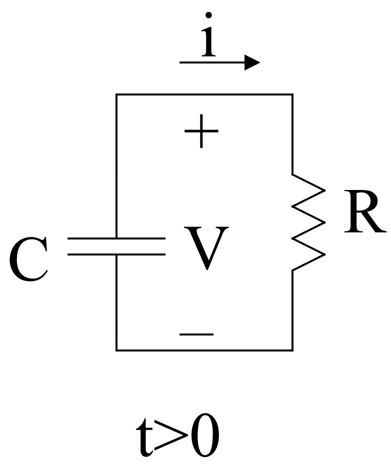
$$= \int_0^\infty 1152e^{-4t}\,dt = 1152\left.\frac{e^{-4t}}{-4}\right|_0^\infty = 288J$$

$$\omega(\infty) - \omega(0) = 322 - 32 = 288J$$

THE NATURAL RESPONSE OF AN RC CIRCUIT



The switch has been in position a for a long time. Then the circuit made up of V_g, R_1 and C reaches a dc steady-state circuit where the capacitor is an open circuit. The voltage across the capacitor at $t=0^-$ is V_g.



Because there can be no instantaneous change in the capacitor voltage, the initial voltage across the capacitor is $V(0^+)=V_g$

$$C\frac{dV}{dt}+\frac{V}{R}=0$$

$$V(t)=V(0)e^{-t/RC} \qquad t\geq 0$$

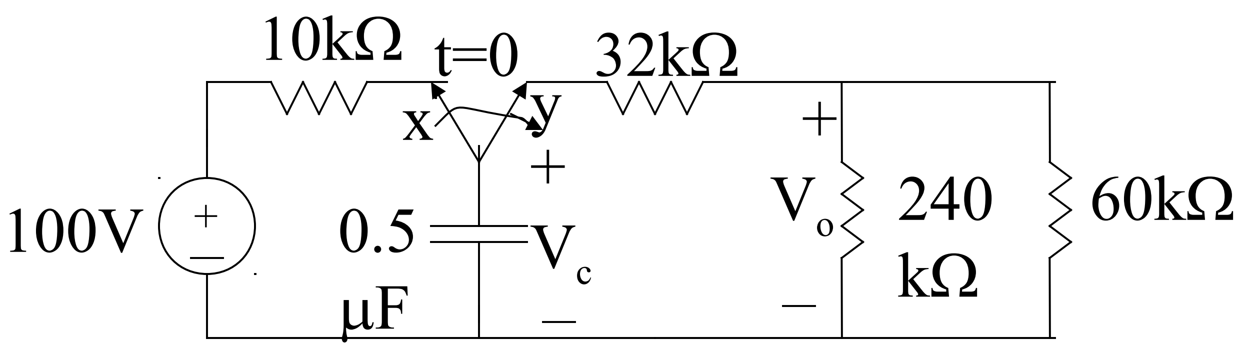
$$V(0^-)=V(0)=V(0^+)=V_g=V_0$$

$$V(t)=V_0e^{-t/\tau} \quad t\geq 0, \qquad \tau=RC$$

$$i(t)=\frac{V(t)}{R}=\frac{V_0}{R}e^{-t/\tau} \qquad t\geq 0^+$$

$$p=Vi=\frac{V_0^2}{R}e^{-2t/\tau} \qquad t\geq 0^+$$

EXAMPLE



The switch in the circuit has been in position x for a long time. At t=0, it moves to position y

Find $V_c(t)$, $V_o(t)$, and $i_o(t)$ for t>0. Find the total energy dissipated in the 60 kΩ resistor.

At $t=0^-$, the circuit reaches to dc steady-state. Capacitor is open circuit and $V_c(0^-)=100V$. Because the voltage across the capacitor cannot change instantaneously $V_c(0^+)=100V$. The equivalent resistance seen by the capacitor is $(240||60)+32=80\ k\ \Omega$

The time constant is $R_{eq}C=80x10^3x0.5x10^{-6}=40$ ms. Then,

$$V_c(t) = 100e^{-25t}V \quad t \geq 0$$

$$V_o(t) = \frac{48}{48+32}V_c(t) = 60e^{-25t}V \quad t \geq 0^+$$

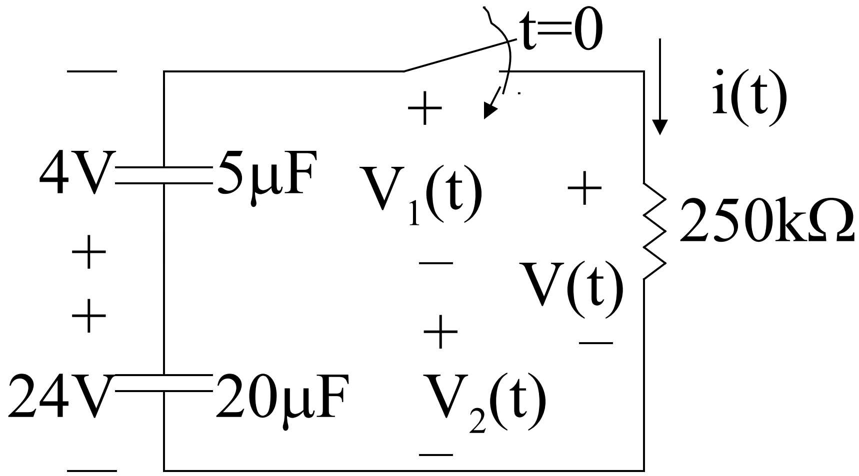
The expression for $V_o(t)$ is valid for $t>0^+$, because $V_o(0^-)=0$. Thus we have an instantaneous change in the voltage across the 240kΩ resistor.

$$i_o(t) = \frac{V_o(t)}{60\times10^3} = e^{-25t}mA \quad t \geq 0^+$$

$$p_{60k} = i_o^2(t)(60\times10^3) = 60e^{-50t}mW \quad t \geq 0^+$$

$$\omega_{60k} = \int_0^\infty p_{60k}dt = 1.2mJ$$

EXAMPLE



Find $V_1(t)$, $V_2(t)$, $V(t)$, and $i(t)$ for $t>0$. Calculate the initial energy stored in the capacitors. Determine the stored energy in the capacitors as $t \to \infty$.

$V(0^+)=V_1(0^+)+V_2(0^+)=24-4=20V$. The equivalent capacitance is $4\mu F$. Therefore the time constant is $Rc_{eq}=1$ s. Then,

$$V(t) = 20e^{-t}V \quad t \geq 0$$

$$i(t) = \frac{V(t)}{250 \times 10^3} = 80e^{-t}\mu A \quad t \geq 0^+$$

$$V_1(t) = -\frac{10^6}{5}\int_0^t 80\times10^{-6}e^{-x}dx - 4$$

$$= (16e^{-t} - 20)V \quad t \ge 0$$

$$V_2(t) = -\frac{10^6}{20}\int_0^t 80\times10^{-6}e^{-x}dx + 24$$

$$= (4e^{-t} + 20)V \quad t \ge 0$$

$$\omega_1(0) = \frac{1}{2}(5\times10^{-6})(16) = 40\mu J$$

$$\omega_2(0) = \frac{1}{2}(20\times10^{-6})(576) = 5760\mu J$$

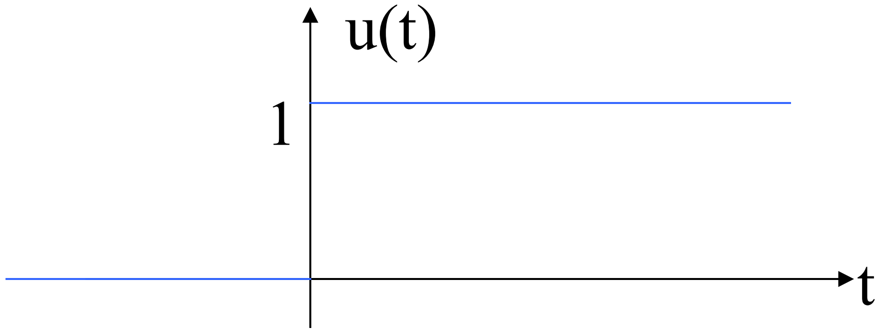
$$\omega_1(\infty) = \frac{1}{2}(5\times 10^{-6})(-20)^2 = 1000\,\mu J$$

$$\omega_2(\infty) = \frac{1}{2}(20\times 10^{-6})(20)^2 = 4000\,\mu J$$

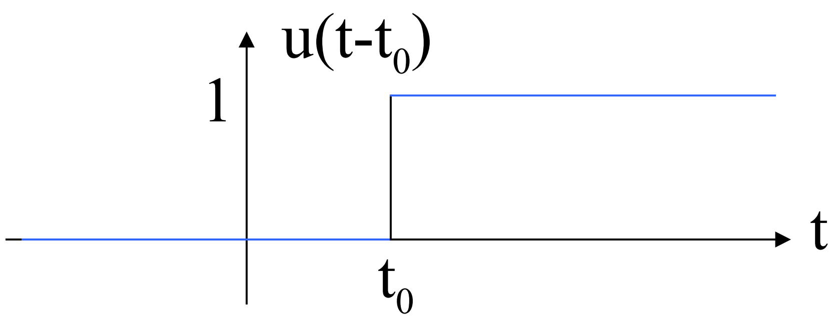
THE UNIT-STEP FUNCTION

The unit-step function is zero for all values of time which are less than zero and unity for positive values of time. The unit-step function is not defined for t=0.

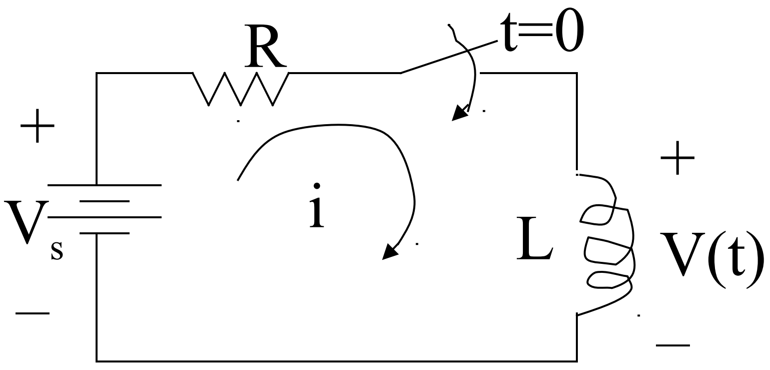
$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



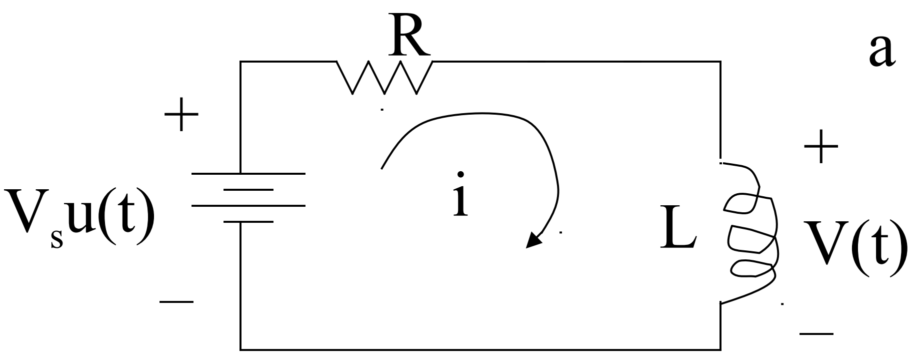
$$u(t - t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$$



THE STEP RESPONSE OF RL AND RC CIRCUITS



The switch in the circuit closes at t=0. Since the voltage applied to the circuit is 0 for t<0 and V_s for t>0, the switch and voltage combination can be represented as a step function $V_s u(t)$.



The initial current through the inductor (circuit) is zero.

After the switch has been closed, KVL requires

$$V_s = Ri + L\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{-Ri + V_s}{L} = -\frac{R}{L}(i - \frac{V_s}{R})$$

$$di = -\frac{R}{L}(i - \frac{V_s}{R})dt$$

$$\frac{di}{i - (V_s / R)} = -\frac{R}{L}dt$$

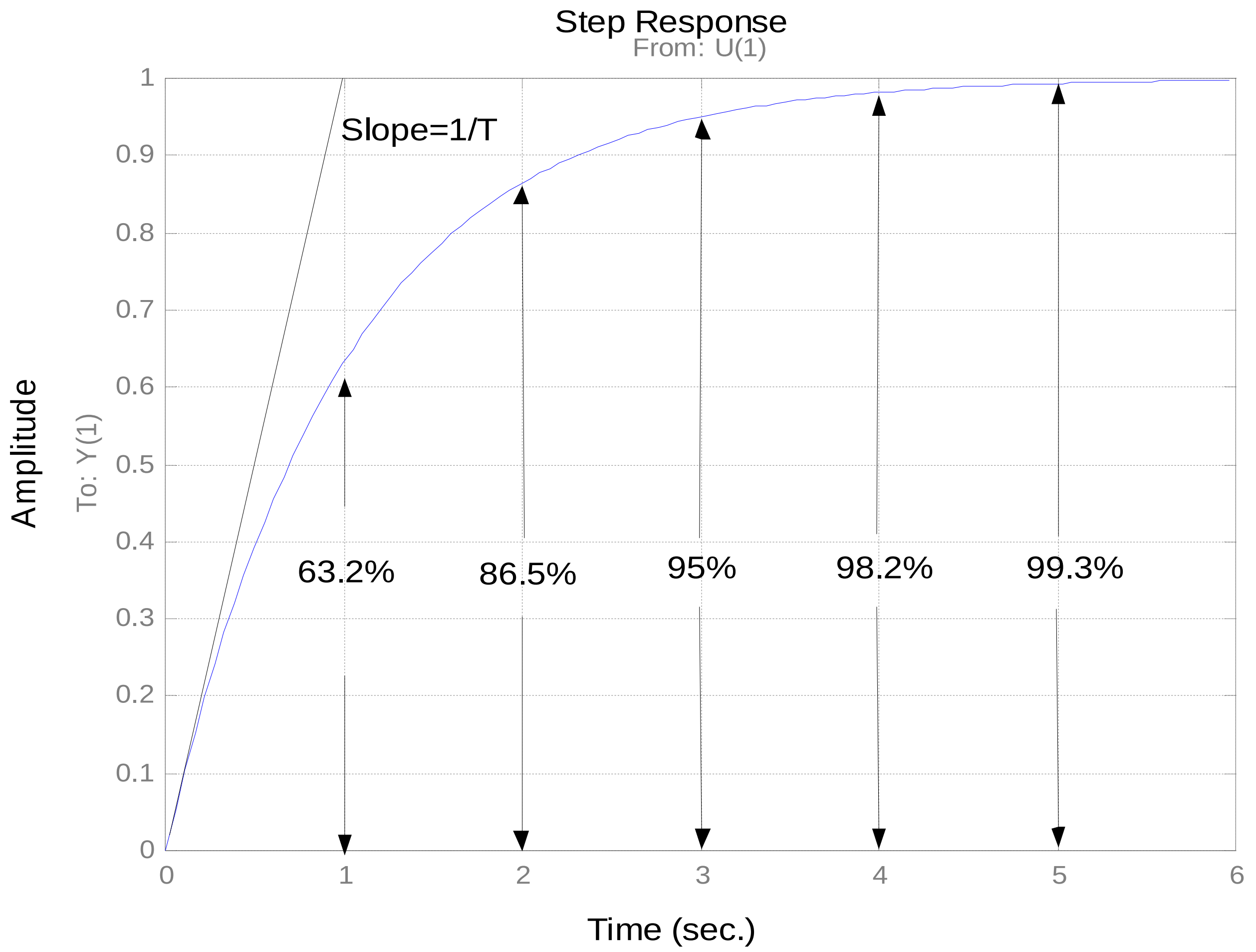
$$\int_{I_0}^{i(t)} \frac{dx}{x-(V_s / R)} = -\frac{R}{L}\int_0^t dy$$

$$\ln \frac{i(t)-(V_s / R)}{I_0-(V_s / R)} = -\frac{R}{L}t$$

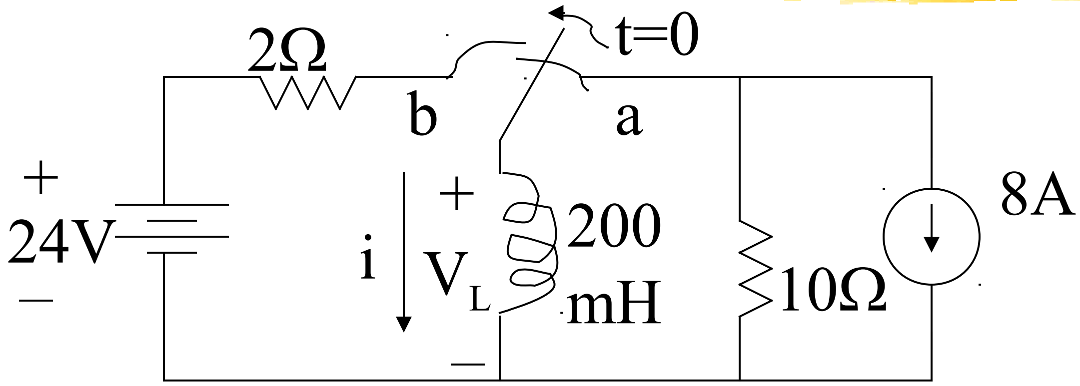
$$\frac{i(t)-(V_s / R)}{I_0-(V_s / R)} = e^{-\frac{R}{L}t}$$

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-\frac{R}{L}t}$$

$$i(t) = \frac{V_s}{R} - \frac{V_s}{R}e^{-\frac{R}{L}t}$$


Step Response
From: U(1)
Slope=1/T
63.2%
86.5%
95%
98.2%
99.3%
Amplitude
To: Y(1)
Time (sec.)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6

EXAMPLE

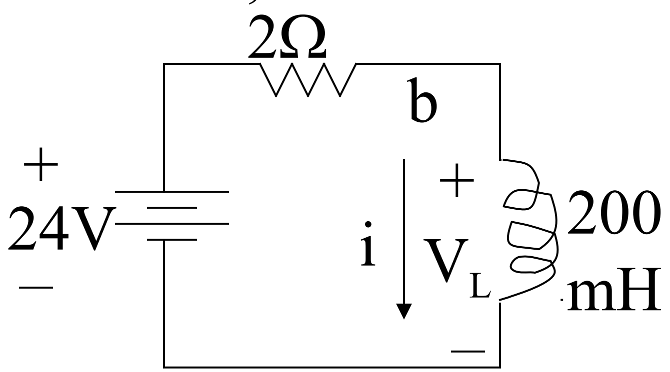


The switch in the circuit has been in position a for a long time. At t=0, the switch moves to position b.

a) Find i(t) for t>0, b) What is the initial voltage across the inductor just after the switch has moved to position b. c) How many milliseconds after the switch has been moved does the inductor voltage equal 24 V? d) Plot both i(t) and $V_L(t)$

Since the switch has been in position a for a long time, the circuit reaches to dc steady-state before the switch is moved to position b. The inductor is short circuit at $t=0^-$ and $I_0=-8A$.

For t>0, circuit becomes



This circuit reaches to steady-state as $t \longrightarrow \infty$. At infinity, inductor is short and current becomes 24/2=12A The time constant of the circuit is 200/2=100 ms.Then,

$$i(t) = 12 + (-8 - 12)e^{-t/0.1}$$

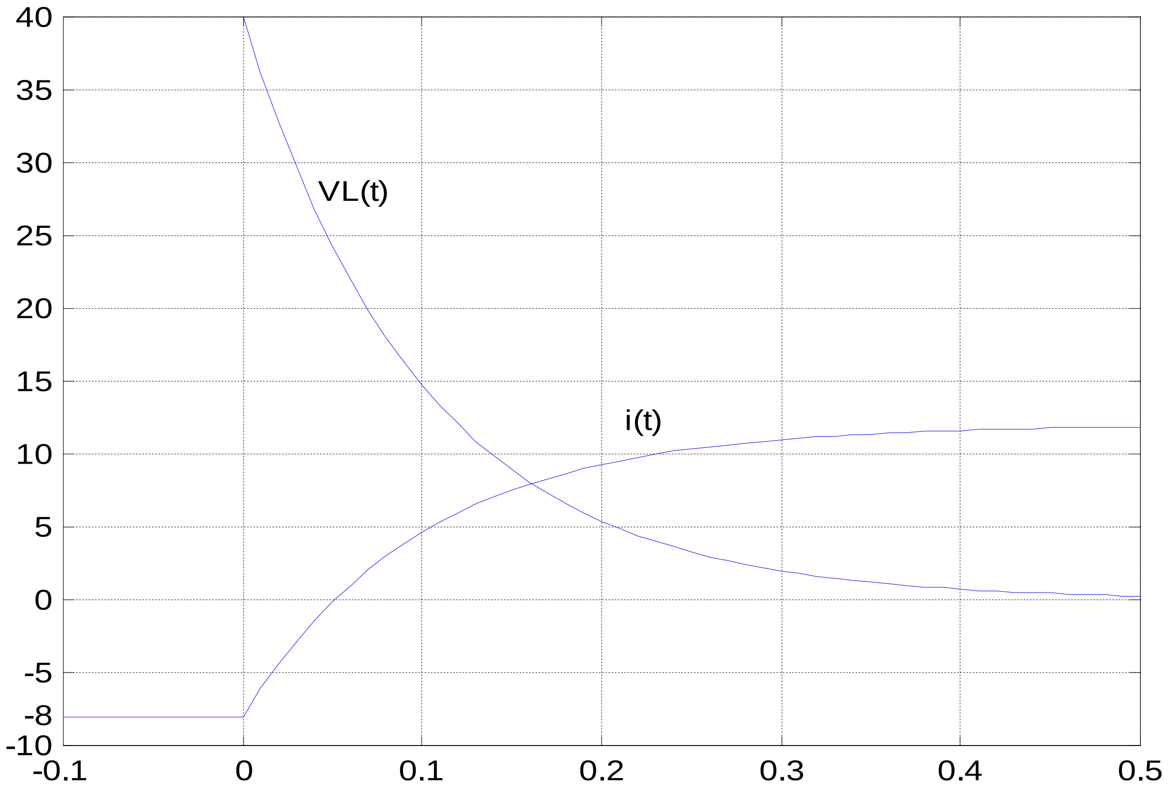
$$= 12 - 20e^{-10t}\,A \quad t \geq 0$$

$$V_L(t) = L\frac{di}{dt} = 0.2(200e^{-10t}) = 40e^{-10t}V \quad t \geq 0^+$$

$$V_L(0^+) = 40V$$

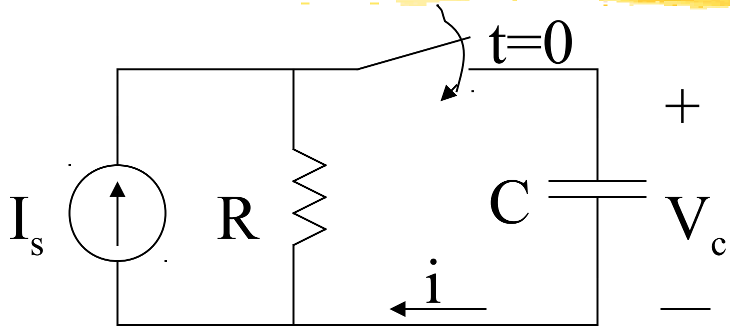
$$24 = 40e^{-10t}$$

$$t = \frac{1}{10}\ln\frac{40}{24} = 51.08ms$$

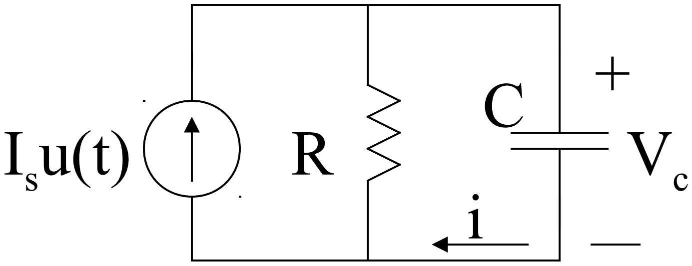


40
35
30
25
20
15
10
5
0
-5
-8
-10
-0.1
0
0.1
0.2
0.3
0.4
0.5
VL(t)
i(t)

THE STEP RESPONSE OF AN RC CIRCUIT



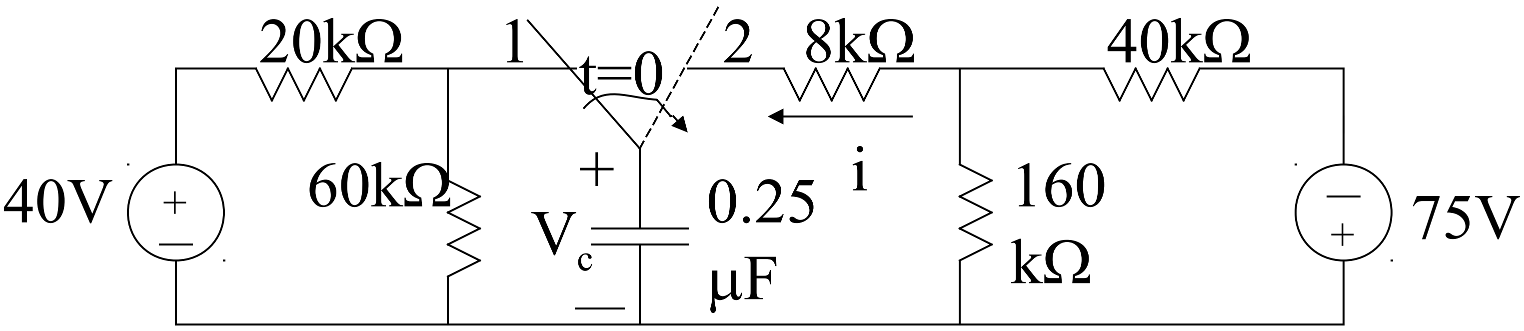
The current through the RC circuit is 0 for t<0 and I_s for t>0, therefore it can be replaced by a step current source as $I_s u(t)$.



$$C\frac{dV_c}{dt}+\frac{V_c}{R}=I_s \Rightarrow \frac{dV_c}{dt}+\frac{V_c}{RC}=\frac{I_s}{C}$$

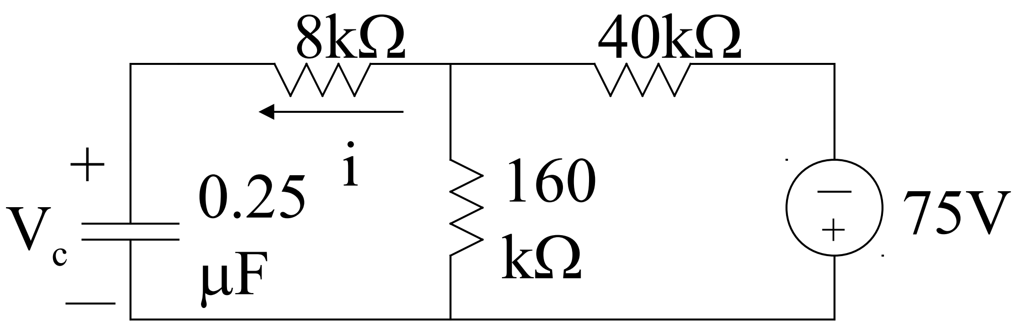
$$V_c(t)=I_sR+(V_0-I_sR)e^{-t/RC} \quad t\geq 0$$

EXAMPLE



The switch in the circuit has been in position 1 for a long time. At t=0, the switch moves to position 2. Find $V_c(t)$ and i(t) for t>0.

$$V_c(0^-) = \frac{60}{20+60}40 = 30V = V_c(0^+)$$

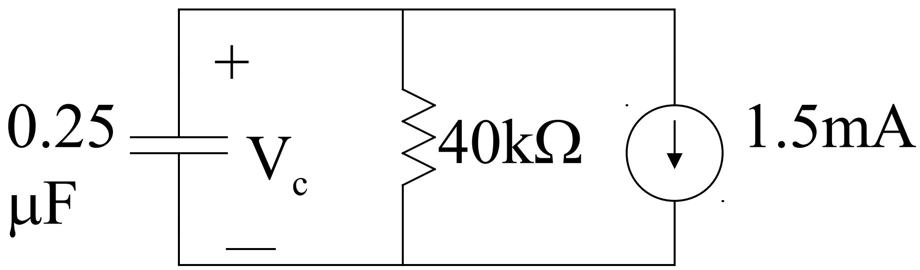


Let us determine the Norton equivalent seen by the capacitor.

$$V_{TH} = \frac{160\times 10^3}{(40+160)\times 10^3}(-75) = -60V$$

$$R_{TH} = 8 + (40 \| 160) = 40k\Omega$$

$$i_{sc} = \frac{V_{TH}}{R_{TH}} = \frac{-60}{40\times 10^3} = -1.5mA$$



Capacitor will be open as $t \rightarrow \infty$. $V_c(\infty)$=-40*1.5=-60V

The time constant is $0.25*10^{-6}*40*10^{3}$=10ms.

Then,

$$V_c(t) = -60 + (30 - (-60))e^{-100t}$$

$$= -60 + 90e^{-100t}V \quad t \geq 0$$

$$i(t) = C\frac{dV_c}{dt} = (0.25 \times 10^{-6})(-9000e^{-100t})$$

$$= -2.25e^{-100t}mA \qquad t \geq 0^+$$

A GENERAL SOLUTION FOR STEP AND NATURAL RESPONSES

The describing equations for RL and RC circuits have a common form as $\frac{dx}{dt} + \frac{x}{\tau} = K$ K could be zero.

Because the sources in the circuit are constant, the final value of x will be constant and dx/dt=0. Then, the final value of x is $x_f = K\tau$. Solution of the differential equation becomes

$$x(t) = x_f + \left[x(t_0) - x_f\right]e^{-(t-t_0)/\tau}$$

t_0 is the time of switching.

PROCEDURE

1) Identify the variable of interest for the circuit.

2) Determine the initial value of the variable.

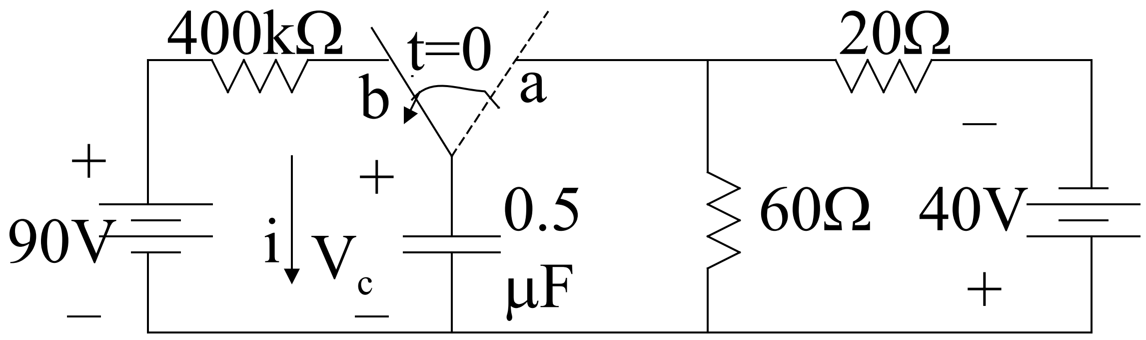
3) Calculate the final value of the variable.

4) Calculate the time constant for the circuit.

5) Then the solution is written as

$$x(t) = x_f + \left[x(t_0) - x_f \right] e^{-(t-t_0)/\tau}$$

EXAMPLE



The switch has been in position a for a long time. At t=0, it moves to b. Find $V_c(t)$ and $i(t)$ for $t>0$.

The switch has been in position a for a long time, circuit reaches to dc steady-state at $t=0^-$ and the capacitor looks like an open circuit. $V_c(0^-)=-[(60)/(60+20)]40=-30V=V_c(0^+)$.

After the switch has been in position b for a long time, circuit reaches to dc steady-state as $t\rightarrow\infty$. Capacitor looks like an open circuit. $V_{cf}=90V$

$\tau = RC = (400 x 10^3)(0.5 x 10^{-6}) = 0.2s$

$V_c(t) = 90 + (-30-90)e^{-5t} = 90 - 120e^{-5t}$ V $t \geq 0$

$i(0^+) = [90-(-30)]/(400 x 10^3) = 300\ \mu A$

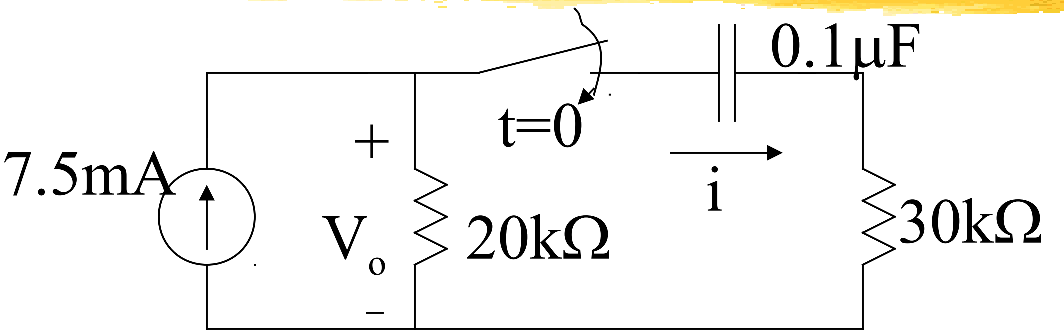
$i_f = 0$ (capacitor is open circuit, no current flows through the circuit).

$i(t) = 0 + (300-0)e^{-5t}\ \mu A$

To find how long the switch must be in position b before the capacitor voltage becomes zero, solve $120e^{-5t} = 90$.

$e^{5t} = (120/90)$. $t = 57.54$ ms.

EXAMPLE



The switch has been open for a long time. The initial charge on the capacitor is zero.

At t=0, the switch is closed. Find i(t) and $V_o(t)$ for t≥0.

$$i(0^+) = \frac{7.5(20)}{20+30} = 3mA \qquad i_f = 0$$

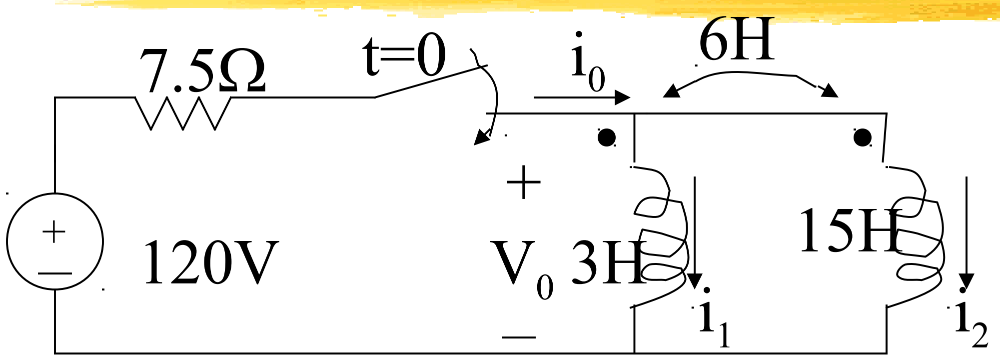
$$\tau = (50\times 10^3)(0.1\times 10^{-6}) = 5ms$$

$$i(t) = 3e^{-200t}mA \qquad t \geq 0^+$$

$$V_0(t) = 20\times 10^3(7.5\times 10^{-3} - i(t))$$

$$= 150 - 60e^{-200t}V \qquad t \geq 0^+$$

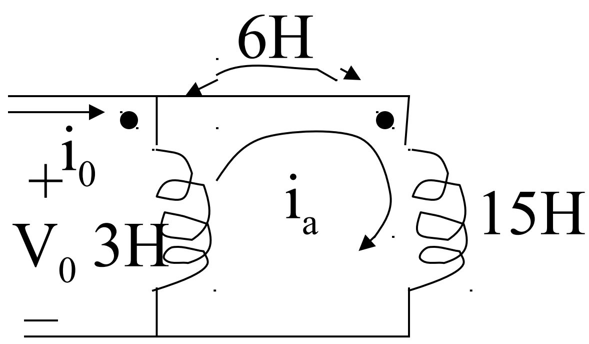
EXAMPLE



There is no energy stored in the circuit at the time the switch is closed.

Find i_0, V_0, i_1, and i_2.

First, we should determine the equivalent inductance of inductors and mutual inductor.



$$-V_o + 3\frac{d(i_o - i_a)}{dt} + 6\frac{di_a}{dt} = 0$$

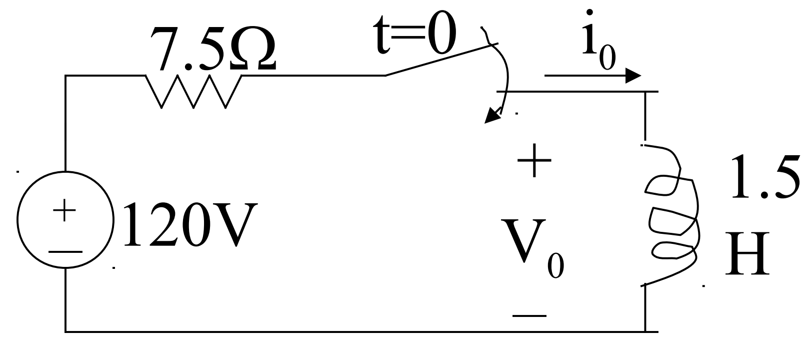
$$15\frac{di_a}{dt} + 3\frac{d(i_a - i_o)}{dt} + 6\frac{d(i_o - i_a)}{dt} - 6\frac{di_a}{dt} = 0$$

$$3\frac{di_0}{dt}+3\frac{di_a}{dt}=V_o$$

$$3\frac{di_0}{dt}+6\frac{di_a}{dt}=0 \Rightarrow \frac{di_a}{dt}=-\frac{1}{2}\frac{di_o}{dt}$$

$$(3-\frac{3}{2})\frac{di_o}{dt}=V_o=\frac{3}{2}\frac{di_o}{dt}$$

$$L_{eq}=\frac{3}{2}H$$



$$\tau=\frac{L_{eq}}{R}=\frac{1.5}{7.5}=0.2s$$

$$i_o=16-16e^{-5t}A \quad t\geq 0$$

$$V_o=120-7.5i_0$$

$$=120e^{-5t}V \quad t\geq 0^+$$

$$3\frac{di_1}{dt}+6\frac{di_2}{dt}=6\frac{di_1}{dt}+15\frac{di_2}{dt}\Rightarrow\frac{di_1}{dt}=-3\frac{di_2}{dt}$$

$$i_o=i_1+i_2\Rightarrow\frac{di_o}{dt}=\frac{di_1}{dt}+\frac{di_2}{dt}$$

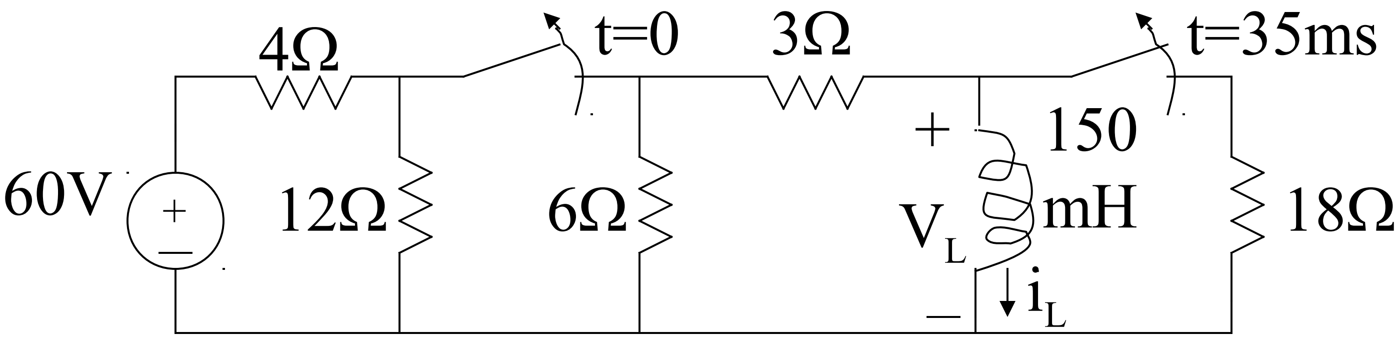
$$80e^{-5t}=-2\frac{di_2}{dt}$$

$$i_2=\int_0^t -40e^{-5x}dx=-8+8e^{-5t}A\quad t\geq 0$$

$$i_1=i_o-i_2=24-24e^{-5t}A\quad t\geq 0$$

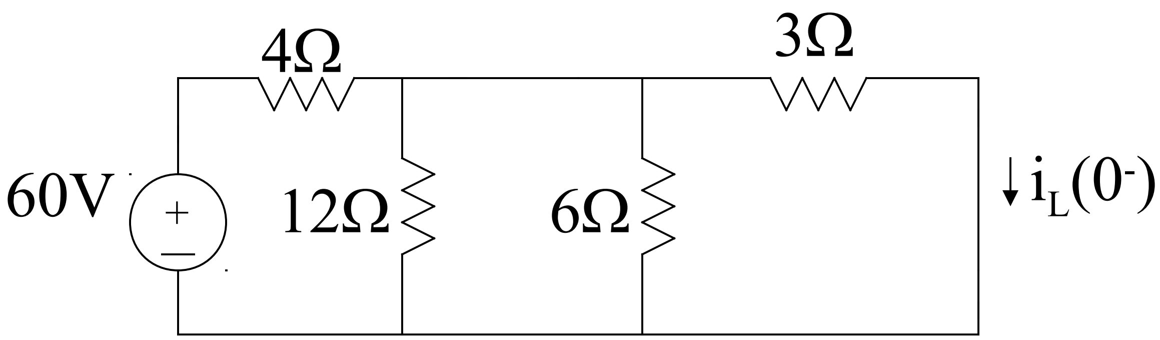
SEQUENTIAL SWITCHING

Whenever switching occurs more than once in a circuit, we have **sequential switching**.

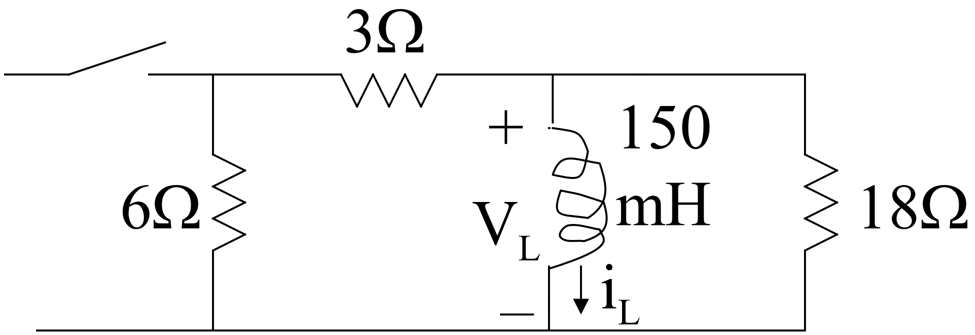


The two switches in the circuit have been closed for a long time. At t=0, switch 1 is opened. The, 35 ms later, switch 2 is opened. Find $i_L(t)$ for $t \geq 0$. What percentage of the initial energy stored in the inductor is dissipated in the 18Ω, 3Ω, and 6Ω resistors.

For $t<0$, both switches are closed for a long time causing the inductor to short-circuit the 18Ω resistor. The circuit at $t=0^-$ is



After making several source transformations $i_L(0^-)$ is determined to be 6A. For $0 \leq t \leq 35$ ms, 4Ω , 12Ω and 60 V source are disconnected from the circuit. The circuit becomes



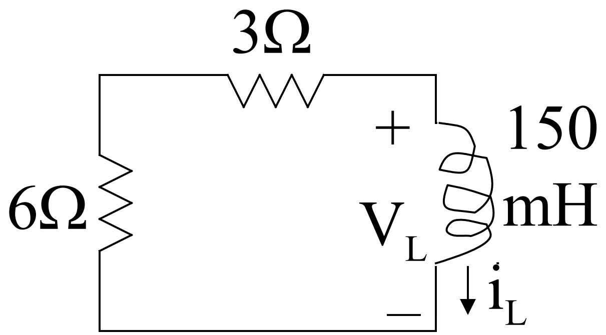
The equivalent resistance seen by the inductor is (3+6)||18=6Ω. The time constant of the circuit is $(150/6)\text{x}10^{-3}$=25 ms

$$i_L(t) = 6e^{-40t}A \quad 0 \le t \le 35ms$$

When t=35ms, the value of the inductor current is

$$i_L(35\times 10^{-3}) = 6e^{-1.4} = 1.48A$$

When switch 2 is opened, the circuit reduces to



The time constant is (150/9)x10^{-3}=16.67ms

$$i_L(t) = 1.48e^{-60(t-0.035)}A \qquad t \geq 35ms$$

The 18Ω resistor is in the circuit when t< 35ms. During this interval

$$V_L(t) = 0.15\frac{d}{dt}(6e^{-40t}) = -36e^{-40t}V \quad 0<t<35ms$$

$$p_{18} = \frac{V_L^2}{18} = 72e^{-80t}W \quad 0 < t < 35ms$$

$$\omega_{18} = \int_0^{0.035} 72e^{-80t}\,dt = \frac{72}{-80} e^{-80t}\Big|_0^{0.035}$$

$$= 0.9(1 - e^{-2.8}) = 845.27mJ$$

The initial energy stored in the 150 mH inductor is

$$\omega_L(0) = \frac{1}{2}(0.15)(36) = 2.7J = 2700mJ$$

$$(845.27 / 2700) \times 100 = 31.31\%$$

For 0<t<35 ms, the voltage across the 3Ω resistor is

$$V_{3\Omega} = \left(\frac{V_L}{9}\right)3 = -12e^{-40t}$$

$$\omega_{3\Omega} = \int_0^{0.035} \frac{144e^{-80t}}{3} dt = 0.6(1 - e^{-2.8}) = 563.51 mJ$$

For t>35 ms, the current in the 3Ω resistor is equal to i_L. Then

$$\omega_{3\Omega} = \int_{0.035}^{\infty} 3 \times (1.48e^{-60(t-0.035)})^2 dt = 54.73 mJ$$

$$\omega_{3\Omega T} = 563.51 + 54.73 = 618.24mJ$$

$$\frac{618.24}{2700} \times 100 = 22.9\%$$

Because the 6Ω resistor is in series with the 3Ω resistor, the energy dissipated and the percentage of the initial energy stored will be twice that of the 3Ω resistor.

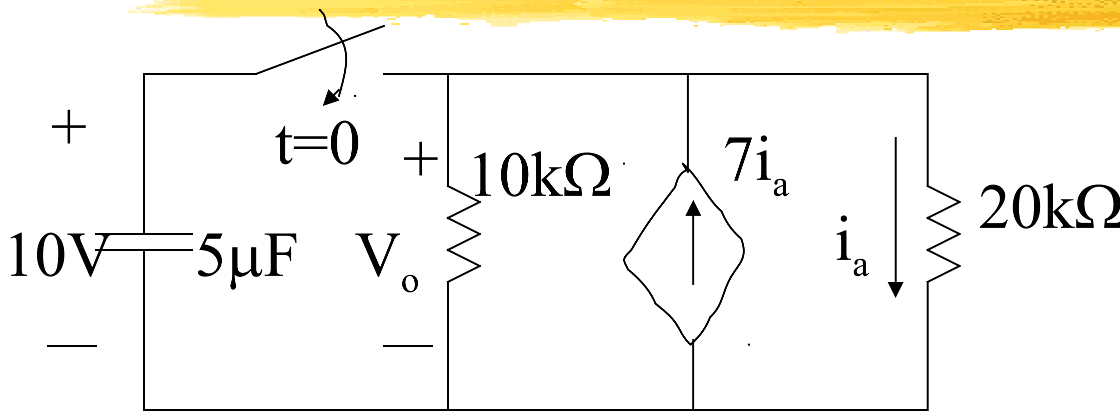
$$\omega_{6\Omega T} = 1236.48mJ$$

$$\frac{1236.48}{2700} \times 100 = 45.8\%$$

UNBOUNDED RESPONSE

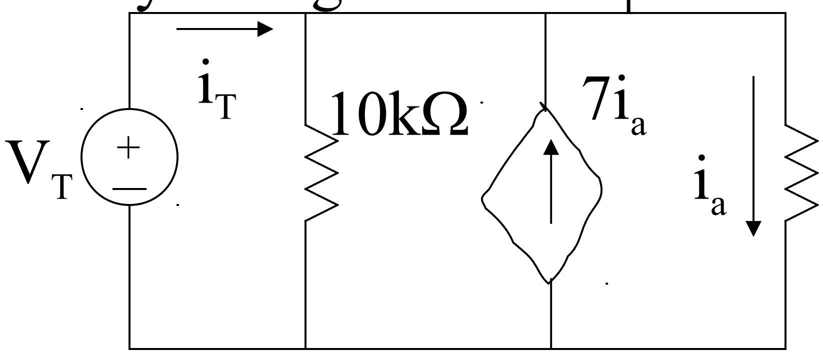
A circuit response may grow, rather than decay, exponentially with time. This type of response is called an **unbounded response** and it is possible if the circuit contains dependent sources. In that case, the Thevenin resistance seen by the inductor and capacitor may be negative. This negative resistance generates a negative time constant, and the resulting currents and voltages increase without limit.

EXAMPLE



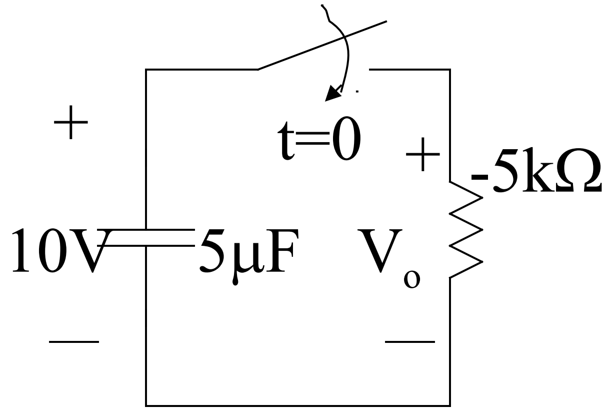
When the switch is closed, the voltage across the capacitor is 10V. Find the expression for V_o for $t \geq 0$

We should determine R_{Th} seen by the capacitor. Connect a dummy voltage source V_T across the terminals of the capacitor.



$$i_T = \frac{V_T}{10} - 7\frac{V_T}{20} + \frac{V_T}{20} mA$$

$$R_{Th} = \frac{V_T}{i_T} = -5k\Omega$$



For t≥0, the differential equation describing the given circuit is

$$(5\times10^{-6})\frac{dV_o}{dt}-\frac{V_o}{5}\times10^{-3}=0$$

$$\frac{dV_o}{dt}-40V_o=0$$

$$V_o(t)=10e^{40t}V \qquad t\geq0$$

Assume that capacitor short-circuits when its terminal voltage reaches 150 V. How many milliseconds elapse before the capacitor short-circuits?

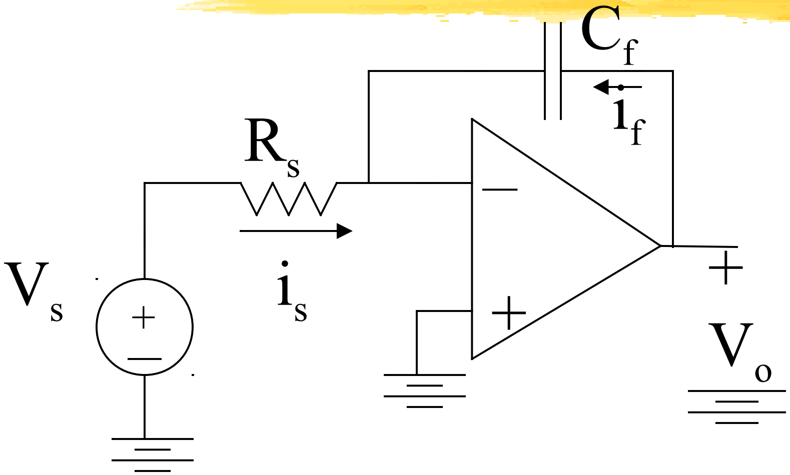
$$150 = 10e^{40t_a}$$

$$e^{40t_a} = 15$$

$$40t_a = \ln 15$$

$$t_a = 67.7ms$$

THE INTEGRATING AMPLIFIER



$$i_f + i_s = 0$$

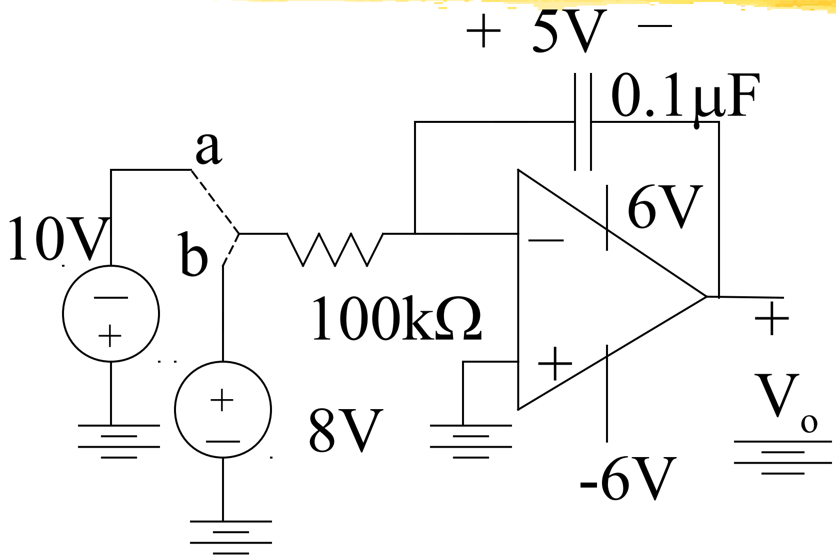
$$v_+ = v_- = 0$$

$$i_s = \frac{V_s}{R_s}, \quad i_f = C_f \frac{dV_o}{dt}$$

$$\frac{dV_o}{dt} = -\frac{1}{R_s C_f} V_s$$

$$V_o(t) = -\frac{1}{R_s C_f} \int_{t_o}^{t} V_s dy + V_o(t_o)$$

EXAMPLE



At the instant the switch makes contact with terminal a, the voltage across the capacitor is 5V. The switch remains at a for 9 ms then moves to terminal b. How many milliseconds after making contact with terminal a does the opamp saturate?

$$V_o = -5 - \frac{1}{10^{-2}} \int_0^t (-10)\,dy = (-5 + 1000t)V$$

$$V_o(9ms) = -5 + 9 = 4V$$

The expression for the output voltage after the switch moves to terminal b

$$V_o = 4 - \frac{1}{10^{-2}} \int_{9\times10^{-3}}^{t} 8dy$$

$$= 4 - 800(t - 9\times10^{-3}) = (11.2 - 800t)V$$

During this interval, the voltage is decreasing. Therefore opamp will saturate when V_o=-6V

$$11.2 - 800t_s = -6 \Rightarrow t_s = 21.5ms$$