ELECTROMAGNETICS-I FINAL EXAM

- Exam time is 90 min.
- Using 3 pages of formula sheet during the exam is permissible
- No problem solution should be written onto the formula sheet.

Dr Salih FADIL **June 09, 2011**

#1) If $V = x - y + xy + 2z$ *volt*, find $\vec{E}$ at (1, 2, 3) and electrostatic energy stored in a cube of side 2 *m* centered at the origin.

#2) The line $y = 1,\ z = -3$ carries charge 30 *nC/m* while the plane $x = 1$ carries charge $20\ nC/m^2$. Find $\vec{E}$ at the origin.

#3) Two point charges $Q_1 = 3\,nC$ and $Q_2 = -2\,nC$ are placed at (0, 0, 0) and (0, 0, -1) respectively. Assuming zero potential at infinity, find the potential at (1, 1, 1).

GOOD LUCK ☺

ELECTROMAGNETICS-I FINAL EXAM

SOLUTION MANUAL

Dr. Salih FADIL

June 09, 2011

#1) $V = x - y + xy + 2z$ volt.

$$\vec{E} = -\nabla V = -\{(1+y)\vec{a}_x + (-1+x)\vec{a}_y + 2\vec{a}_z\}$$

$$\vec{E}(1,2,3) = -3\vec{a}_x - 2\vec{a}_z \quad V/m.$$

$$W_E = \frac{1}{2}\int_v (\vec{E}\cdot\vec{D})\,dv = \frac{1}{2}\varepsilon_0 \int_v |\vec{E}|^2\,dv.$$

$$|\vec{E}|^2 = (1+y)^2 + (x-1)^2 + 4.$$

$$W_E = \int\int\int_{-1}^{+1} \left[(1+y)^2 + (x-1)^2 + 4\right] dx\,dy\,dz.$$

$$\Rightarrow \int_{-1}^{+1} (1+y)^2\,dy \quad x\Big|_{-1}^{1} \; z\Big|_{-1}^{1} = \frac{(1+y)^3}{3}\Big|_{-1}^{1} 2\times 2 = \frac{8}{3}\times 4 = \frac{32}{3}$$

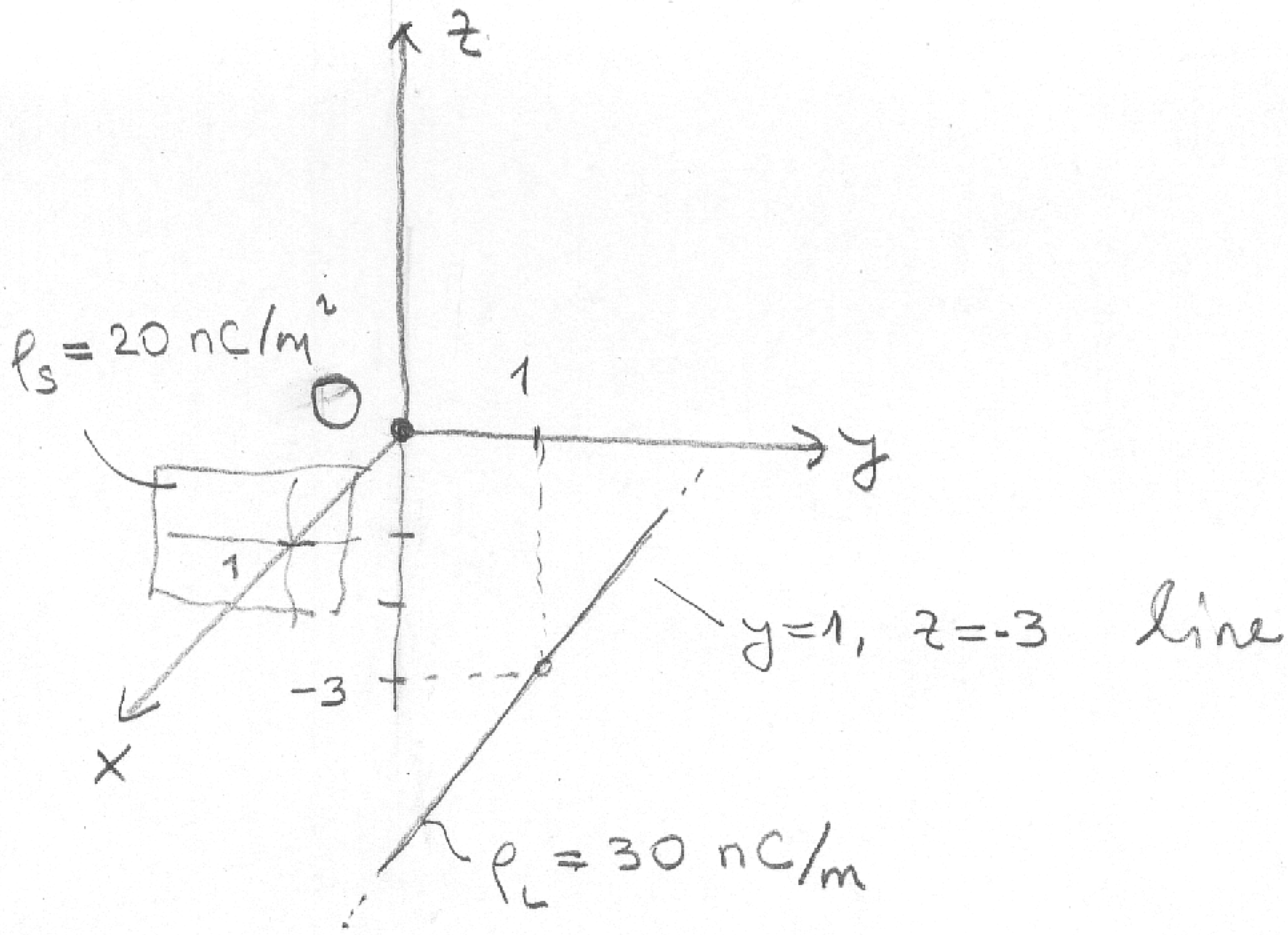
$$\int_{-1}^{1} (x-1)^2\,dx \quad 2.2 = 4.\frac{(x-1)^3}{3}\Big|_{-1}^{1} = \frac{4}{3}\left[0-(-8)\right] = \frac{32}{3}$$

$$4\times 2\times 2\times 2 = 32$$

$$W_E = \frac{1}{2}\,\frac{10^{-9}}{36\pi}\left[\frac{2\times 32}{3} + 32\right] = \frac{10^{-9}}{72\pi}\,\frac{5\times 32}{3} = 0,235\; 10^{-9}\; J$$

$$= 0,235\; nJ$$

#2)



$$\vec{E}_o = \vec{E}_L + \vec{E}_s$$

$$\vec{E}_s = \frac{\rho_s}{2\varepsilon_0}\vec{a}_n \qquad \vec{a}_n = -\vec{a}_x$$

$$\vec{E}_s = \frac{20\ \cancel{10^{-9}}}{2\ \frac{\cancel{10^{-9}}}{36\pi}}(-\vec{a}_x) = \frac{36\pi \times \overset{10}{\cancel{20}}}{\cancel{2}}(-\vec{a}_x) = -1130{,}97\ \vec{a}_x \ \text{V/m.}$$

$$\vec{E}_L = \frac{\rho_L}{2\pi\varepsilon_0\rho}\vec{a}_\rho$$

$$\vec{\rho} = (0,0,0) - (0,1,-3)$$

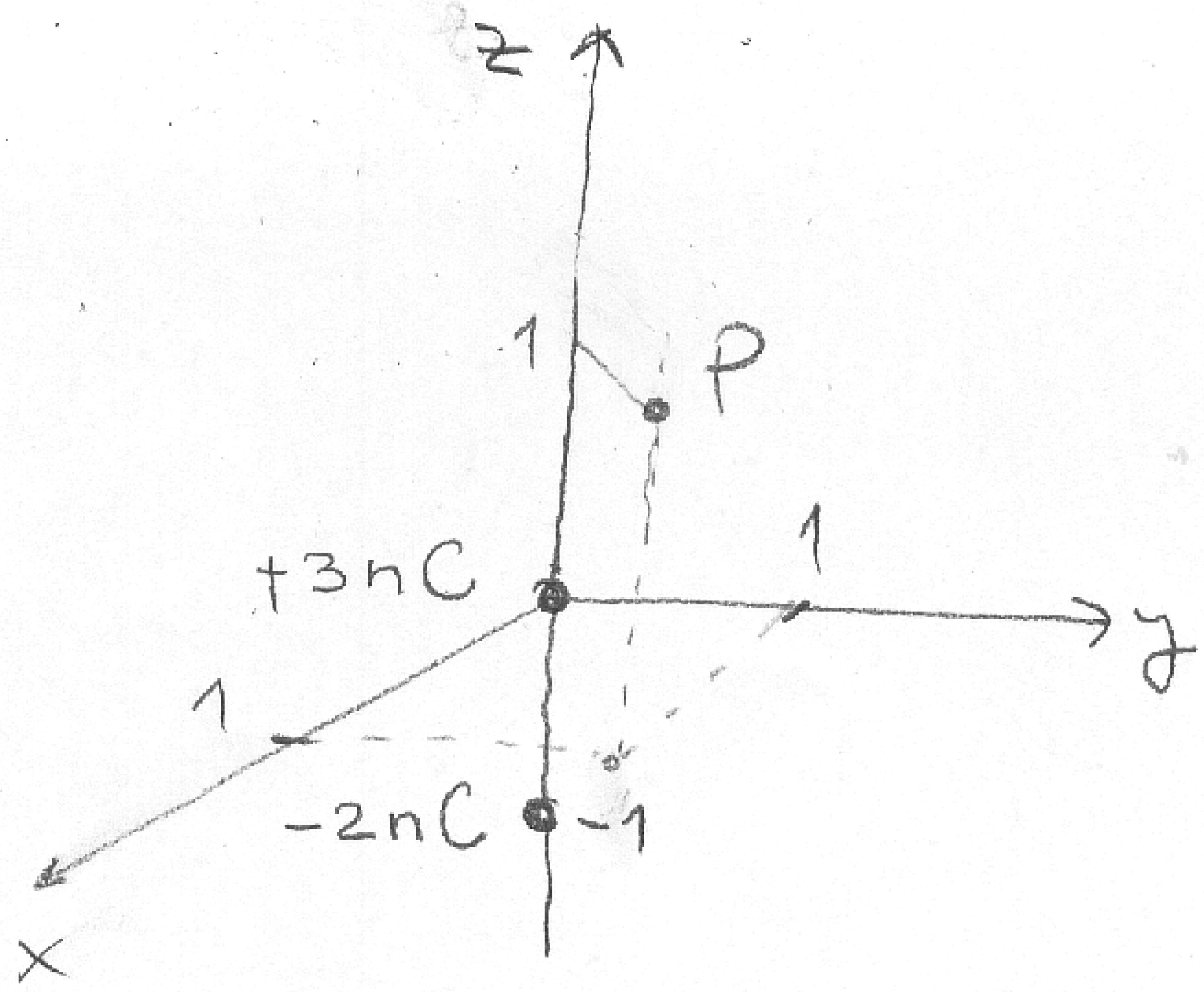
$$\vec{\rho} = (0,-1,3)$$

$$\rho = (9+1)^{1/2} \qquad \vec{a}_\rho = \frac{-\vec{a}_y + 3\vec{a}_z}{\sqrt{10}}$$

$$\vec{E}_L = \frac{30\ \cancel{10^{-9}}}{\cancel{2\pi}\ \frac{\cancel{10^{-9}}}{\cancel{36\pi}}_{18}}\ \frac{-\vec{a}_y + 3\vec{a}_z}{\sqrt{10}\sqrt{10}} = \frac{3\cancel{0} \times 18}{1\cancel{0}}(-\vec{a}_y + 3\vec{a}_z) = -54\vec{a}_y + 162\vec{a}_z$$

$$\vec{E}_o = -1130\ \vec{a}_x - 54\ \vec{a}_y + 162\ \vec{a}_z \ \text{V/m.}$$

#3)



$$V_p = V_{+3} + V_{-2} \qquad V = \frac{Q}{4\pi \varepsilon_0 |\vec{R}|}$$

$$V_{+3} = \frac{3\,\cancel{10^{-9}}}{\cancel{4\pi}\,\underbrace{\frac{\cancel{10^{-9}}}{\cancel{36\pi}}}_{9}\,|(1,1,1)-(0,0,0)|} = \frac{27}{|(1,1,1)|} = \frac{27}{\sqrt{3}} \text{ volt.}$$

$$V_{-2} = \frac{-2\,\cancel{10^{-9}}}{\cancel{4\pi}\,\underbrace{\frac{\cancel{10^{-9}}}{\cancel{36\pi}}}_{9}\,|(1,1,1)-(0,0,-1)|} = \frac{-18}{|(1,1,2)|} = \frac{-18}{\sqrt{4+1+1}}$$

$$V_{-2} = \frac{-18}{\sqrt{6}}$$

$$V_p = \frac{27}{\sqrt{3}} - \frac{18}{\sqrt{6}} = \underline{\underline{8,24}}\ V$$