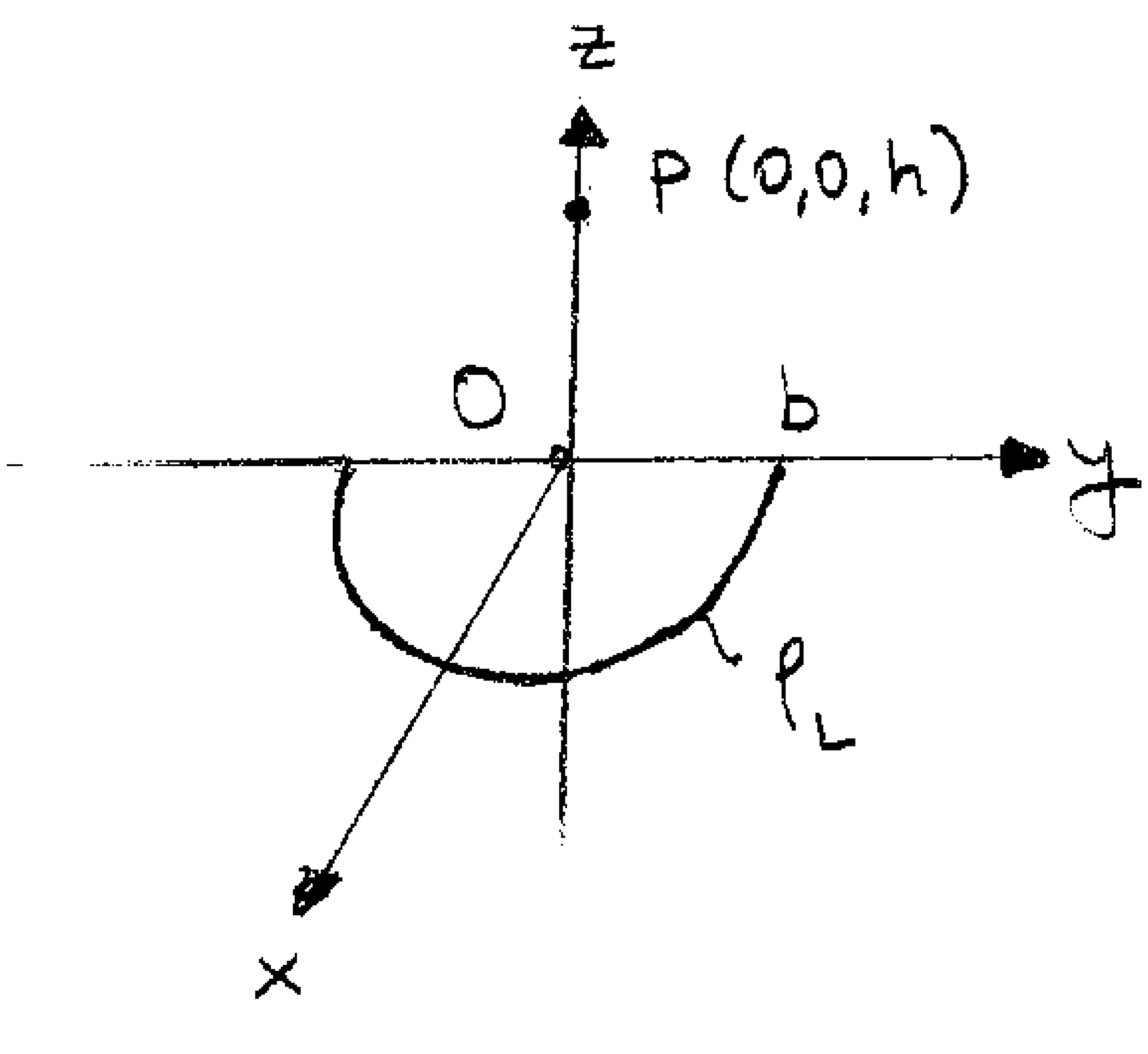
ELECTROMAGNETICS – I FINAL EXAM

Dr. Salih FADIL **June 07, 2012**

#1) The cylindrical surface $\rho = 3$ separates two homogenous dielectric regions; region-1 , $(\rho \le 3)$) and region-2 $(\rho \ge 3)$ with $\varepsilon_{r1} = 2.5$ and $\varepsilon_{r2} = 1.0$ respectively. Given that $\vec{E}_1 = 2\vec{a}_\rho + 5\vec{a}_\phi - 4\vec{a}_z \; kV/m$, find $\vec{D}_2$.

#2) A potential field is given by $V = 3x^2y - yz$. Find the electric energy stored in the cube whose center in the origin and length of side 2 m.

#3) A thin line charge of density ρ_L is in the form of semicircle of radius *b* lying on xy-plane with its center located at the origin as shown in the figure. If $\rho_L = \rho_0 \sin\phi$, find $\vec{E}$ at points P and O.



Good Luck ☺

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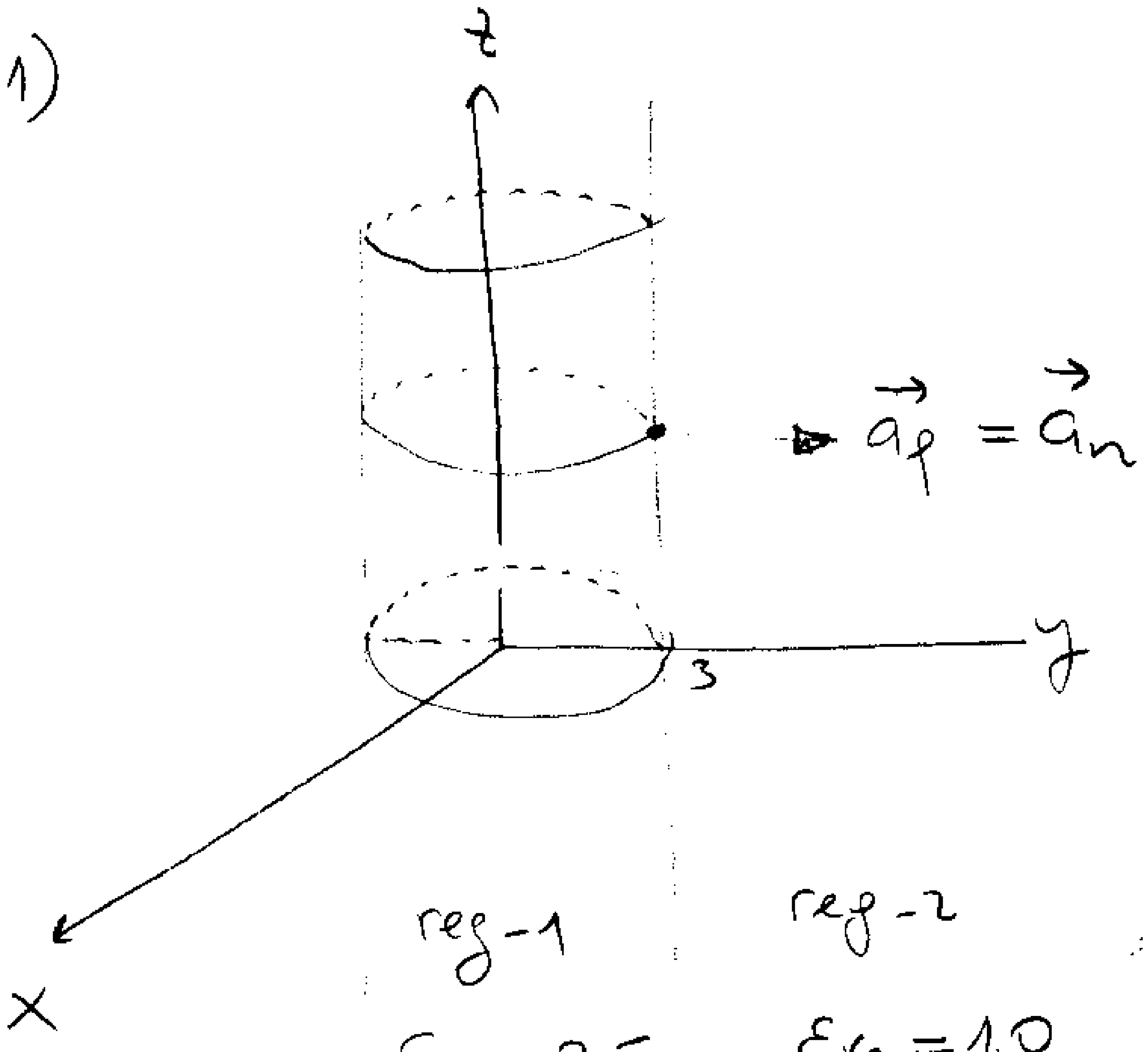
#1)



$\varepsilon_{r1} = 2{,}5$ $\quad$ $\varepsilon_{r2} = 1{,}0$

$$\vec{E}_1 = 2\vec{a}_\rho + 5\vec{a}_\phi - 4\vec{a}_z \ \text{kV/m}$$

$$\vec{E}_{1n} = 2\vec{a}_\rho \ \text{kV/m}$$

$$\vec{E}_{1t} = 5\vec{a}_\phi - 4\vec{a}_z \ \text{kV/m}$$

$\vec{E}_1$

$$\vec{E}_{1t} = \vec{E}_{2t} = 5\vec{a}_\phi - 4\vec{a}_z \ \text{kV/m}$$

$$\vec{D}_{1n} = \vec{D}_{2n} \Rightarrow \varepsilon_{r1} \vec{E}_{1n} = \varepsilon_{r2} \vec{E}_{2n} \qquad \vec{E}_{2n} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}} \cdot E_{1n}$$

$$\vec{E}_{2n} = \frac{2{,}5}{1.} \, 2\vec{a}_\rho \ \text{kV/m}$$

$$\vec{E}_{2n} = 5\vec{a}_\rho \ \text{kV/m}$$

$$\vec{D}_2 = \varepsilon_0 \cdot 1 \left[5\vec{a}_\rho + 5\vec{a}_\phi - 4\vec{a}_z\right] 10^3 \ \text{C/m}^2$$

$$= \frac{10^{-9}\, 10^3}{36\pi} \left[5\vec{a}_\rho + 5\vec{a}_\phi - 4\vec{a}_z\right] \ \text{C/m}^2$$

$$\vec{D}_2 = \frac{5}{36\pi}\vec{a}_\rho + \frac{5}{36\pi}\vec{a}_\phi - \frac{4}{36\pi}\vec{a}_z \ \mu\text{C/m}^2$$

$$= 0{,}0442\,\vec{a}_\rho + 0{,}0442\,\vec{a}_\phi - 0{,}03536\,\vec{a}_z \ \mu\text{C/m}^2$$

#2)

$$V = 3x^2y - yz$$

$$\vec{E} = -\nabla V = -\left[6xy\,\vec{a_x} + (3x^2 - z)\,\vec{a_y} - y\,\vec{a_z}\right] \text{ V/m}$$

$$|\vec{E}|^2 = 36x^2y^2 + (3x^2 - z)^2 + y^2 \quad ; \quad W_E = \frac{1}{2}\varepsilon_0 \int_v |\vec{E}|^2\, dv$$

$$= 36x^2y^2 + 9x^4 - 6x^2z + z^2 + y^2$$

$$W_E = \frac{\varepsilon_0}{2}\Big\{36\iiint_{-1}^{+1} x^2y^2\,dx\,dy\,dz + 9\iiint_{-1}^{+1} x^4\,dx\,dy\,dz - 6\iiint_{-1}^{+1} x^2z\,dx\,dy\,dz$$

$$+ \iiint_{-1}^{+1} z^2\,dx\,dy\,dz + \iiint_{-1}^{+1} y^2\,dx\,dy\,dz\Big\}$$

$$= \frac{1}{2}\varepsilon_0\Big\{36\,\frac{x^3}{3}\Big|_{-1}^{+1}\,\frac{y^3}{3}\Big|_{-1}^{+1}\,z\Big|_{-1}^{+1} + 9\,\frac{x^5}{5}\Big|_{-1}^{+1}\,y\Big|_{-1}^{+1}\,z\Big|_{-1}^{+1} - 6\,\frac{x^3}{3}\Big|_{-1}^{+1}\,\frac{z^2}{2}\Big|_{-1}^{+1}\,y\Big|_{-1}^{+1}$$

$$+ x\Big|_{-1}^{+1}\,y\Big|_{-1}^{+1}\,\frac{z^3}{3}\Big|_{-1}^{+1} + x\Big|_{-1}^{+1}\,z\Big|_{-1}^{+1}\,\frac{y^3}{3}\Big|_{-1}^{+1}\Big\}$$

$$= \frac{1}{2}\varepsilon_0\Big\{\frac{36}{9}\,(1-(-1))(1-(-1))\,(1-(-1) + \frac{9}{5}\,(1-(-1))(1-(-1))\,(1-(-1))$$

$$- \frac{6}{6}\,(1-(-1))\underbrace{(1-1)}_{0}\,(1-(-1)) + \frac{1}{3}(1-(-1))\,(1-(-1))\,(1-(-1))$$

$$+ \frac{1}{3}(1-(-1))\,(1-(-1))\,(1-(-1))\Big\}$$

$$= \frac{1}{2}\varepsilon_0\Big\{\frac{36}{9}\,2^3 + \frac{9}{5}\,2^3 - 1.0 + \frac{1}{3}\,2^3 + \frac{1}{3}\,2^3\Big\}$$

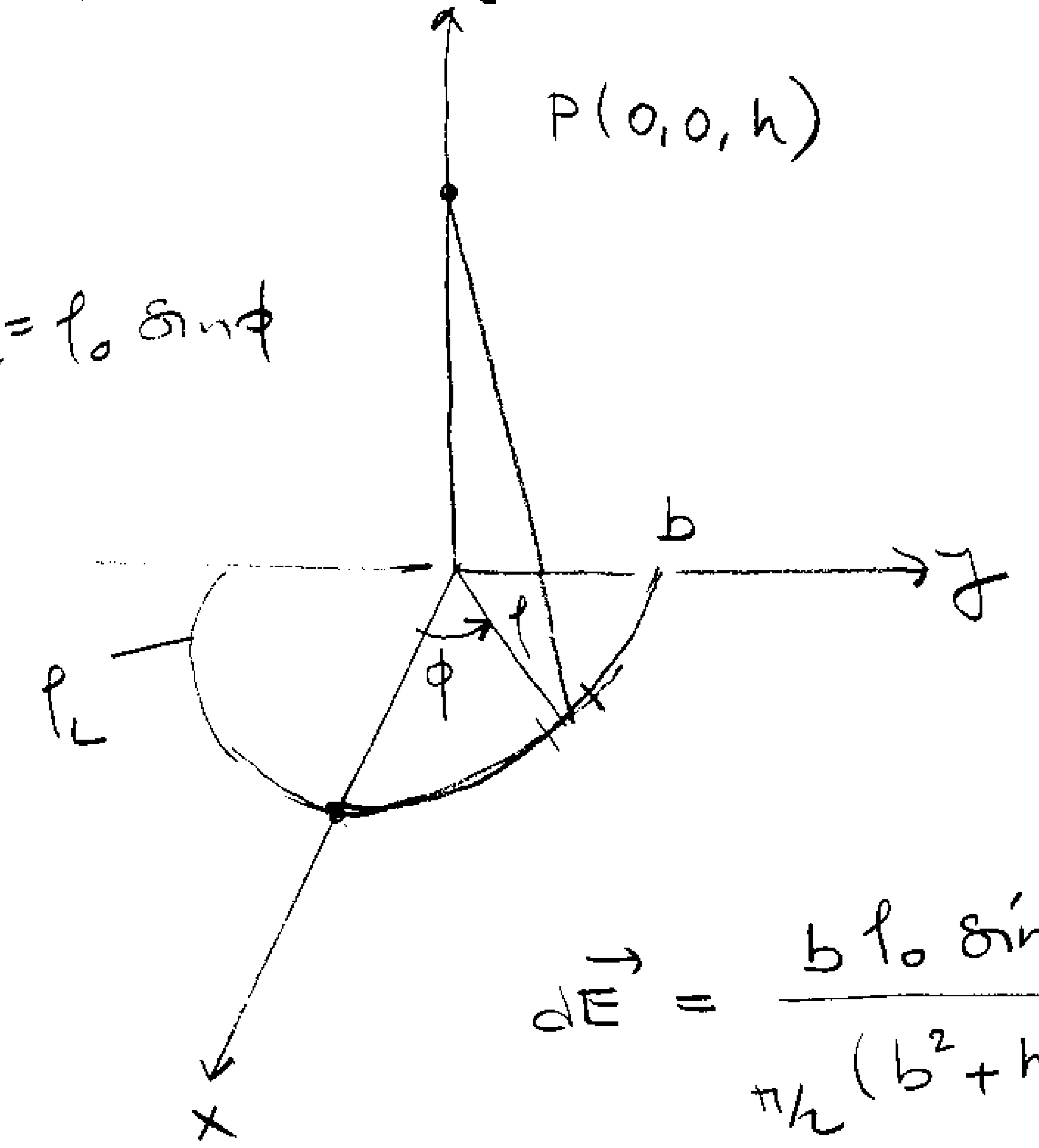
$$= \frac{\varepsilon_0}{2}\Big\{\frac{36\times 8}{9} + \frac{9\times 8}{5} + \frac{1}{3}\,2\times 8\Big\}$$

$$= \frac{51.7333}{2}\,\frac{10^{-9}}{36\pi} = 2.28711\ 10^{-10}\text{ J}$$

$$= 0{,}2287\cdot n\text{ J}$$

#3)

$\rho_L = \rho_0 \sin\phi$



$$d\vec{E} = \frac{dQ\,\vec{R}}{4\pi\varepsilon_0 R^3} \quad V/m$$

$$dQ = \rho\, d\phi\, \rho_L = b\, d\phi\, \rho_L$$

$$\vec{R} = -\rho\,\vec{a}_\rho + h\,\vec{a}_z = -b\,\vec{a}_\rho + h\,\vec{a}_z$$

$$|\vec{R}| = (b^2+h^2)^{1/2}$$

$$d\vec{E} = \frac{b\,\rho_0 \sin\phi\, d\phi}{(b^2+h^2)^{3/2}\,4\pi\varepsilon_0}\left(-b\,\vec{a}_\rho + h\,\vec{a}_z\right)$$

$$\vec{E} = \frac{b\,\rho_0}{4\pi\varepsilon_0 (b^2+h^2)^{3/2}}\left[-b\int_{\phi=-\pi/2}^{\pi/2} \sin\phi\, d\phi\, \vec{a}_\rho + h\,\vec{a}_z \int_{-\pi/2}^{\pi/2} \sin\phi\, d\phi\right].$$

$$\vec{a}_\rho = \cos\phi\,\vec{a}_x + \sin\phi\,\vec{a}_y$$

$$\int_{\phi=-\pi/2}^{\pi/2} \sin\phi\left[\cos\phi\,\vec{a}_x + \sin\phi\,\vec{a}_y\right] d\phi = \vec{a}_x \int_{-\pi/2}^{\pi/2} \sin\phi\cos\phi\, d\phi + \vec{a}_y \int_{-\pi/2}^{\pi/2} \underbrace{\sin^2\phi}_{\frac{1}{2}(1-\cos 2\phi)}\, d\phi$$

$$= \vec{a}_x\, \frac{\sin^2\phi}{2}\Big|_{-\pi/2}^{\pi/2} + \vec{a}_y\left[\frac{1}{2}\phi\Big|_{-\pi/2}^{\pi/2} - \frac{1}{4}\sin 2\phi\Big|_{-\pi/2}^{\pi/2}\right]$$

$$= \frac{\vec{a}_x}{2}(1-1) + \vec{a}_y\left[\frac{1}{2}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) - \frac{1}{4}(0-0)\right] = \vec{a}_y \cdot (+\pi/2)$$

$$\vec{E} = \frac{b\,\rho_0}{4\pi\varepsilon_0 (b^2+h^2)^{3/2}}\left[-b\,\vec{a}_y\,(+\pi/2) + h\,\vec{a}_z \underbrace{\left(-\cos\phi\Big|_{-\pi/2}^{\pi/2}\right)}_{0}\right]$$

$$\boxed{\vec{E} = \frac{-b^2\rho_0\cancel{\pi}}{8\cancel{\pi}\varepsilon_0 (b^2+h^2)^{3/2}}\vec{a}_y = \frac{-b^2\rho_0}{8\varepsilon_0 (b^2+h^2)^{3/2}}\vec{a}_y \text{ at point } P}$$

At point O, $h=0$

$$\boxed{\vec{E} = \frac{-b^2\rho_0}{8\varepsilon_0 b^2}\vec{a}_y = \frac{-\rho_0}{8\varepsilon_0 b}\vec{a}_y \quad V/m.}$$