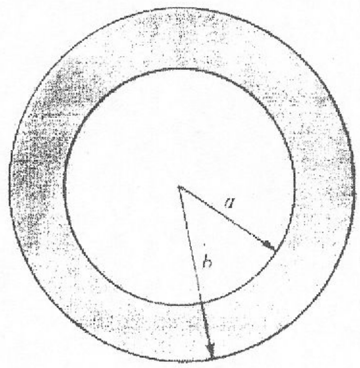
ELECTROMAGNETICS I FINAL EXAM

Dr. Salih FADIL **August 24, 2010**

#1) Consider the figure as a spherical dielectric shell so that $\varepsilon = \varepsilon_0 \varepsilon_r$ for $a < r < b$ and $\varepsilon = \varepsilon_0$ for $0 < r < a$. If a charge Q is placed at the center of the shell, find

a) $\vec{P}$ for $a < r < b$
b) ρ_{Pv} for $a < r < b$
c) ρ_{Ps} at $r = a$ and $r = b$.



#2) Given that $\vec{E} = (3x^2 + y)\vec{a}_x + x\vec{a}_y \ kV/m$, find the work done in moving a $-2\ \mu C$ charge from $(0, 5, 0)$ to $(2, -1, 0)$ by taking the path

a) $(0, 5, 0) \rightarrow (2, 5, 0) \rightarrow (2, -1, 0)$
b) $y = 5 - 3x$

#3) If $\vec{D} = (2y^2 + z)\vec{a}_x + 4xy\vec{a}_y + x\vec{a}_z \ C/m^2$, find:

a) the volume charge density at $(-1, 0, 3)$
b) the flux through the cube defined by $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1$
c) the total charge enclosed by the cube.

GOOD LUCK.....☺

ELECTROMAGNETICS-I FINAL EXAM
SOLUTION MANUAL

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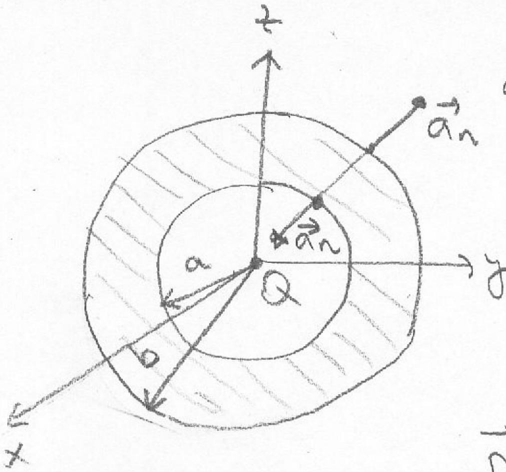
#1)



a) $\vec{E}$ due to point charge located at the center of the shell is

$$\vec{E} = \frac{Q\,\vec{r}}{4\pi\varepsilon|\vec{r}|^3} \qquad \varepsilon = \varepsilon_0 \cdot \varepsilon_r \qquad a<r<b$$

$$\vec{P} = \chi_e\,\varepsilon_0\,\vec{E} = (\varepsilon_r - 1)\,\varepsilon_0\,\vec{E}$$

$$\vec{P} = (\varepsilon_r - 1)\cancel{\varepsilon_0}\frac{Q\,\vec{r}}{4\pi\cancel{\varepsilon_0}\varepsilon_r\,r^3} = \frac{Q}{4\pi r^3}\left(1-\frac{1}{\varepsilon_r}\right)\vec{r} \quad C/m^2$$

$$\boxed{\vec{P} = \frac{Q}{4\pi r^2}\left(1-\frac{1}{\varepsilon_r}\right)\vec{a}_r \;,\quad a<r<b}$$

b) $$\rho_{pv} = -\nabla P = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 P_r) = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(\cancel{r^2}\frac{Q}{4\pi\cancel{r^2}}\left(1-\frac{1}{\varepsilon_r}\right)\right)$$

$$\rho_{pv} = 0 \quad C/m^3$$

c) $$\rho_{ps} = \vec{P}\cdot\vec{a}_n \quad C/m^2$$

$$\rho_{ps}\Big|_{r=a} = \frac{Q}{4\pi r^2}\left(1-\frac{1}{\varepsilon_r}\right)(+\vec{a}_r)\cdot(-\vec{a}_r)\Big|_{r=a} = \frac{-Q}{4\pi a^2}\left(1-\frac{1}{\varepsilon_r}\right) \; C/m^2$$

$$\rho_{ps}\Big|_{r=b} = \frac{Q}{4\pi r^2}\left(1-\frac{1}{\varepsilon_r}\right)(\vec{a}_r)\cdot(\vec{a}_r)\Big|_{r=b} = \frac{Q}{4\pi b^2}\left(1-\frac{1}{\varepsilon_r}\right) \; C/m^2$$

So, the total surface charge

$$Q_{ps} = \cancel{4\pi a^2}\left(\frac{-Q}{\cancel{4\pi a^2}}\right)\left(1-\frac{1}{\varepsilon_r}\right) + \cancel{4\pi b^2}\,\frac{Q}{\cancel{4\pi b^2}}\left(1-\frac{1}{\varepsilon_r}\right) = 0$$

So, $Q_{ps} + Q_{pv} = 0$ is satisfied (Neutral object)

#2) a) $W = -Q\int \vec{E}\cdot\vec{dl}$

$$\underset{A}{(0,5,0)} \longrightarrow \underset{B}{(2,5,0)} \longrightarrow \underset{C}{(2,-1,0)}$$

A → B: $\vec{dl} = dx\,\vec{a_x}$, $y=5$, $z=0$

B → C: $\vec{dl} = dy\,\vec{a_y}$, $x=2$, $z=0$

$$10^3\int_A^B \vec{E}\cdot\vec{dl} = 10^3\int_A^B (3x^2+y)\,dx = 10^3\int_{x=0}^{2}(3x^2+5)\,dx = \left[\cancel{3}\frac{x^3}{\cancel{3}}\Big|_0^2 + 5x\Big|_0^2\right]10^3$$

$$= (8+10)\,10^3 = 18\,10^3\ \text{V}$$

$$10^3\int_B^C \vec{E}\cdot\vec{dl} = 10^3\int_B^C x\,dy = 2\,10^3\,y\Big|_5^{-1} = 2\,10^3(-1-5) = -12\,10^3\ \text{V}$$

$$W = -Q\int_A^C \vec{E}\cdot\vec{dl} = 2\,10^{-6}\,(\underbrace{18-12}_{6})\,10^3 = 12\,10^{-3}\ \text{Joule}$$

An external agent does the work

b) $$W = 2\,10^{-6}\,10^3\int_A^C [(3x^2+y)\,dx + x\,dy]$$

$y = 5-3x$

$dy = -3\,dx$

$$= 2\,10^{-3}\int_{x=0}^{2}[(3x^2+5-3x+x(-3)]dx = 2\,10^{-3}\int_0^2 (3x^2-6x+5)\,dx$$

$$= 2\,10^{-3}\left\{\cancel{3}\frac{x^3}{\cancel{3}}\Big|_0^2 - \frac{x^2}{\cancel{2}}\cancel{6}^{3}\Big|_0^2 + 5x\Big|_0^2\right\} = 2\,10^{-3}(8-12+10)$$

$$= 12\,10^{-3}\ \text{J}$$

$$= 12\ \text{mJ}$$ (the same!)

3)

a) $$\rho_v = \nabla \cdot \vec{D} = \frac{\partial}{\partial x}(2y^2+z) + \frac{\partial}{\partial y}(4xy) + \frac{\partial}{\partial z}(x)$$

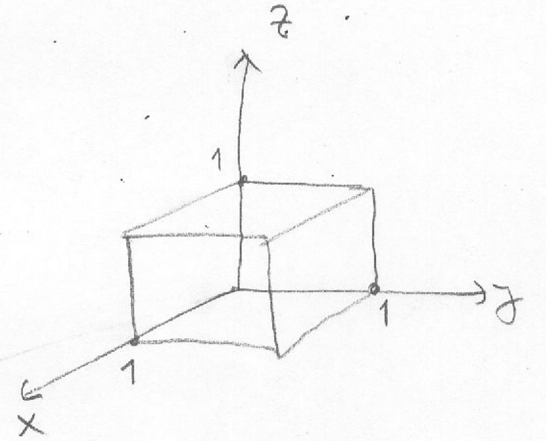
$$= 4x \Big|_{(-1,0,3)} = -4 \quad C/m^3$$

b) $$\Psi = \left\{ \int_{top} + \int_{bottom} + \int_{front} + \int_{rear} + \int_{left} + \int_{righ} \right\} \vec{D} \cdot d\vec{S}$$

$$\int_{top} \vec{D} \cdot d\vec{S} = \int_{top} \vec{D} \cdot (dx\, dy\, \vec{a}_z) = \int_{x=0}^{1}\int_{y=0}^{1} x \, dx\, dy = \frac{x^2}{2}\Big|_0^1 y\Big|_0^1 = \frac{1}{2} C$$

$$\int_{bottom} \vec{D} \cdot d\vec{S} = \int_{bottom} \vec{D} \cdot (dx\, dy\, (-\vec{a}_z)) = -\int_{x=0}^{1}\int_{y=0}^{1} x \, dx\, dy = -1/2\, C$$

$$\int_{front} \vec{D} \cdot d\vec{S} = \int_{front} \vec{D} \cdot (dy\, dz)\, \vec{a}_x$$



$$= \int_{front} (2y^2+z)\, dy\, dz$$

$$2\,\frac{y^3}{3}\Big|_0^1 z\Big|_0^1 + \frac{z^2}{2}\Big|_0^1 y\Big|_0^1$$

$$= \underset{(2)}{\frac{2}{3}} + \underset{(3)}{\frac{1}{2}} = \frac{7}{6}\, C$$

$$\int_{rear} \vec{D} \cdot d\vec{S} = -\int_{y=0}^{1}\int_{z=0}^{1} (2y^2+z)\, dy\, dz = -7/6$$

$$\int_{left} \vec{D} \cdot d\vec{S} = \int_{left} \vec{D} \cdot (dx\, dz\, (-\vec{a}_y)) = -\int_{left} 4xy\, dx\, dz \Big|_{y=0} = 0.$$

$$\int_{\substack{right \\ y=1}} \vec{D}\cdot d\vec{S} = \int_{\substack{right \\ y=1}} \vec{D}\,(dx\,dz\,\vec{a}_y) = +\int_{\substack{right \\ y=1}} 4xy\,dx\,dz = 4\int_0^1 x\,dx \int_0^1 dy$$

$$= 4\,\frac{x^2}{2}\Big|_0^1\, y\Big|_0^1$$

$$= 2\,C$$

$$\Psi = \oint_S \vec{D}\cdot d\vec{S} = \cancel{-\tfrac{1}{2}} + \cancel{\tfrac{1}{2}} + \cancel{\tfrac{7}{6}} - \cancel{\tfrac{7}{6}} + 0 + 2 = 2\,C$$

c) By applying Gauss' law

$$\Psi = \oint_S \vec{D}\cdot d\vec{S} = \int_v \rho_v\,dv = Q_{enc} = 2\,C.$$

or

$$Q_{enc} = \int_v 4x\,dx\,dy\,dz = 4\,\frac{x^2}{2}\Big|_0^1\, y\Big|_0^1\, z\Big|_0^1 = \frac{4}{2} = 2\,C$$