THEVENIN AND NORTON EQUIVALENTS

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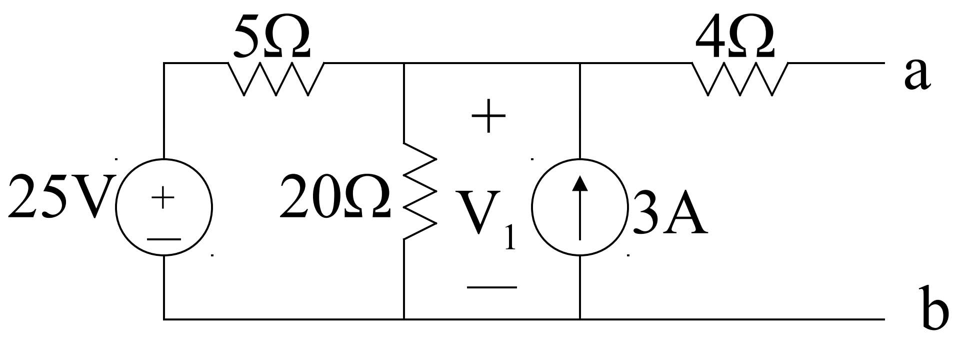
THEVENIN EQUIVALENT

- At times in circuit analysis, we want to determine terminal behavior of a circuit.
- Thevenin and Norton equivalents are circuit simplification techniques that focus on terminal behavior.
- The Thevenin equivalent circuit is an independent voltage source V_{TH} is series with a resistor R_{TH} which replaces an interconnection of sources and resistors.

- This series combination is equivalent to the original circuit such that if we connect a load across the terminals of each circuit, we get the same voltage and current at the terminals of the load.
- To calculate the Thevenin voltage V_{TH}, we simply calculate the open-circuit voltage at the terminals of the original circuit.

- If we place a short circuit across the terminals of the Thevenin equivalent circuit, the short-circuit current is $i_{sc}=V_{TH}/R_{TH}$.
- This short-circuit current must be identical to the short-circuit current in the original network. Thus, $R_{TH}=V_{TH}/i_{sc}$

EXAMPLE

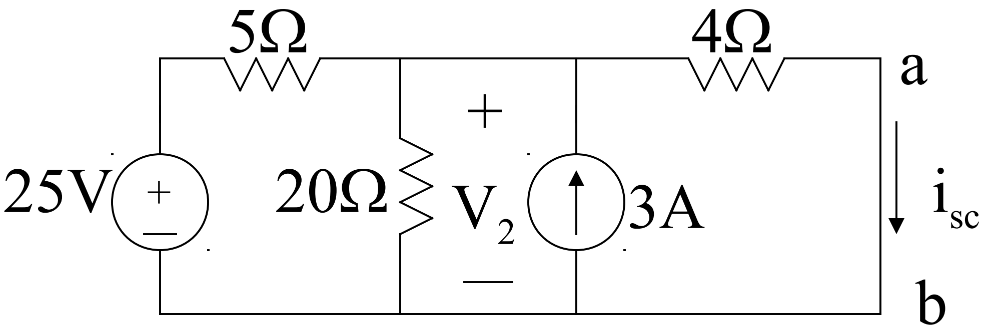


Find the Thevenin equivalent of the circuit at the terminals a and b.

$$V_{TH} = V_{ab}$$

$$\frac{V_1 - 25}{5} + \frac{V_1}{20} - 3 = 0 \Rightarrow V_1 = 32V$$

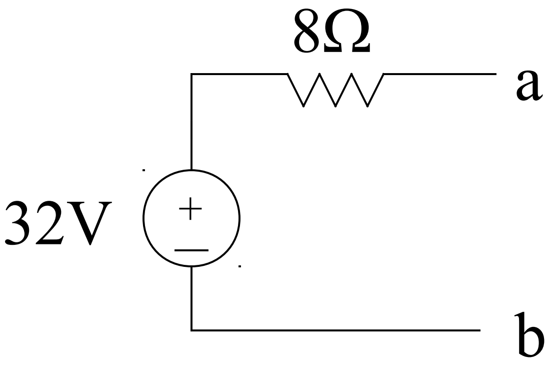
$$V_{TH} = V_1 = 32V$$



$$\frac{V_2 - 25}{5} + \frac{V_2}{20} - 3 + \frac{V_2}{4} = 0$$

$$V_2 = 16V \qquad i_{sc} = \frac{16}{4} = 4A$$

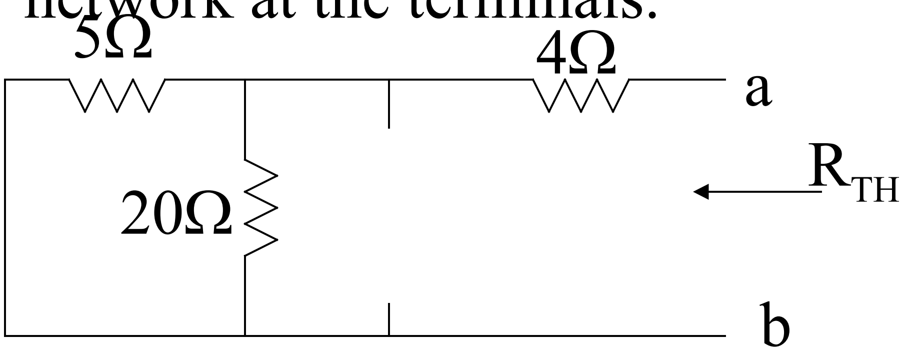
$$R_{TH} = \frac{V_{TH}}{i_{sc}} = \frac{32}{4} = 8\Omega$$



Thevenin Equivalent

CALCULATING R_{TH}

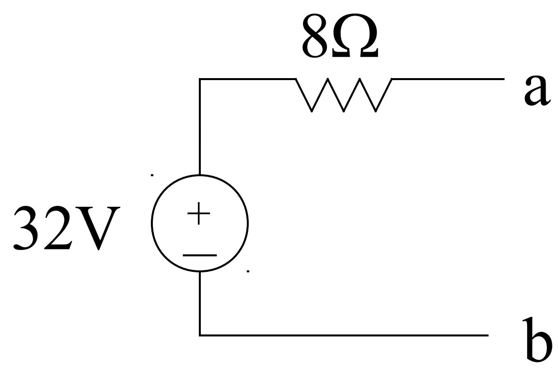
When the circuit has only independent sources, it is possible to calculate R_{TH} by killing the sources. A voltage source is killed by short circuiting and a current source is killed by open circuiting. Then R_{TH} is the equivalent resistance of the dead network at the terminals.



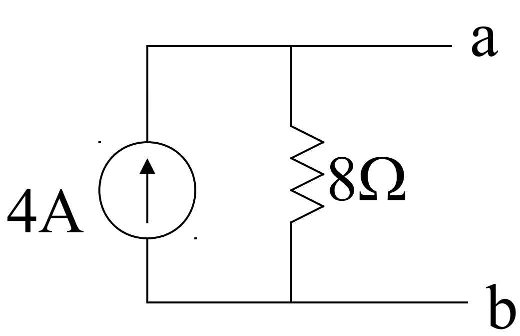
$$R_{TH} = (5||20) + 4 = 8\Omega$$

NORTON EQUIVALENT

A **Norton Equivalent Circuit** consists of an independent current source in parallel with the Norton equivalent resistance. It can be derived from a Thevenin equivalent circuit simply by making a source transformation.

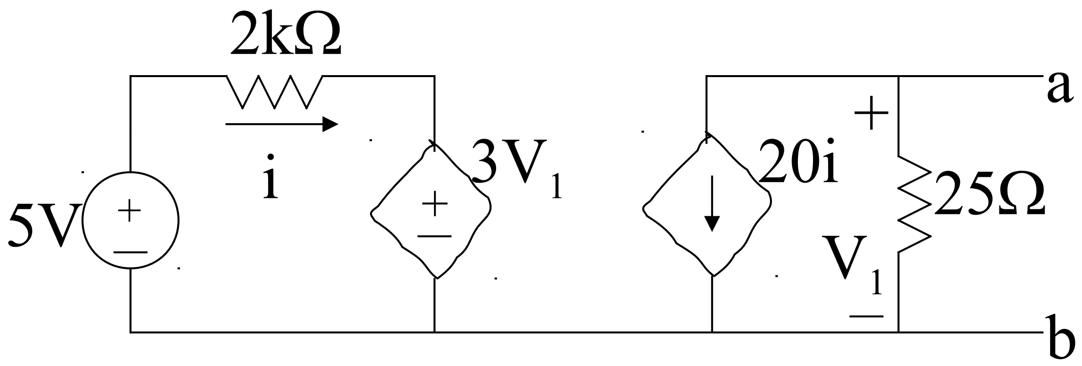


Thevenin equivalent



Norton equivalent

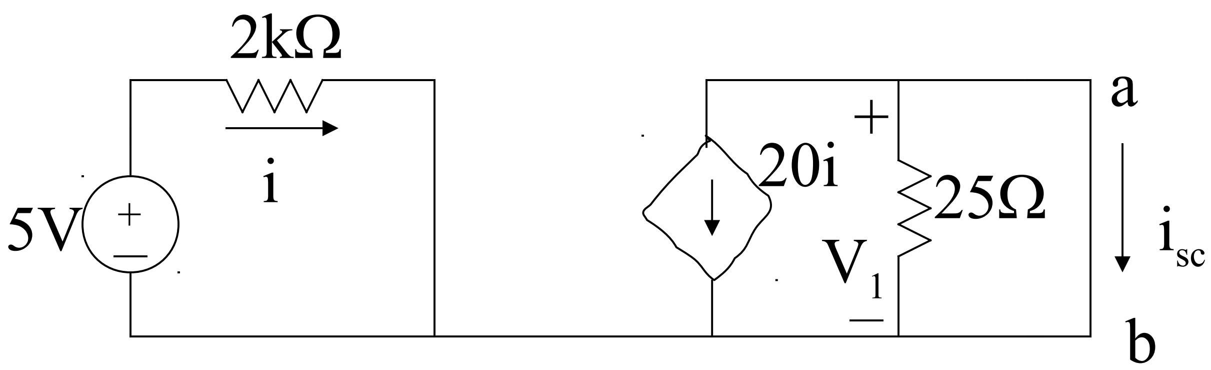
EXAMPLE



$$V_{TH} = V_{ab} = (-20i)(25) = -500i$$

$$i = \frac{5 - 3V_1}{2000} = \frac{5 - 3V_{TH}}{2000}$$

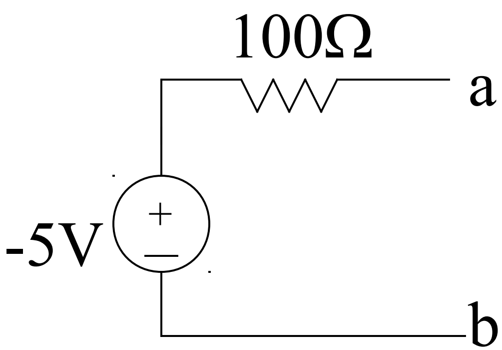
$$V_{TH} = -5V$$



$$i_{sc} = -20i \qquad i = \frac{5}{2000} = 2.5mA$$

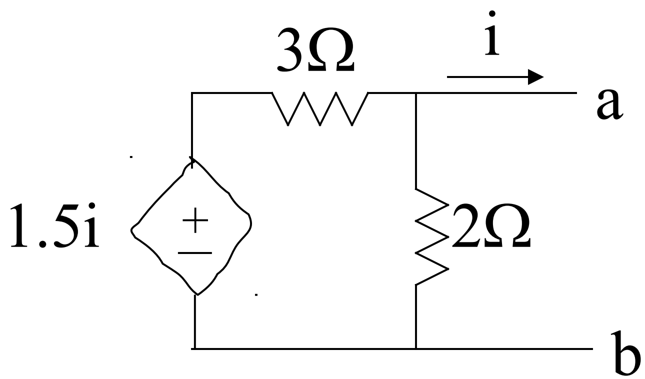
$$i_{sc} = -20(2.5) = -50mA$$

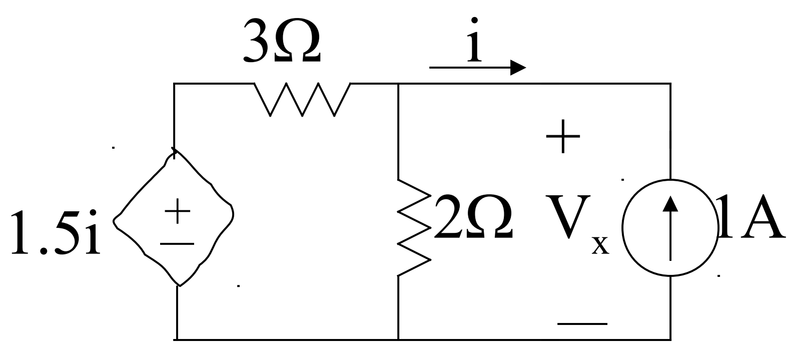
$$R_{TH} = \frac{V_{TH}}{i_{sc}} = \frac{-5}{-50\times10^{-3}} = 100\Omega$$



Thevenin equivalent

NO INDEPENDENT SOURCE CASE

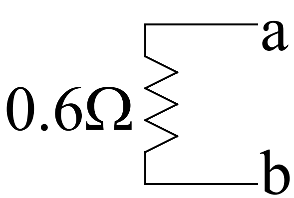




Since there is no independent source, both V_{TH} and i_{sc} are zero. In this case, the equivalent circuit is only a resistor equal to R_{TH}. To determine R_{TH}, we connect a source (voltage or current) across the terminals. The value of the source is not important.

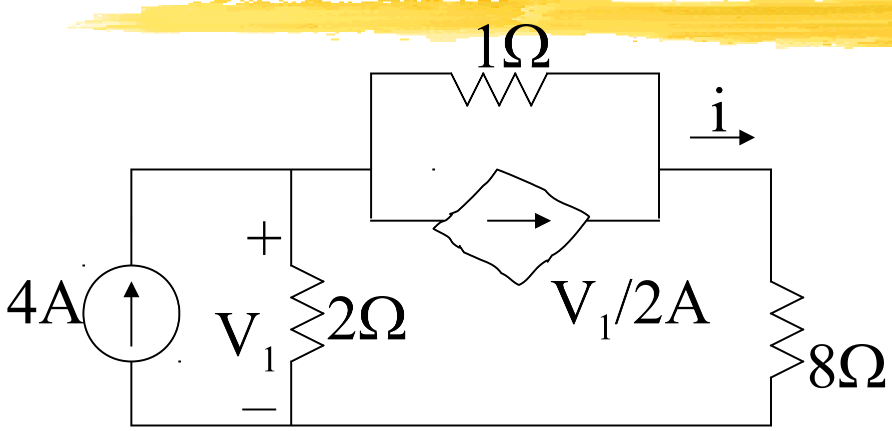
$$\frac{V_x - 1.5(-1)}{3} + \frac{V_x}{2} - 1 = 0$$

$$V_x = 0.6V \Rightarrow R_{TH} = \frac{0.6}{1} = 0.6\Omega$$

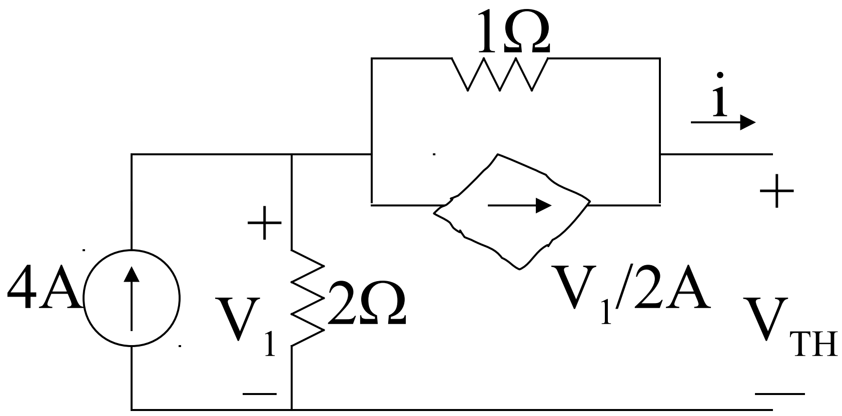


Thevenin equivalent

EXAMPLE

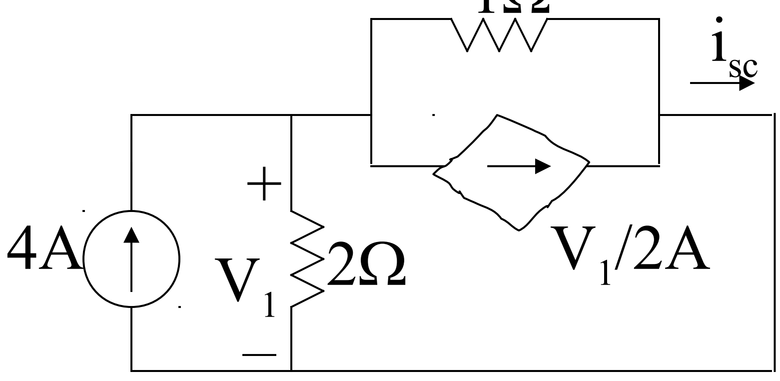


Find i by using the Norton equivalent circuit seen by 8Ω resistor.



$$V_1 = 4(2) = 8V$$

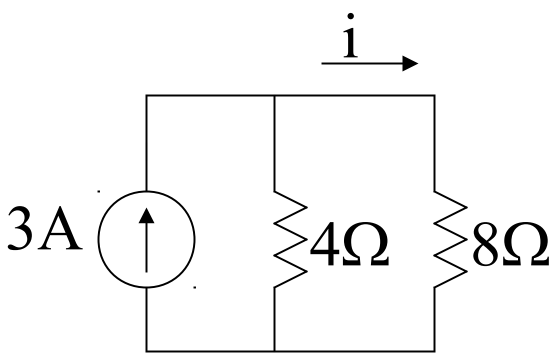
$$V_{TH} = 1\left(\frac{V_1}{2}\right) + V_1 = 4 + 8 = 12V$$



$$-4+\frac{V_1}{2}+\frac{V_1}{2}+\frac{V_1}{1}=0$$

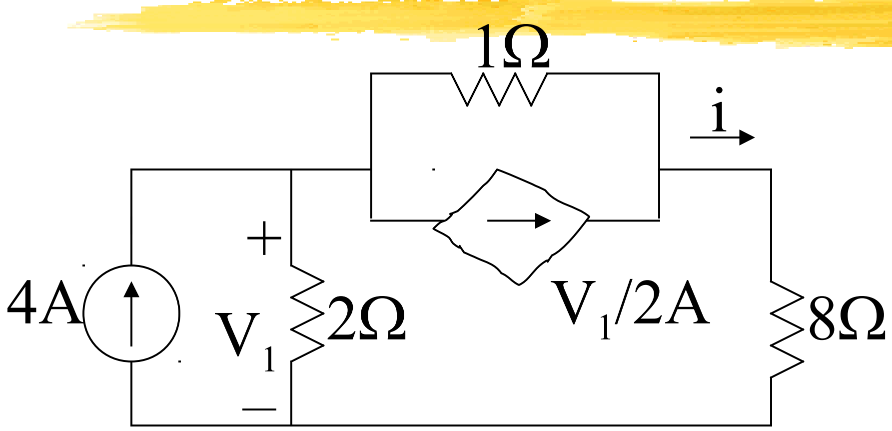
$$V_1=2V \Rightarrow i_{sc}=\frac{V_1}{2}+\frac{V_1}{1}=1+2=3A$$

$$R_{TH}=\frac{V_{TH}}{i_{sc}}=\frac{12}{3}=4\Omega$$

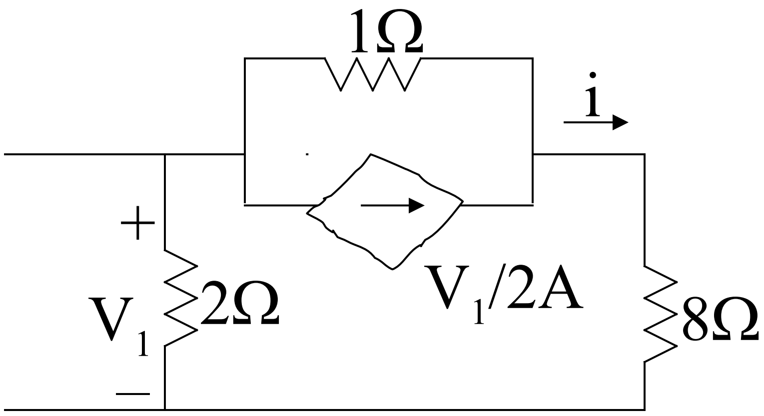


$$i=\frac{4}{4+8}3=1A$$

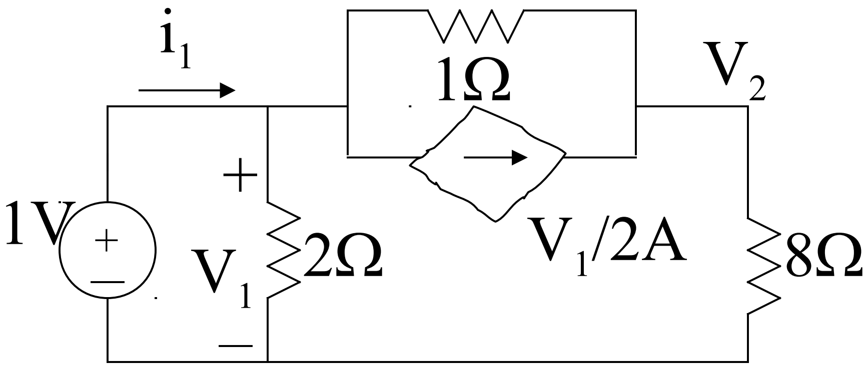
EXAMPLE



Find V_1 by using the Thevenin equivalent seen by 4A current source.



When we remove the current source to find V_{TH}, no independent source is present in the circuit. Therefore the Thevenin equivalent is only R_{TH}.

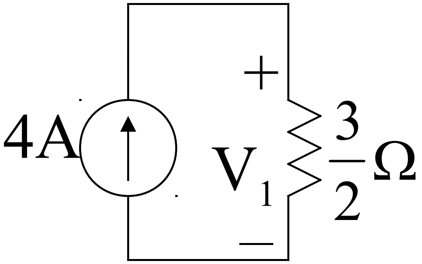


Connect a 1V voltage source and determine the current through it.

$$\frac{V_2-1}{1}-\frac{V_1}{2}+\frac{V_2}{8}=0 \Rightarrow V_2=\frac{4}{3}V$$

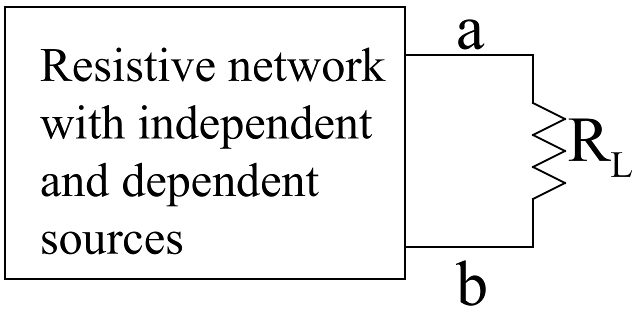
$$-i_1+\frac{1}{2}+\frac{1}{2}+\frac{1-\frac{4}{3}}{1}=0 \Rightarrow i_1=\frac{2}{3}A$$

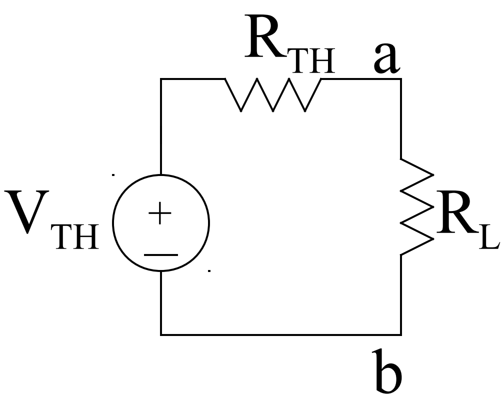
$$R_{TH}=\frac{1}{i_1}=\frac{3}{2}\Omega$$



$$V_1=4\left(\frac{3}{2}\right)=6V$$

MAXIMUM POWER TRANSFER





Determine the value of R_L that permits maximum power delivery to R_L.

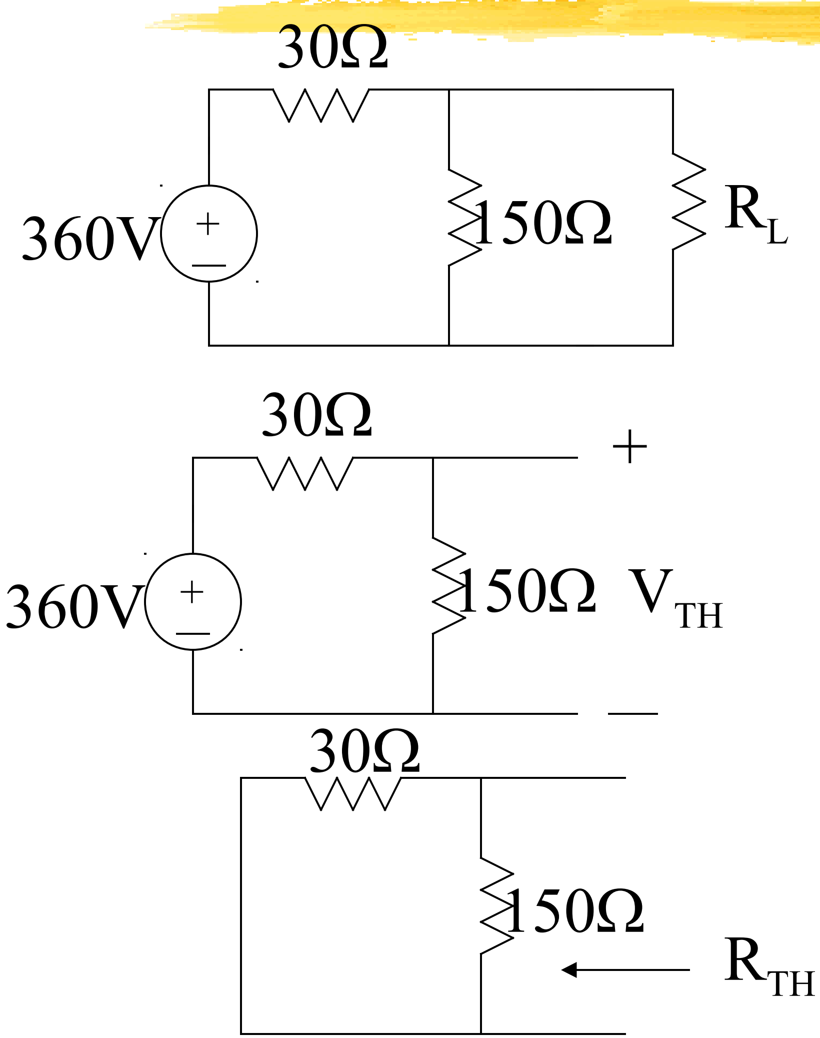
$$p = i^2 R_L = \left(\frac{V_{TH}}{R_{TH} + R_L} \right)^2 R_L$$

$$\frac{dp}{dR_L} = V_{TH}^2 \left[\frac{(R_{TH} + R_L)^2 - R_L \cdot 2(R_{TH} + R_L)}{(R_{TH} + R_L)^4} \right]$$

$$(R_{TH} + R_L)^2 = R_L \cdot 2(R_{TH} + R_L)$$

$$R_L = R_{TH} \Rightarrow p_{\max} = \frac{V_{TH}^2 R_L}{(2R_L)^2} = \frac{V_{TH}^2}{4R_L}$$

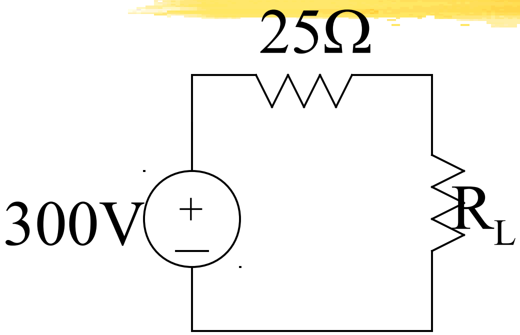
EXAMPLE

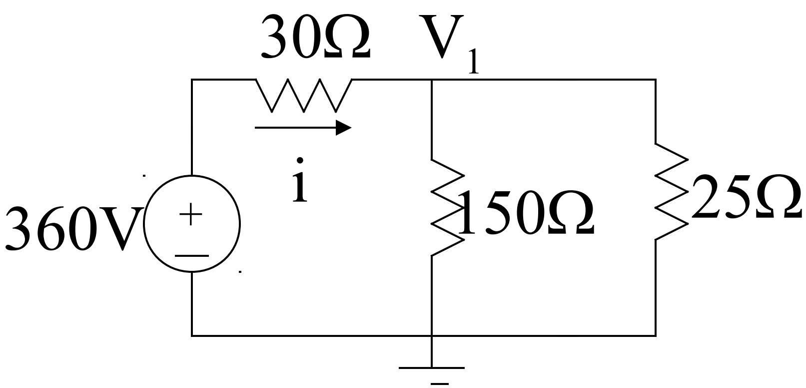


Determine R_L for maximum power transfer. When R_L is adjusted for maximum power transfer, what percentage of the power delivered by 360V source reaches R_L?

$$V_{TH} = \frac{150}{150+30}360 = 300V$$

$$R_{TH} = 30||150 = 25\Omega$$





For maximum power transfer R_L=25Ω.

$$p_{max} = \frac{(300)^2}{4(25)} = 900W$$

$$V_1 = \frac{(150||25)}{(150||25)+30}360 = 150V$$

$$i = \frac{360-150}{30} = 7A$$

$$p_{360V} = (-7)360 = -2520W$$

$$\frac{900}{2520}\times 100 = 35.71\%$$