Top surface

$z=4, \quad d\vec{S} = \rho\, d\phi\, d\rho\, \vec{a}_z$

$$\int_{top} \vec{F}\cdot d\vec{S} = 8\int_{top} \rho\, d\phi\, d\rho = \cancel{8}^{4}\, \frac{\rho^2}{\cancel{2}}\Big|_0^5 \cdot 2\pi = 8\pi(25-0) = 200\pi$$

Bottom surface

$z=0, \quad d\vec{S} = \rho\, d\phi\, d\rho\, (-\vec{a}_z)$

$$\int_{bottom} \vec{F}\cdot d\vec{S} = 0$$

1a

Curved surface

$d\vec{S} = \rho\, d\phi\, dz\, \vec{a}_\rho, \quad \rho = 5$

$$\int_{curved} \vec{F}\cdot d\vec{S} = \int_{curved} \rho^2 \rho\, d\phi\, dz = 125 \; 2\pi \; 4 = 1000\pi$$

$$\oint_S \vec{F}\cdot d\vec{S} = 1000\pi + 200\pi = 1200\pi$$

1b

$$\nabla\cdot\vec{F} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\cdot\rho^2) + \frac{\partial}{\partial z}(2z) = \frac{1}{\rho}\times 3\rho^2 + 2 = 3\rho + 2$$

$$\int_v (\nabla\cdot\vec{F})\, dv = \int_v (3\rho+2)\rho\, d\phi\, d\rho\, dz = \int_{\rho=0} (3\rho^2 + 2\rho)\, d\rho \int_{\phi=0}^{2\pi} d\phi \int_0^4 dz$$

$$= \left(\frac{\cancel{3}\rho^3}{\cancel{3}} + \cancel{2}\frac{\rho^2}{\cancel{2}}\right)\Big|_0^5 2\pi\; 4 = (125 + 25)\, 8\pi = 1200\pi$$

the same result

So

$$\oint_S \vec{F}\cdot d\vec{S} = \int_v (\nabla\cdot\vec{F})\, dv \text{ is satisfied !}$$

ELECTROMAGNETICS – I FIRST EXAM

Dr. Salih FADIL **July 20, 2010**

Find the lenght of each side of the triangle

They are the corners of a triangle

#1) Three points in Cartesian space are given as $P_1(0,0,2)$, $P_2(0,0,5)$ and $P_3(0,2,0)$.

a) ~~Show that those points are the corners of a triangle~~ **by using only vector algebra.**

b) Find the all inner angles of the triangle **by using only vector algebra.** What kind of triangle is it? Why?

c) Find the area of the triangle **by using only vector algebra.**

#2) Let $\vec{A} = \rho\cos\phi\,\vec{a}_\rho + z\sin\phi\,\vec{a}_\phi - \rho z^2\vec{a}_z$ and $\vec{B} = r\sin\theta\,\vec{a}_r + r^2\cos\theta\sin\phi\,\vec{a}_\theta$

a) Calculate the vector component (in spherical coordinates) of $\vec{A}$ along $\vec{B}$ at $P(3,-2,6)$

~~**b)** Calculate a unit vector (in cylindrical coordinates) perpendicular to both $\vec{A}$ and $\vec{B}$ at $P(3,-2,6)$.~~

canceled

#3) For the vector function $\vec{F} = \rho^2\vec{a}_\rho + 2z\,\vec{a}_z$, verify the divergence theorem for the circular cylindrical region enclosed by $\rho = 5$, $z = 0$ and $z = 4$ surfaces.

ELECTROMAGNETICS-I FIRST EXAM SOLUTION MANUAL

(Summer school - 2010)

Dr. Salih FADIL

July 20, 2010

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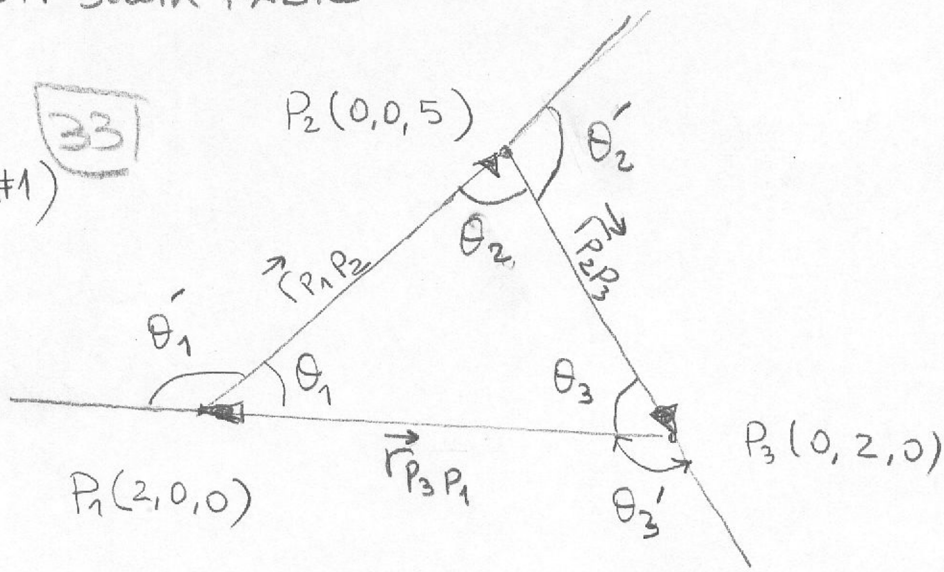
#1)



a) $\vec{r}_{P_1P_2} = (0,0,5)-(2,0,0) = (-2,0,5)$

$\vec{r}_{P_2P_3} = (0,2,0)-(0,0,5) = (0,2,-5)$

$\vec{r}_{P_3P_1} = (2,0,0)-(0,2,0) = (2,-2,0)$

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$\overline{P_1P_2} = |\vec{r}_{P_1P_2}| = \sqrt{4+25} = \sqrt{29} = 5.385$ unit

$\overline{P_2P_3} = |\vec{r}_{P_2P_3}| = \sqrt{4+25} = 5.385$ unit

$\overline{P_3P_1} = |\vec{r}_{P_3P_1}| = \sqrt{4+4} = \sqrt{8} = 2.828$ unit

b) $\vec{r}_{P_1P_2} \cdot \vec{r}_{P_1P_3} = -25 = \sqrt{29}\,\sqrt{29}\,\cos\theta_2'$ $\quad \theta_2' = \cos^{-1}\left(\frac{-25}{29}\right)$

$\theta_2 = 180 - \theta_2'$ $\quad \theta_2' = 149.55^\circ$

$\boxed{\theta_2 = 30.45}$

$\vec{r}_{P_2P_3} \cdot \vec{r}_{P_3P_1} = -4 = \sqrt{29}\,\sqrt{8}\,\cos\theta_3'$ $\quad \theta_3' = \cos^{-1}\left(\frac{-4}{\sqrt{232}}\right) = 105.225^\circ$

$\boxed{\theta_3 = 180 - \theta_3' = 74.774^\circ}$

$\vec{r}_{P_3P_1} \cdot \vec{r}_{P_1P_2} = \sqrt{8}\,\sqrt{29}\,\cos\theta_1' = -4 \rightarrow \boxed{\theta_1 = 74.774^\circ}$

Since $\theta_1 = \theta_3 = 74.774$ (also $\overline{P_2P_1} = \overline{P_2P_3}$), the triangle is a isosceles triangle.

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c) $\text{Area} = \frac{1}{2}\left|\vec{r}_{P_1P_2} \times \vec{r}_{P_2P_3}\right| = \frac{1}{2}\left(100+100+16\right)^{1/2} = 7.348\ (\text{unit})^2$

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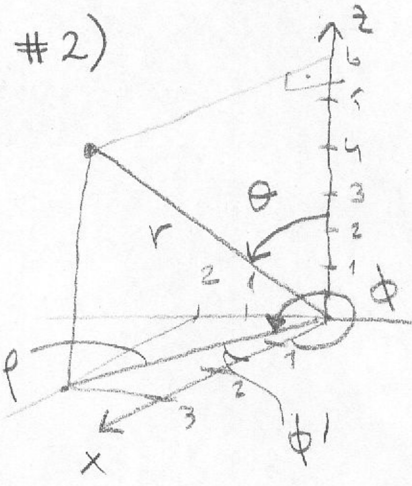
$$\vec{r}_{P_1P_2} \times \vec{r}_{P_2P_3} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ -2 & 0 & 5 \\ 0 & 2 & -5 \end{vmatrix} = \vec{a}_x(0-10) - \vec{a}_y(10-0) + \vec{a}_z(-4-0)$$

$$= -10\vec{a}_x - 10\vec{a}_y - 4\vec{a}_z$$

#2)



Cartesian

$x = 3$

$y = -2$

$z = 6$

Cylindrical

$\rho = \sqrt{4+9} = \sqrt{13}$

$\phi = 360 - \tan^{-1}\left(\frac{2}{3}\right)$

$\phi = 326.3$

$z = 6$

spherical

$r = (4+9+36)^{1/2} = 7$

$\theta = \tan^{-1}\left(\frac{\sqrt{13}}{6}\right) = 31^\circ$

$\phi = 326.3^\circ$

$A_\rho = \sqrt{13}\cos(326.3) = 3.0$

$A_\phi = 6\sin(326.3) = -3.329$

$A_z = -\sqrt{13} \times 36 = -129.8$

$B_r = 7 \times \sin(31^\circ) = 3.6052$

$B_\theta = 49\cos(31^\circ)\sin(326.3^\circ)$

$B_\theta = -23.304$

$B_\phi = 0$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin(31^\circ) & 0 & \cos(31^\circ) \\ \cos(31^\circ) & 0 & -\sin(31^\circ) \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3.0 \\ -3.329 \\ -129.8 \end{bmatrix} = \begin{bmatrix} -109.715 \\ 69.423 \\ -3.329 \end{bmatrix}$$

$$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\alpha = A_B\,|\vec{B}| = (-109.715)\times(3.6052)$$

$$(-23.304)\times(69.423) = -2013.35$$

$$A_B = \frac{-2013.35}{\left[(3.6052)^2 + (23.304)^2\right]^{1/2}} = -85.38$$

$$\vec{A}_B = (-85.38)\,\frac{3.6052\,\vec{a}_r - 23.304\,\vec{a}_\theta}{\sqrt{556}} = -3.62\,(3.6052\,\vec{a}_r - 23.304\,\vec{a}_\theta)$$

$$\boxed{\vec{A}_B = -13.05\,\vec{a}_r + 84.36\,\vec{a}_\theta}$$

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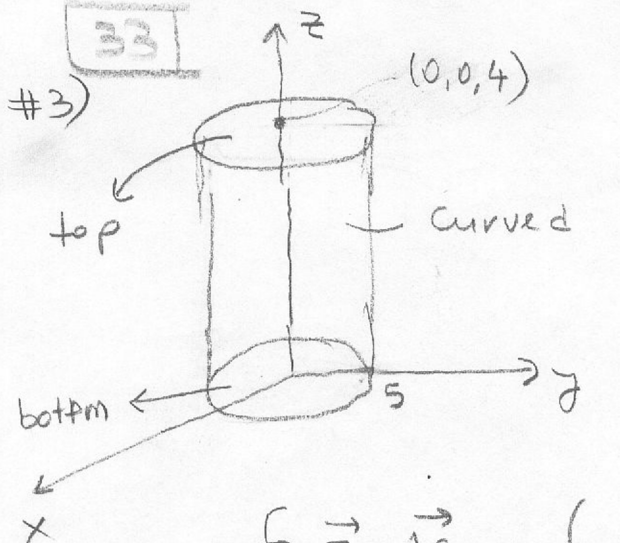
#3)



$$\vec{F} = \rho^2\,\vec{a}_\rho + 2z\,\vec{a}_z$$

Divergence theorem

$$\oint_S \vec{F}\cdot d\vec{S} = \int_v (\nabla\cdot\vec{F})\,dv$$

$$\oint_S \vec{F}\cdot d\vec{S} = \left(\int_{top} + \int_{bottom} + \int_{curved}\right)\vec{F}\cdot d\vec{S}$$