ELECTROMAGNETICS - I FINAL EXAM

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#1) A homogeneous dielectric ($\varepsilon_r = 2.5$) fills region 1 ($x \leq 0$) while region 2 ($x \geq 0$) is free space.

a) If $\vec{E}_1 = 10\vec{a}_x + 6\vec{a}_y - 8\vec{a}_z \ V/m$, find $\vec{E}_2$ and θ_2.

b) If $E_2 = 12 \ V/m$, $\theta_2 = 60$, find E_1 and θ_1. Take θ_1 and θ_2 as the angles between $\vec{E}_1$ and normal to the boundary surface, $\vec{E}_2$ and normal to the boundary surface, respectively.

#2) Given that $\vec{E} = (3x^2 + y)\vec{a}_x + x\vec{a}_y \ kV/m$, find the work done in moving a $-2 \ \mu C$ charge from $(0,5,0)$ to $(2,-1,0)$ by taking the path

a) $(0,5,0) \to (2,5,0) \to (2,-1,0)$

b) $y = 5 - 3x$.

#3) For $\vec{E} = 20r\sin\theta\,\vec{a}_r - 10r\cos\theta\,\vec{a}_\theta \ V/m$ in free space, find the energy stored in conical region $0 \leq r \leq 5m$, $0 \leq \theta \leq 60$

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SOLUTION MANUAL

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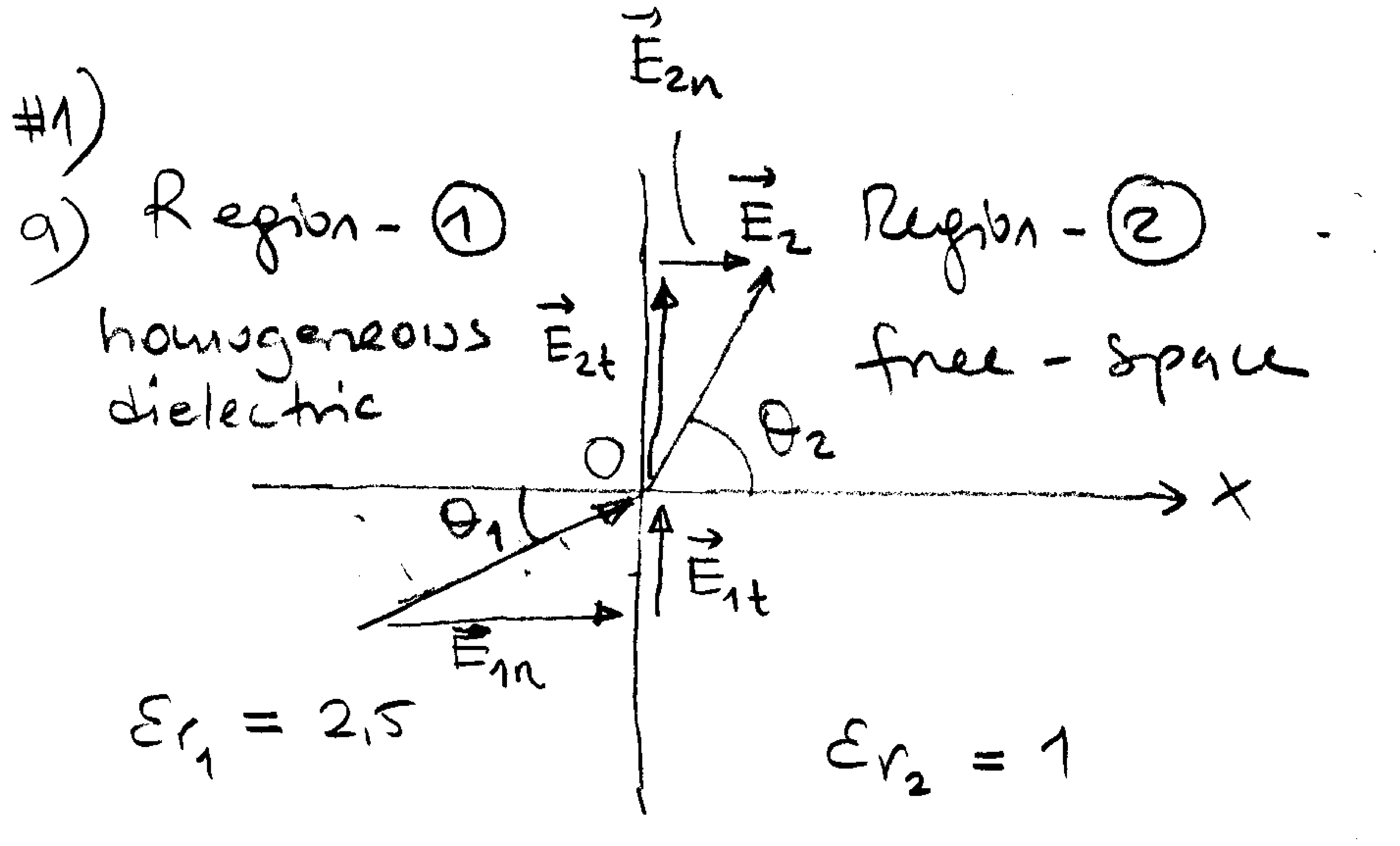
#1)

a)



$$\vec{E}_1 = 10\,\vec{a}_x + 6\,\vec{a}_y - 8\,\vec{a}_z \ \text{V/m} \qquad \vec{E}_{1n} = 10\,\vec{a}_x \ \text{V/m}$$

$$\vec{E}_{1t} = 6\,\vec{a}_y - 8\,\vec{a}_z \ \text{V/m}$$

$$\vec{E}_{1t} = \vec{E}_{2t} = 6\,\vec{a}_y - 8\,\vec{a}_z \ \text{V/m}$$

$\vec{D}_{1n} = \vec{D}_{2n}$ (no surface charge density on the boundary surface)

$$\varepsilon_1 \vec{E}_{1n} = \varepsilon_2 \vec{E}_{2n} \qquad \vec{E}_{2n} = \frac{2{,}5\,\cancel{\varepsilon_0}}{1\,\cancel{\varepsilon_0}}\vec{E}_{1n} = 2{,}5 \cdot 10\,\vec{a}_x = 25\,\vec{a}_x$$

$$\boxed{\vec{E}_2 = 25\,\vec{a}_x + 6\,\vec{a}_y - 8\,\vec{a}_z \ \text{V/m}}$$

$$\tan\theta_2 = \frac{E_{2t}}{E_{2n}} = \frac{(64+36)^{1/2}}{25} \qquad \boxed{\theta_2 = \tan^{-1}\left(\frac{10}{25}\right) = 21{,}8^\circ}$$

b) $E_2 = 12\,\text{V}, \quad \theta_2 = 60^\circ$

$$E_{2t} = E_2 \sin\theta_2 = 12 \sin 60^\circ = 10{,}392 \ \text{V/m}$$

$$E_{2n} = E_2 \cos\theta_2 = 12 \cos 60^\circ = 6 \ \text{V/m}.$$

$E_{1t} = E_{2t} = 10.392$

$E_{2n} = 2.5\, E_{1n} \Rightarrow \quad E_{1n} = \frac{1}{2.5}\, 6 = 2.4 \text{ V/m}.$

$$E_1 = \left(E_{1t}^2 + E_{1n}^2\right)^{1/2} = \left[(10.392)^2 + (2.4)^2\right]^{1/2} = \boxed{10.665 \text{ V/m}}$$

$$\cos\theta_1 = \frac{E_{1n}}{E_1} = \frac{2.4}{10.665} \qquad \boxed{\theta_1 = \cos^{-1}\left(\frac{2.4}{10.665}\right) = 76.99^\circ}$$

#2) $\vec{E} = (3x^2 + y)\vec{a}_x + x\,\vec{a}_y$ kV/m $\quad Q = -2\,10^{-6}$ C

a) $\overset{A}{(0,5,0)} \xrightarrow[\vec{dl} = dx\,\vec{a}_x,\ y=5]{} \overset{B}{(2,5,0)} \xrightarrow[\vec{dl} = dy\,\vec{a}_y,\ x=2]{} \overset{C}{(2,-1,0)}$

$$W = -Q\int_A^C \vec{E}\cdot\vec{dl} = -Q\left\{\int_A^B \vec{E}\cdot\vec{dl} + \int_B^C \vec{E}\cdot\vec{dl}\right\}$$

$$\int_A^B \vec{E}\cdot\vec{dl} = \int_{x=0}^{2} (3x^2+y)\Big|_{y=5} dx = \cancel{3}\frac{x^3}{\cancel{3}}\Big|_0^2 + 5x\Big|_0^2 = (8-0) + (10-0) = 18\text{ k}$$

$$\int_B^C \vec{E}\cdot\vec{dl} = \int_{y=5}^{-1} x\Big|_{x=2} dy = 2y\Big|_5^{-1} = 2(-1-5) = -12\text{ k}.$$

$W = +2\,10^{-6}\,(18-12)\,10^{3} = +12\,10^{-3}$ J $= +12$ mJ. external agent does the work

b) $y = 5 - 3x \qquad dy = -3dx. \qquad \vec{dl} = dx\,\vec{a}_x + dy\,\vec{a}_y$

$$W = -Q\int_A^C \left\{(3x^2+y)\,dx + x\,dy\right\} = -Q\int_A^B \left\{(3x^2+5-3x) + x(-3dx)\right\}$$

$$= -Q\int_{x=0}^{2} (3x^2 - 3x + 5 - 3x)\,dx = -Q\int_{x=0}^{2} (3x^2 - 6x + 5)\,dx \qquad \overbrace{8-12+10}$$

$$= 2\,10^{-6}\,10^{3}\left(\cancel{3}\frac{x^3}{\cancel{3}}\Big|_0^2 - 6\frac{x^2}{2}\Big|_0^2 + 5x\Big|_0^2\right) = 2\,10^{-3}\left[(8-0) - 3(4-0) + 5(2-0)\right]$$

$= 12$ mJ $\quad$ the same!

#3) $\vec{E} = 20\, r \sin\theta\, \vec{a}_r - 10\, r \cos\theta\, \vec{a}_\theta$ V/m in free-space.

$$W_E = \frac{1}{2} \int_{v} \varepsilon_0 |\vec{E}|^2 dv \qquad dv = r^2 \sin\theta\, d\theta\, d\phi\, dr$$

in spherical coordinates.

$$|\vec{E}|^2 = (20\, r \sin\theta)^2 + (10\, r \cos\theta)^2$$
$$= 400\, r^2 \sin^2\theta + 100\, r^2 \cos^2\theta$$

$$W_E = \frac{\varepsilon_0}{2} \Big\{ 400 \underbrace{\int_v r^4 \sin^3\theta\, d\theta\, d\phi\, dr}_{I_1} + 100 \underbrace{\int_v r^4 \cos^2\theta \sin\theta\, d\theta\, d\phi\, dr}_{I_2} \Big\}$$

$$I_1 = \int_{r=0}^{5} r^4 dr \cdot \int_{\theta=0}^{60^\circ} \underbrace{(1-\cos^2\theta)}_{\sin^2\theta} \sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$= \frac{r^5}{5}\Big|_0^5 \cdot \Big\{ -\cos\theta \Big|_0^{60^\circ} + \frac{\cos^3\theta}{3}\Big|_0^{60^\circ} \Big\} 2\pi = (625-0)\Big\{(-\cos 60^\circ + 1) + \frac{1}{3}(\cos^3 60 - 1)\Big\} 2\pi$$

$$= 1250\pi \Big\{ (\overbrace{-0{,}5+1}^{0{,}5}) + \frac{1}{3}(0{,}125 - 1) \Big\}$$

$$= 1250\pi\, (0{,}20833) = \underline{\underline{260.416\,\pi}}$$

$$I_2 = \int_{r=0}^{5} r^4 dr \cdot \int_{\theta=0}^{60^\circ} \cos^2\theta \sin\theta\, d\theta \cdot \int_{\phi=0}^{2\pi} d\phi$$

$$= \frac{r^5}{5}\Big|_0^5 \cdot \Big\{ -\frac{\cos^3\theta}{3} \Big|_0^{60^\circ} \Big\} 2\pi = 1250\pi \left(-\frac{1}{3}(0{,}125-1)\right)$$
$$= 364.58\,\pi$$

$$W_E = \frac{10^{-9}}{36\cancel{\pi}} \frac{\cancel{\pi}}{2} \left[400 \times 260.416 + 100 \times 364.58 \right] = 1.953\, 10^{-6}\ J$$
$$= \underline{\underline{1{,}953\ \mu J.}}$$