ELECTROMAGNETICS-I FINAL EXAM

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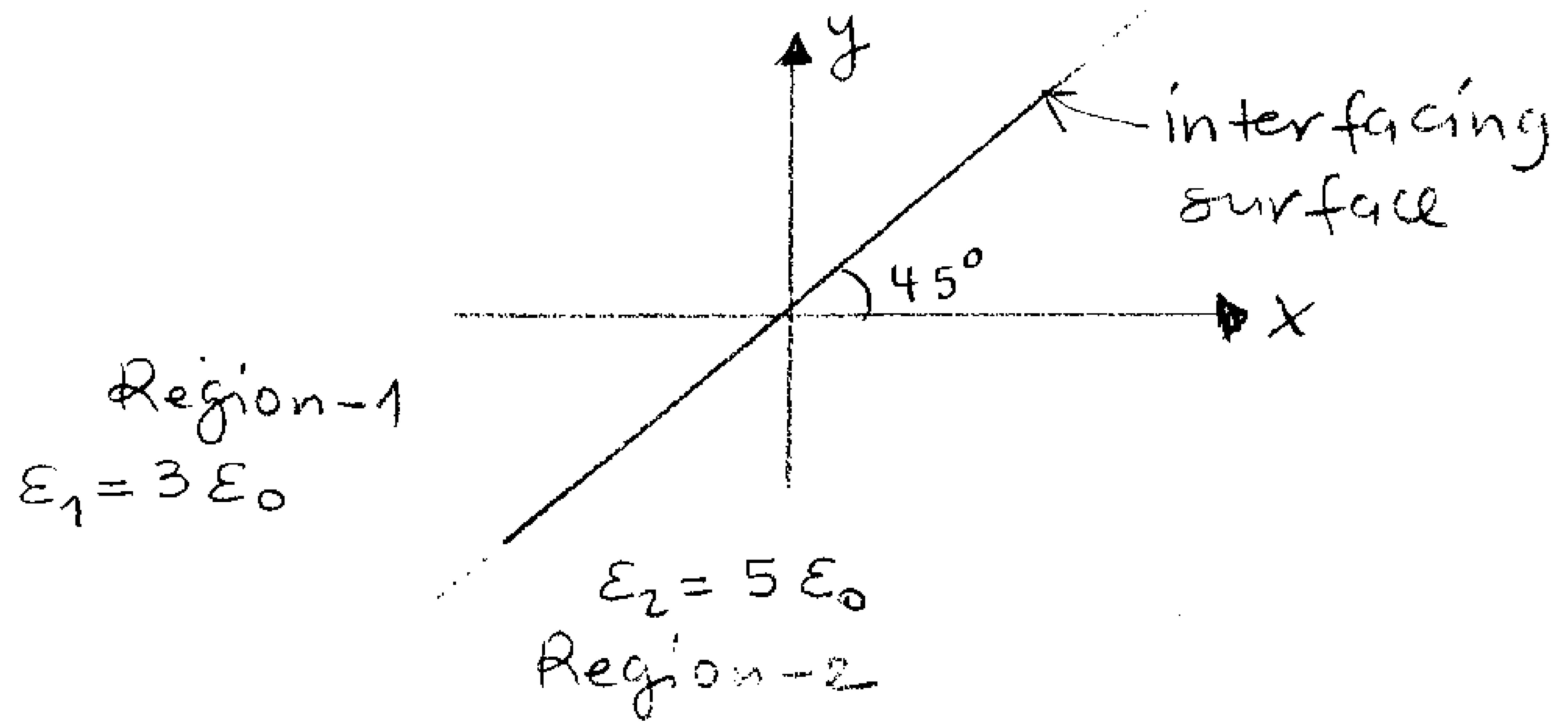
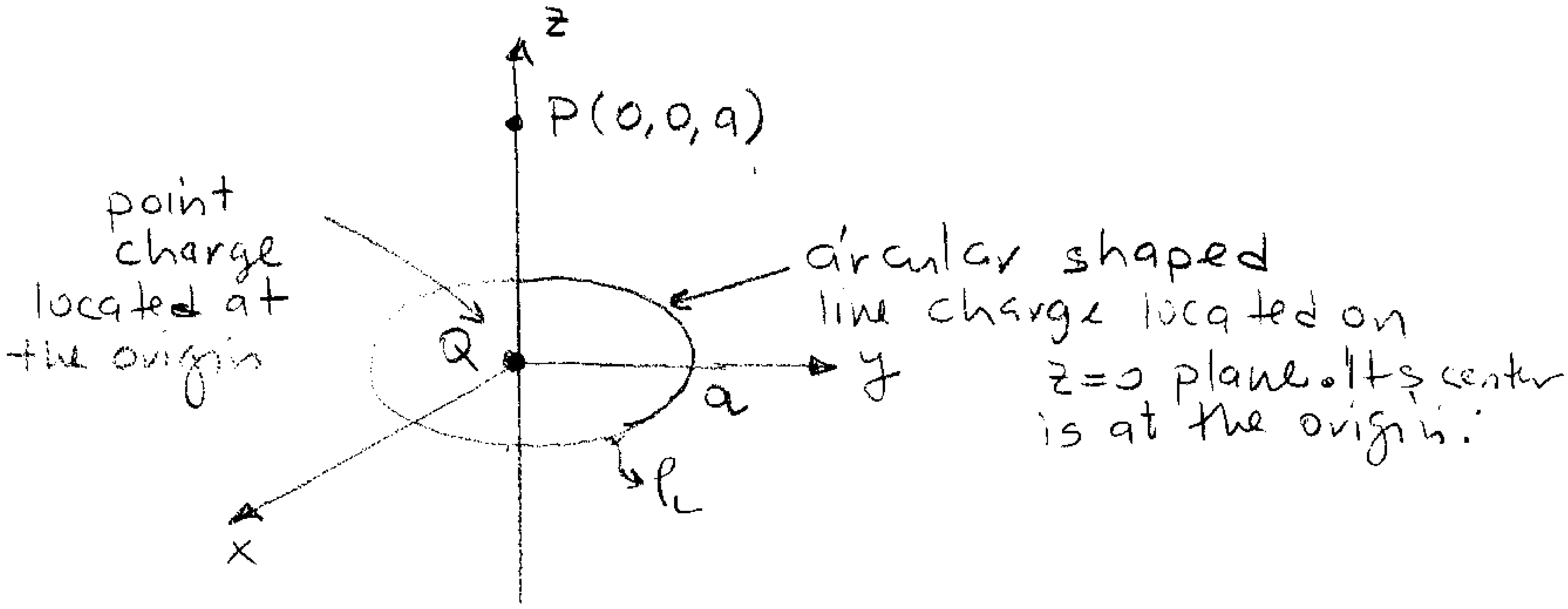
August 17, 2012

#1) Consider the following figure. If electric field intensity vector in region 1 is given as $\vec{E}_1 = 4\vec{a}_x + 6\vec{a}_y + 3\vec{a}_z$ V/m, calculate $\vec{E}_2$.



#2) Consider the following figure. If the absolute potential at point P(0,0,a) is zero, express ρ_L in terms of Q.



#3) The finite sheet $0 \le x \le 1,\ 0 \le y \le 1$ on the $z = 0$ plane has a charge density $\rho_S = xy(x^2 + y^2 + 25)^{3/2}\ nC/m^2$. Find:

- **a)** The electric field intensity vector at (0, 0, 5)
- **b)** The force experienced by -1 *mC* charge located at (0, 0, 5).

Good luck...☺

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SOLUTION MANUAL

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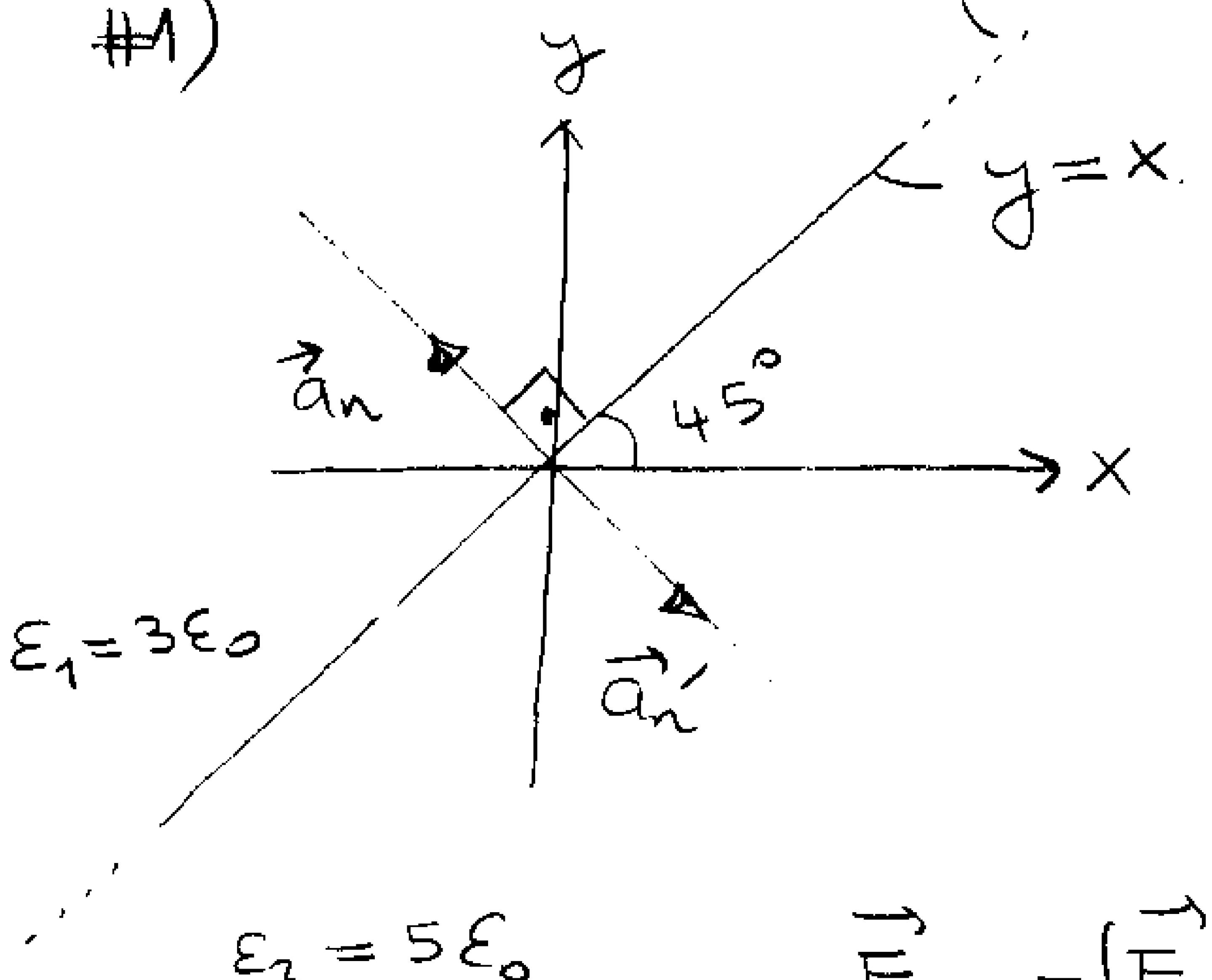
#1)

interfacing surface



$y-x=0$

$$\vec{a}_n = \pm \frac{\nabla f}{|\nabla f|} = \pm \frac{\vec{a}_y - \vec{a}_x}{\sqrt{2}}$$

$$\vec{a}_n = \frac{1}{\sqrt{2}}(\vec{a}_y - \vec{a}_x)$$ the other one can also be selected.

$$\vec{E}_{1n} = (\vec{E}_1 \cdot \vec{a}_n)\,\vec{a}_n = \frac{(6-4)}{2}(-\vec{a}_x + \vec{a}_y)$$

$$\vec{E}_1 = 4\vec{a}_x + 6\vec{a}_y + 3\vec{a}_z$$

$$\vec{E}_{1n} = -\vec{a}_x + \vec{a}_y$$

$$\vec{E}_{1t} = \vec{E}_1 - \vec{E}_{1n} = (4,6,3)-(-1,1,0)$$

$$\vec{E}_{1t} = 5\vec{a}_x + 5\vec{a}_y + 3\vec{a}_z$$

$$\vec{E}_{2t} = \vec{E}_{1t} = 5\vec{a}_x + 5\vec{a}_y + 3\vec{a}_z$$

$$\vec{D}_{1n} = \vec{D}_{2n} \rightarrow 3\cancel{\varepsilon_0}(-\vec{a}_x + \vec{a}_y) = 5\cancel{\varepsilon_0}\vec{E}_{2n} \Rightarrow \vec{E}_{2n} = \frac{3}{5}(-\vec{a}_x + \vec{a}_y) \text{ V/m}$$

$$\vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n} = (5-0{,}6)\vec{a}_x + (+5+0{,}6)\vec{a}_y + 3\vec{a}_z$$

$$\boxed{\vec{E}_2 = 4.4\,\vec{a}_x + 5.6\,\vec{a}_y + 3\vec{a}_z \text{ V/m}}$$

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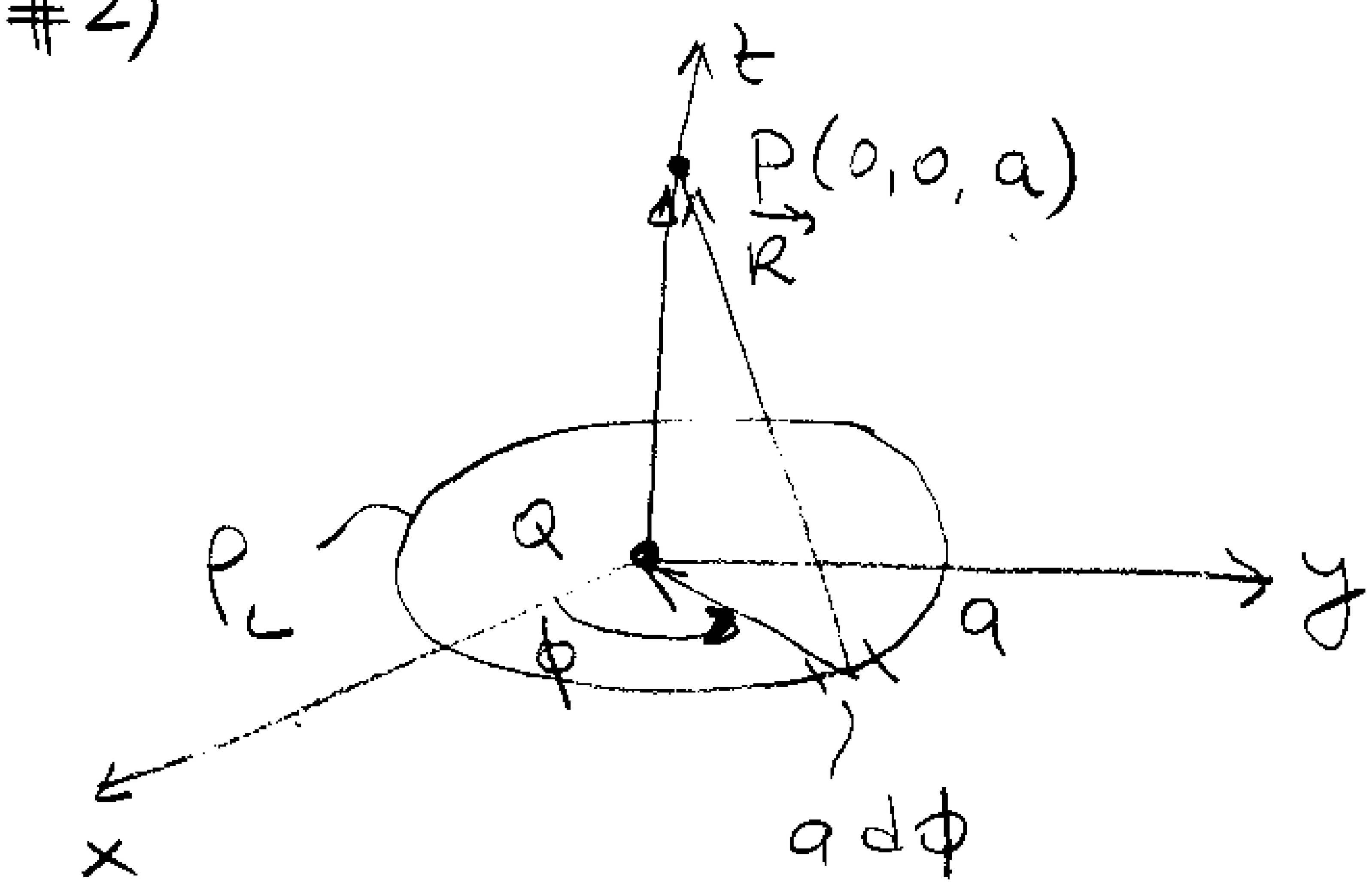
#2)



$$V_P = V_Q + V_L$$

$$dV_L = \frac{\rho_L\, a\, d\phi}{4\pi\varepsilon_0 \left| -a\vec{a}_\rho + a\vec{a}_z \right|}$$

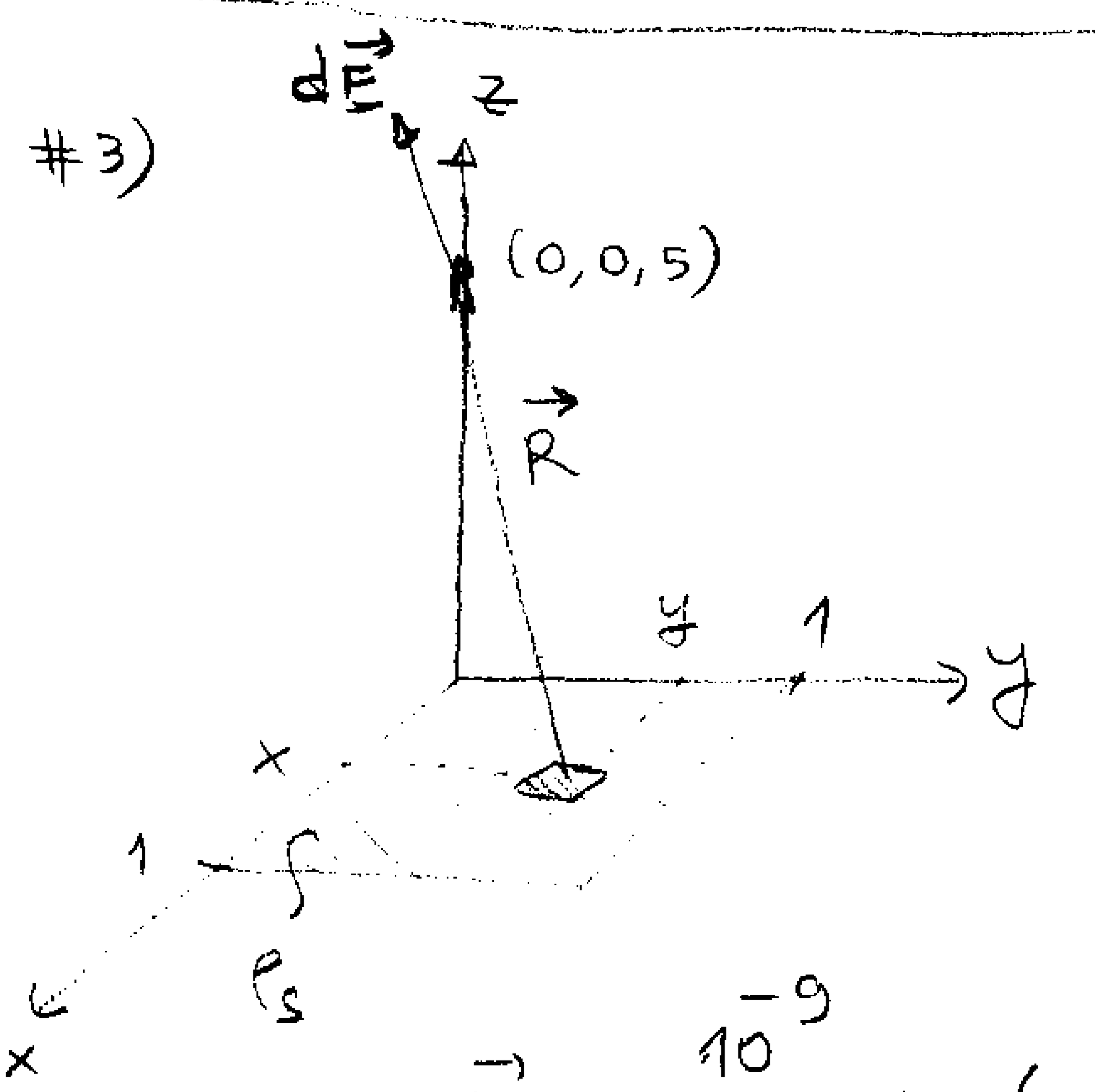
$$dV_L = \frac{\rho_L\, a}{4\pi\varepsilon_0} \frac{d\phi}{\sqrt{2a^2}}$$

$$V_L = \frac{\rho_L\, a}{4\pi\varepsilon_0 \sqrt{2}\, a}\ \phi\Big|_0^{2\pi} = \frac{\rho_L \cancel{a}\ \cancel{2\pi}}{\cancel{4\pi}_{2}\,\varepsilon_0 \sqrt{2}\, \cancel{a}} = \frac{\rho_L}{2\sqrt{2}\,\varepsilon_0}$$

$$V_Q = \frac{Q}{4\pi\varepsilon_0\, a} \ ; \qquad \frac{\rho_L}{2\sqrt{2}\,\cancel{\varepsilon_0}} + \frac{Q}{4\pi \cancel{\varepsilon_0}\, a} = 0$$

$$\boxed{\rho_L = \frac{-Q\,\cancel{2}\sqrt{2}}{\cancel{4}_{2}\pi a} = -\frac{Q}{\sqrt{2}\,\pi a}\ \mathrm{C/m}}$$

#3)



$$\vec{R} = (0-x)\vec{a}_x + (0-y)\vec{a}_y + (5-0)\vec{a}_z$$

$$R = (x^2+y^2+25)^{1/2}$$

$$d\vec{E} = \frac{\rho_s\, dx\, dy\, \vec{R}}{4\pi\varepsilon_0\, R^3}$$

$$d\vec{E} = \frac{dx\, dy}{4\pi\varepsilon_0}\ \frac{x\, y\, \cancel{(x^2+y^2+25)^{3/2}}}{\cancel{(x^2+y^2+25)^{3/2}}}\,(-x\vec{a}_x - y\vec{a}_y + 5\vec{a}_z)$$

$$d\vec{E} = \frac{10^{-9}}{4\pi\varepsilon_0}\, x y\,(-x\vec{a}_x - y\vec{a}_y + 5\vec{a}_z)\, dx\, dy$$

$$\vec{E} = \frac{10^{-9}}{4\pi\varepsilon_0}\left\{\left(-\int_0^1 x^2 dx\right)\left(\int_0^1 y\, dy\right)\vec{a}_x - \left[\int_0^1 x\, dx \int_0^1 y^2 dy\right]\vec{a}_y + 5\,\vec{a}_z\, \frac{x^2}{2}\Big|_0^1 \frac{y^2}{2}\Big|_0^1\right\}$$

$$= \frac{\cancel{10^{-9}}}{4\pi \frac{\cancel{10^{-9}}}{\cancel{36\pi}_{9}}}\left\{-\frac{x^3}{3}\Big|_0^1 , \frac{y^2}{2}\Big|_0^1 \vec{a}_x - \frac{x^2}{2}\Big|_0^1 , \frac{y^3}{3}\Big|_0^1 \vec{a}_y + 5\ 1{,}1\ \vec{a}_z\right\}$$

$$= 9\left\{\left(-\frac{1}{3}\right)\left(\frac{1}{2}\right)\vec{a}_x - \frac{1}{2}\cdot\frac{1}{3}\,\vec{a}_y + \frac{5}{4}\vec{a}_z\right\}$$

$$\vec{E} = -\frac{9}{6}\vec{a}_x - \frac{9}{6}\vec{a}_y + 45\,\vec{a}_z = -1{,}5\,\vec{a}_x - 1{,}5\,\vec{a}_y + 11{,}25\,\vec{a}_z \ \ \mathrm{V/m.}$$

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b) $\vec{F} = -10^{-3}\,\vec{E}$ N.

$$\boxed{\vec{F} = 1{,}5\,\vec{a}_x + 1{,}5\,\vec{a}_y - 11{,}25\,\vec{a}_z\ \ \mathrm{mN}}$$

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