Logic and Computer Design Fundamentals

Chapter 2 – Combinational Logic Circuits

Part 1 – Gate Circuits and Boolean Equations

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Overview

- **Part 1 – Gate Circuits and Boolean Equations**
  - Binary Logic and Gates
  - Boolean Algebra
  - Standard Forms
- **Part 2 – Circuit Optimization**
  - Two-Level Optimization
  - Map Manipulation
  - Multi-Level Circuit Optimization
- **Part 3 – Additional Gates and Circuits**
  - Other Gate Types
  - Exclusive-OR Operator and Gates
  - High-Impedance Outputs

Binary Logic and Gates

- **Binary variables** take on one of two values.
- **Logical operators** operate on binary values and binary variables.
- Basic logical operators are the **logic functions** AND, OR and NOT.
- **Logic gates** implement logic functions.
- **Boolean Algebra**: a useful mathematical system for specifying and transforming logic functions.
- We study Boolean algebra as foundation for designing and analyzing digital systems!

Binary Variables

- Recall that the two binary values have different names:
  - True/False
  - On/Off
  - Yes/No
  - 1/0
- We use 1 and 0 to denote the two values.
- Variable identifier examples:
  - A, B, y, z, or X_1 for now
  - RESET, START_IT, or ADD1 later

Logical Operations

- The three basic logical operations are:
  - AND
  - OR
  - NOT
- AND is denoted by a dot (·).
- OR is denoted by a plus (+).
- NOT is denoted by an overbar (¯), a single quote mark (') after, or (~) before the variable.

Notation Examples

- Examples:
  - $Y = A \cdot B$ is read "Y is equal to A AND B."
  - $z = x + y$ is read "z is equal to x OR y."
  - $X = \overline{A}$ is read "X is equal to NOT A."
- Note: The statement:

  1 + 1 = 2 (read "one plus one equals two")

  is not the same as

  1 + 1 = 1 (read "1 or 1 equals 1").

Operator Definitions

- **Operations are defined on the values "0" and "1" for each operator:**

AND	OR	NOT
$0 \cdot 0 = 0$	$0 + 0 = 0$	$\overline{0} = 1$
$0 \cdot 1 = 0$	$0 + 1 = 1$	$\overline{1} = 0$
$1 \cdot 0 = 0$	$1 + 0 = 1$	
$1 \cdot 1 = 1$	$1 + 1 = 1$	

Truth Tables

- ***Truth table*** **– a tabular listing of the values of a function for all possible combinations of values on its arguments**
- **Example: Truth tables for the basic logic operations:**

AND		
X	Y	$Z = X \cdot Y$
0	0	0
0	1	0
1	0	0
1	1	1

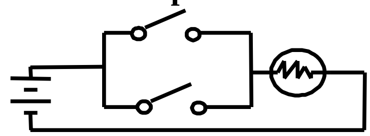
OR		
X	Y	$Z = X + Y$
0	0	0
0	1	1
1	0	1
1	1	1

NOT	
X	$Z = \overline{X}$
0	1
1	0

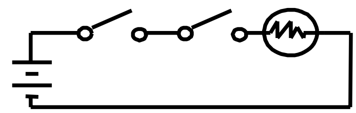
Logic Function Implementation

- **Using Switches**
  - **For inputs:**
    - **logic 1 is switch closed**
    - **logic 0 is switch open**
  - **For outputs:**
    - **logic 1 is light on**
    - **logic 0 is light off.**
  - **NOT uses a switch such that:**
    - **logic 1 is switch open**
    - **logic 0 is switch closed**

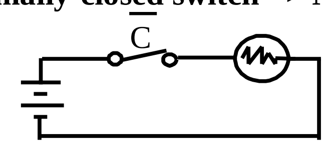
Switches in parallel => OR



Switches in series => AND

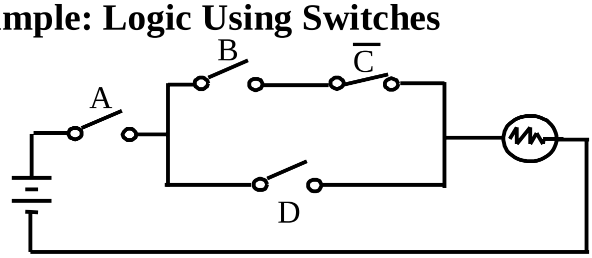


Normally-closed switch => NOT



Logic Function Implementation (Continued)

- **Example: Logic Using Switches**



- **Light is on (L = 1) for**

  L(A, B, C, D) =

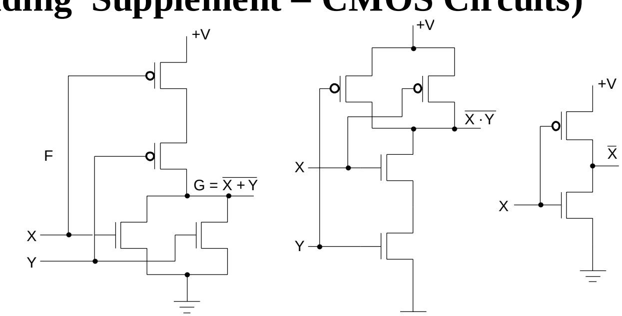
  and off (L = 0), otherwise.
- **Useful model for relay circuits and for CMOS gate circuits, the foundation of current digital logic technology**

Logic Gates

- **In the earliest computers, switches were opened and closed by magnetic fields produced by energizing coils in *relays*. The switches in turn opened and closed the current paths.**
- **Later, *vacuum tubes* that open and close current paths electronically replaced relays.**
- **Today, *transistors* are used as electronic switches that open and close current paths.**

Logic Gates (continued)

- **Implementation of logic gates with transistors (See Reading Supplement – CMOS Circuits)**

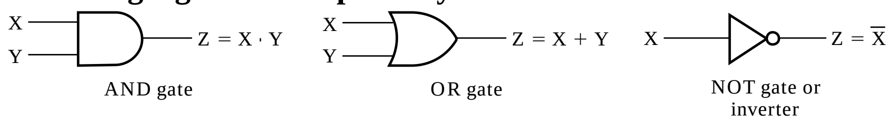


(a) NOR (b) NAND (c) NOT

- **Transistor or tube implementations of logic functions are called <u>logic gates</u> or just <u>gates</u>**
- **Transistor gate circuits can be modeled by switch circuits**

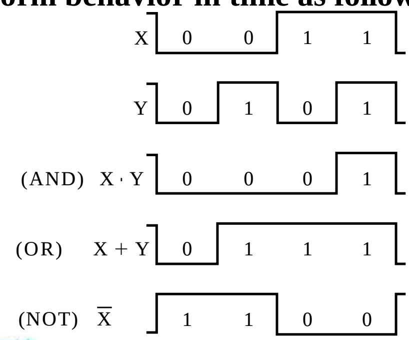
Logic Gate Symbols and Behavior

- **Logic gates have special symbols:**



(a) Graphic symbols

- **And waveform behavior in time as follows:**



(b) Timing diagram

Logic Diagrams and Expressions

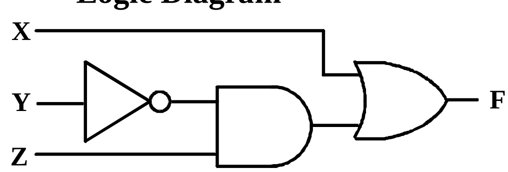
Truth Table

X Y Z	$F = X + \overline{Y} \cdot Z$
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	1
1 1 0	1
1 1 1	1

Equation

$$F = X + \overline{Y} Z$$

Logic Diagram



- **Boolean equations, truth tables and logic diagrams describe the same function!**
- **Truth tables are unique; expressions and logic diagrams are not. This gives flexibility in implementing functions.**

Boolean Algebra

- An algebraic structure defined on a set of at least two elements, B, together with three binary operators (denoted +, · and ‾) that satisfies the following basic identities:

1.	$X + 0 = X$	2.	$X \cdot 1 = X$	
3.	$X + 1 = 1$	4.	$X \cdot 0 = 0$	
5.	$X + X = X$	6.	$X \cdot X = X$	
7.	$X + \overline{X} = 1$	8.	$X \cdot \overline{X} = 0$	
9.	$\overline{\overline{X}} = X$			
10.	$X + Y = Y + X$	11.	$XY = YX$	Commutative
12.	$(X + Y) + Z = X + (Y + Z)$	13.	$(XY)Z = X(YZ)$	Associative
14.	$X(Y + Z) = XY + XZ$	15.	$X + YZ = (X + Y)(X + Z)$	Distributive
16.	$\overline{X + Y} = \overline{X} \cdot \overline{Y}$	17.	$\overline{X \cdot Y} = \overline{X} + \overline{Y}$	DeMorgan's

Some Properties of Identities & the Algebra

- If the meaning is unambiguous, we leave out the symbol "·"
- The identities above are organized into pairs. These pairs have names as follows:

  1-4 Existence of 0 and 1 — 5-6 Idempotence
  7-8 Existence of complement — 9 Involution
  10-11 Commutative Laws — 12-13 Associative Laws
  14-15 Distributive Laws — 16-17 DeMorgan's Laws

- The dual of an algebraic expression is obtained by interchanging + and · and interchanging 0's and 1's.
- The identities appear in dual pairs. When there is only one identity on a line the identity is self-dual, i. e., the dual expression = the original expression.

Some Properties of Identities & the Algebra (Continued)

- **Unless it happens to be self-dual, the dual of an expression does not equal the expression itself.**
- **Example: $F = (A + \overline{C}) \cdot B + 0$**

  dual $F = (A \cdot \overline{C} + B) \cdot 1 = A \cdot \overline{C} + B$
- **Example: $G = X \cdot Y + (\overline{W + Z})$**

  dual G =
- **Example: $H = A \cdot B + A \cdot C + B \cdot C$**

  dual H =
- **Are any of these functions self-dual?**

Some Properties of Identities & the Algebra (Continued)

- **There can be more that 2 elements in B, i. e., elements other than 1 and 0. What are some common useful Boolean algebras with more than 2 elements?**
  1. **Algebra of Sets**
  2. **Algebra of n-bit binary vectors**
- **If B contains only 1 and 0, then B is called the <u>switching algebra</u> which is the algebra we use most often.**

Boolean Operator Precedence

- **The order of evaluation in a Boolean expression is:**
  1. Parentheses
  2. NOT
  3. AND
  4. OR
- **Consequence: Parentheses appear around OR expressions**
- **Example:** $F = A(B + C)(C + \overline{D})$

Example 1: Boolean Algebraic Proof

- **$A + A \cdot B = A$ (Absorption Theorem)**

Proof Steps	**Justification (identity or theorem)**
$A + A \cdot B$	
$= A \cdot 1 + A \cdot B$	$X = X \cdot 1$
$= A \cdot (1 + B)$	$X \cdot Y + X \cdot Z = X \cdot (Y + Z)$ (Distributive Law)
$= A \cdot 1$	$1 + X = 1$
$= A$	$X \cdot 1 = X$

- **Our primary reason for doing proofs is to learn:**
  - **Careful and efficient use of the identities and theorems of Boolean algebra, and**
  - **How to choose the appropriate identity or theorem to apply to make forward progress, irrespective of the application.**

Example 2: Boolean Algebraic Proofs

- $AB + \overline{A}C + BC = AB + \overline{A}C$ **(Consensus Theorem)**

Proof Steps **Justification (identity or theorem)**

$AB + \overline{A}C + BC$

$= AB + \overline{A}C + 1 \cdot BC$ **?**

$= AB + \overline{A}C + (A + \overline{A}) \cdot BC$ **?**

=

Example 3: Boolean Algebraic Proofs

- $(\overline{X + Y})Z + X\overline{Y} = \overline{Y}(X + Z)$

Proof Steps **Justification (identity or theorem)**

$(\overline{X + Y})Z + X\overline{Y}$

=

Useful Theorems

- $x \cdot y + \overline{x} \cdot y = y \quad (x + y)(\overline{x} + y) = y$ **Minimization**
- $x + x \cdot y = x \quad x \cdot (x + y) = x$ **Absorption**
- $x + \overline{x} \cdot y = x + y \quad x \cdot (\overline{x} + y) = x \cdot y$ **Simplification**
- $x \cdot y + \overline{x} \cdot z + y \cdot z = x \cdot y + \overline{x} \cdot z$ **Consensus**
  $(x + y) \cdot (\overline{x} + z) \cdot (y + z) = (x + y) \cdot (\overline{x} + z)$
- $\overline{x + y} = \overline{x} \cdot \overline{y} \quad \overline{x \cdot y} = \overline{x} + \overline{y}$ **DeMorgan's Laws**

Proof of Simplification

$$x \cdot y + \overline{x} \cdot y = y \quad (x + y)(\overline{x} + y) = y$$

Proof of DeMorgan's Laws

$$\overline{x + y} = \overline{x} \cdot \overline{y} \qquad \overline{x \cdot y} = \overline{x} + \overline{y}$$

Boolean Function Evaluation

$F1 = xy\overline{z}$
$F2 = x + \overline{y}z$
$F3 = \overline{x}\,\overline{y}\,\overline{z} + \overline{x}\,y\,z + x\overline{y}$
$F4 = x\overline{y} + \overline{x}\,z$

x	y	z	F1	F2	F3	F4
0	0	0	0	0		
0	0	1	0	1		
0	1	0	0	0		
0	1	1	0	0		
1	0	0	0	1		
1	0	1	0	1		
1	1	0	1	1		
1	1	1	0	1		

Expression Simplification

- **An application of Boolean algebra**
- **Simplify to contain the smallest number of <u>literals</u> (complemented and uncomplemented variables):**

$$AB + \overline{A}CD + \overline{A}BD + \overline{A}C\overline{D} + ABCD$$

$= AB + ABCD + \overline{A}CD + \overline{A}C\overline{D} + \overline{A}BD$

$= AB + AB(CD) + \overline{A}C(D + \overline{D}) + \overline{A}BD$

$= AB + \overline{A}C + \overline{A}BD = B(A + \overline{A}D) + \overline{A}C$

$= B(A + D) + \overline{A}C$ **5 literals**

Complementing Functions

- **Use DeMorgan's Theorem to complement a function:**
  1. **Interchange AND and OR operators**
  2. **Complement each constant value and literal**
- **Example: Complement F =** $\overline{x}y\overline{z} + x\overline{y}\overline{z}$

  $\overline{F} = (x + \overline{y} + z)(\overline{x} + y + z)$
- **Example: Complement G =** $(\overline{a} + bc)\overline{d} + e$

  $\overline{G} =$

Overview – Canonical Forms

- **What are Canonical Forms?**
- **Minterms and Maxterms**
- **Index Representation of Minterms and Maxterms**
- **Sum-of-Minterm (SOM) Representations**
- **Product-of-Maxterm (POM) Representations**
- **Representation of Complements of Functions**
- **Conversions between Representations**

Canonical Forms

- **It is useful to specify Boolean functions in a form that:**
  - **Allows comparison for equality.**
  - **Has a correspondence to the truth tables**
- **Canonical Forms in common usage:**
  - **Sum of Minterms (SOM)**
  - **Product of Maxterms (POM)**

Minterms

- **<u>Minterms</u> are AND terms with every variable present in either true or complemented form.**
- **Given that each binary variable may appear normal (e.g., x) or complemented (e.g., $\overline{x}$), there are 2^n minterms for n variables.**
- **<u>Example:</u> Two variables (X and Y)produce 2 x 2 = 4 combinations:**

  XY **(both normal)**

  $X\overline{Y}$ **(X normal, Y complemented)**

  $\overline{X}Y$ **(X complemented, Y normal)**

  $\overline{X}\,\overline{Y}$ **(both complemented)**
- **Thus there are <u>four minterms</u> of two variables.**

Maxterms

- **<u>Maxterms</u> are OR terms with every variable in true or complemented form.**
- **Given that each binary variable may appear normal (e.g., x) or complemented (e.g., $\overline{x}$), there are 2^n maxterms for n variables.**
- **<u>Example:</u> Two variables (X and Y) produce 2 x 2 = 4 combinations:**

  $X + Y$ **(both normal)**

  $X + \overline{Y}$ **(x normal, y complemented)**

  $\overline{X} + Y$ **(x complemented, y normal)**

  $\overline{X} + \overline{Y}$ **(both complemented)**

Maxterms and Minterms

- **Examples: Two variable minterms and maxterms.**

Index	Minterm	Maxterm
0	$\bar{x}\,\bar{y}$	$x + y$
1	$\bar{x}\,y$	$x + \bar{y}$
2	$x\,\bar{y}$	$\bar{x} + y$
3	$x\,y$	$\bar{x} + \bar{y}$

- **The index above is important for describing which variables in the terms are true and which are complemented.**

Standard Order

- **Minterms and maxterms are designated with a subscript**
- **The subscript is a number, corresponding to a binary pattern**
- **The bits in the pattern represent the complemented or normal state of each variable listed in a standard order.**
- **All variables will be present in a minterm or maxterm and will be listed in the same order (usually alphabetically)**
- **Example: For variables a, b, c:**
  - **Maxterms: $(a + b + \bar{c})$, $(a + b + c)$**
  - **Terms: $(b + a + c)$, $a\,\bar{c}\,b$, and $(c + b + a)$ are NOT in standard order.**
  - **Minterms: $a\,\bar{b}\,c$, $a\ b\ c$, $\bar{a}\,\bar{b}\ c$**
  - **Terms: $(a + c)$, $\bar{b}\ c$, and $(\bar{a} + b)$ do not contain all variables**

Purpose of the Index

- **The <u>index</u> for the minterm or maxterm, expressed as a binary number, is used to determine whether the variable is shown in the true form or complemented form.**
- **For Minterms:**
  - **"1" means the variable is "Not Complemented" and**
  - **"0" means the variable is "Complemented".**
- **For Maxterms:**
  - **"0" means the variable is "Not Complemented" and**
  - **"1" means the variable is "Complemented".**

Index Example in Three Variables

- **Example: (for three variables)**
- **Assume the variables are called X, Y, and Z.**
- **The standard order is X, then Y, then Z.**
- **The <u>Index 0</u> (base 10) = 000 (base 2) for three variables). All three variables are complemented for <u>minterm 0</u> ($\overline{X}, \overline{Y}, \overline{Z}$) and no variables are complemented for <u>Maxterm 0</u> (X,Y,Z).**
  - **Minterm 0, called m_0 is $\overline{X}\,\overline{Y}\,\overline{Z}$.**
  - **Maxterm 0, called M_0 is (X + Y + Z).**
  - **Minterm 6 ?**
  - **Maxterm 6 ?**

Index Examples – Four Variables

Index i	Binary Pattern	Minterm m_i	Maxterm M_i
0	0000	$\bar{a}\bar{b}\bar{c}\bar{d}$	$a+b+c+d$
1	0001	$\bar{a}\bar{b}\bar{c}d$	?
3	0011	?	$a+b+\bar{c}+\bar{d}$
5	0101	$\bar{a}b\bar{c}d$	$a+\bar{b}+c+\bar{d}$
7	0111	?	$a+\bar{b}+\bar{c}+\bar{d}$
10	1010	$a\bar{b}c\bar{d}$	$\bar{a}+b+\bar{c}+d$
13	1101	$ab\bar{c}d$	?
15	1111	$abcd$	$\bar{a}+\bar{b}+\bar{c}+\bar{d}$

Minterm and Maxterm Relationship

- **Review: DeMorgan's Theorem**
  $\overline{x \cdot y} = \bar{x} + \bar{y}$ **and** $\overline{x + y} = \bar{x} \cdot \bar{y}$
- **Two-variable example:**
  $M_2 = \bar{x} + y$ **and** $m_2 = x \cdot \bar{y}$
  Thus M_2 is the complement of m_2 and vice-versa.
- **Since DeMorgan's Theorem holds for *n* variables, the above holds for terms of *n* variables**
- **giving:**

  $$M_i = \overline{m_i} \text{ and } m_i = \overline{M_i}$$

  Thus M_i is the complement of m_i.

Function Tables for Both

- **Minterms of 2 variables**

x y	m_0	m_1	m_2	m_3
0 0	1	0	0	0
0 1	0	1	0	0
1 0	0	0	1	0
1 1	0	0	0	1

Maxterms of 2 variables

x y	M_0	M_1	M_2	M_3
0 0	0	1	1	1
0 1	1	0	1	1
1 0	1	1	0	1
1 1	1	1	1	0

- **Each column in the maxterm function table is the complement of the column in the minterm function table since M_i is the complement of m_i.**

Observations

- **In the function tables:**
  - **Each <u>min</u>term has one and only one 1 present in the 2^n terms (a <u>minimum</u> of 1s). All other entries are 0.**
  - **Each <u>max</u>term has one and only one 0 present in the 2^n terms All other entries are 1 (a <u>max</u>imum of 1s).**
- **We can implement any function by "ORing" the minterms corresponding to "1" entries in the function table. These are called the minterms of the function.**
- **We can implement any function by "ANDing" the maxterms corresponding to "0" entries in the function table. These are called the maxterms of the function.**
- **This gives us two <u>canonical forms</u>:**
  - **<u>Sum of Minterms (SOM)</u>**
  - **<u>Product of Maxterms (POM)</u>**

for stating any Boolean function.

Minterm Function Example

- **Example: Find $F_1 = m_1 + m_4 + m_7$**
- **$F1 = \overline{x}\ \overline{y}\ z + x\ \overline{y}\ \overline{z} + x\ y\ z$**

x y z	index	m_1	+	m_4	+	m_7	$= F_1$
0 0 0	0	0	+	0	+	0	= 0
0 0 1	1	1	+	0	+	0	= 1
0 1 0	2	0	+	0	+	0	= 0
0 1 1	3	0	+	0	+	0	= 0
1 0 0	4	0	+	1	+	0	= 1
1 0 1	5	0	+	0	+	0	= 0
1 1 0	6	0	+	0	+	0	= 0
1 1 1	7	0	+	0	+	1	= 1

Minterm Function Example

- **$F(A, B, C, D, E) = m_2 + m_9 + m_{17} + m_{23}$**
- **F(A, B, C, D, E) =**

Maxterm Function Example

- **Example: Implement F1 in maxterms:**

$F_1 = M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$

$F_1 = (x + y + z) \cdot (x + \bar{y} + z) \cdot (x + \bar{y} + \bar{z}) \cdot (\bar{x} + y + \bar{z}) \cdot (\bar{x} + \bar{y} + z)$

x y z	i	$M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$ = F1
0 0 0	0	0 · 1 · 1 · 1 · 1 = 0
0 0 1	1	1 · 1 · 1 · 1 · 1 = 1
0 1 0	2	1 · 0 · 1 · 1 · 1 = 0
0 1 1	3	1 · 1 · 0 · 1 · 1 = 0
1 0 0	4	1 · 1 · 1 · 1 · 1 = 1
1 0 1	5	1 · 1 · 1 · 0 · 1 = 0
1 1 0	6	1 · 1 · 1 · 1 · 0 = 0
1 1 1	7	1 · 1 · 1 · 1 · 1 = 1

Maxterm Function Example

- $F(A,B,C,D) = M_3 \cdot M_8 \cdot M_{11} \cdot M_{14}$
- F(A, B,C,D) =

Canonical Sum of Minterms

- Any Boolean function can be expressed as a Sum of Minterms.
  - For the function table, the minterms used are the terms corresponding to the 1's
  - For expressions, expand all terms first to explicitly list all minterms. Do this by "ANDing" any term missing a variable v with a term $(v+\bar{v})$.
- Example: Implement $f = x + \bar{x}\,\bar{y}$ as a sum of minterms.

  First expand terms: $f = x(y+\bar{y}) + \bar{x}\,\bar{y}$

  Then distribute terms: $f = xy + x\bar{y} + \bar{x}\,\bar{y}$

  Express as sum of minterms: $f = m_3 + m_2 + m_0$

Another SOM Example

- Example: $F = A + \bar{B}\,C$
- There are three variables, A, B, and C which we take to be the standard order.
- Expanding the terms with missing variables:

- Collect terms (removing all but one of duplicate terms):
- Express as SOM:

Shorthand SOM Form

- From the previous example, we started with:

  $F = A + \overline{B}\,C$

- We ended up with:

$F = m_1+m_4+m_5+m_6+m_7$

- This can be denoted in the formal shorthand:

  $F(A,B,C) = \Sigma_m(1,4,5,6,7)$

- Note that we explicitly show the standard variables in order and drop the "m" designators.

Canonical Product of Maxterms

- Any Boolean Function can be expressed as a Product of Maxterms (POM).
  - For the function table, the maxterms used are the terms corresponding to the 0's.
  - For an expression, expand all terms first to explicitly list all maxterms. Do this by first applying the second distributive law, "ORing" terms missing variable v with a term equal to $v \cdot \overline{v}$ and then applying the distributive law again.
- Example: Convert to product of maxterms:

  $f(x, y, z) = x + \overline{x}\,\overline{y}$

  Apply the distributive law:

  $x + \overline{x}\,\overline{y} = (x + \overline{x})(x + \overline{y}) = 1 \cdot (x + \overline{y}) = x + \overline{y}$

  Add missing variable z:

  $x + \overline{y} + z \cdot \overline{z} = (x + \overline{y} + z)\,(x + \overline{y} + \overline{z})$

  Express as POM: $f = M_2 \cdot M_3$

Another POM Example

- **Convert to Product of Maxterms:**

$$f(A,B,C) = A\overline{C} + BC + \overline{A}\,\overline{B}$$

- **Use** $x + yz = (x+y)\cdot(x+z)$ **with** $x = (A\overline{C} + BC)$, $y = \overline{A}$, **and** $z = \overline{B}$ **to get:**

$$f = (A\overline{C} + BC + \overline{A})(A\overline{C} + BC + \overline{B})$$

- **Then use** $x + \overline{x}y = x + y$ **to get:**

$$f = (\overline{C} + BC + \overline{A})(A\overline{C} + C + \overline{B})$$

and a second time to get:

$$f = (\overline{C} + B + \overline{A})(A + C + \overline{B})$$

- **Rearrange to standard order,**

$$f = (\overline{A} + B + \overline{C})(A + \overline{B} + C) \text{ to give } f = M_5 \cdot M_2$$

Function Complements

- **The complement of a function expressed as a sum of minterms is constructed by selecting the minterms missing in the sum-of-minterms canonical forms.**
- **Alternatively, the complement of a function expressed by a Sum of Minterms form is simply the Product of Maxterms with the same indices.**
- **Example: Given** $F(x,y,z) = \Sigma_m(1,3,5,7)$

$$\overline{F}(x,y,z) = \Sigma_m(0,2,4,6)$$

$$\overline{F}(x,y,z) = \Pi_M(1,3,5,7)$$

Conversion Between Forms

- To convert between sum-of-minterms and product-of-maxterms form (or vice-versa) we follow these steps:
  - Find the function complement by swapping terms in the list with terms not in the list.
  - Change from products to sums, or vice versa.
- Example:Given F as before: $F(x, y, z) = \Sigma_m(1,3,5,7)$
- Form the Complement: $\overline{F}(x, y, z) = \Sigma_m(0,2,4,6)$
- Then use the other form with the same indices – this forms the complement again, giving the other form of the original function: $F(x, y, z) = \Pi_M(0,2,4,6)$

Standard Forms

- <u>Standard Sum-of-Products (SOP) form:</u> equations are written as an OR of AND terms
- <u>Standard Product-of-Sums (POS) form:</u> equations are written as an AND of OR terms
- Examples:
  - SOP: $A\,B\,C + \overline{A}\,\overline{B}\,C + B$
  - POS: $(A + B) \cdot (A + \overline{B} + \overline{C}) \cdot C$
- These "mixed" forms are <u>neither SOP nor POS</u>
  - $(A\,B + C)\,(A + C)$
  - $A\,B\,\overline{C} + A\,C\,(A + B)$

Standard Sum-of-Products (SOP)

- **A sum of minterms form for *n* variables can be written down directly from a truth table.**
  - **Implementation of this form is a two-level network of gates such that:**
  - **The first level consists of *n*-input AND gates, and**
  - **The second level is a single OR gate (with fewer than 2^n inputs).**
- **This form often can be simplified so that the corresponding circuit is simpler.**

Standard Sum-of-Products (SOP)

- **A Simplification Example:**
- $F(A, B, C) = \Sigma m(1,4,5,6,7)$
- **Writing the minterm expression:**

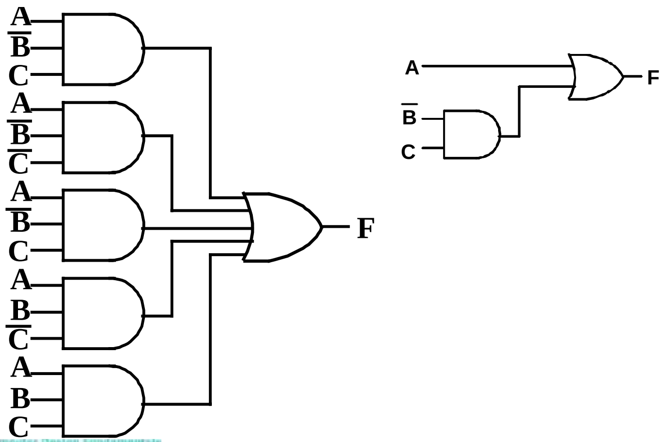
  $F = \overline{A}\,\overline{B}\,C + A\,\overline{B}\,\overline{C} + A\,\overline{B}\,C + AB\overline{C} + ABC$
- **Simplifying:**

  F =

- **Simplified F contains 3 literals compared to 15 in minterm F**

AND/OR Two-level Implementation of SOP Expression

- **The two implementations for F are shown below – it is quite apparent which is simpler!**



SOP and POS Observations

- **The previous examples show that:**
  - **Canonical Forms (Sum-of-minterms, Product-of-Maxterms), or other standard forms (SOP, POS) differ in complexity**
  - **Boolean algebra can be used to manipulate equations into simpler forms.**
  - **Simpler equations lead to simpler two-level implementations**
- **Questions:**
  - **How can we attain a “simplest” expression?**
  - **Is there only one minimum cost circuit?**
  - **The next part will deal with these issues.**

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