ELECTROMAGNETICS - I FIRST MIDTERM EXAM

- *90 minutes exam.*
- *One page A4 sized formula sheet is allowed.*

Dr. Salih FADIL **March 25, 2010**

#1) If P_1 is $(1,2,-3)$ and P_2 is $(-4,0,5)$, find:

a) the distance P_1P_2

b) the shortest distance between the line P_1P_2 and point $P_3(7,-1,2)$

#2) Given vectors $\vec{A} = 2\vec{a}_x + 4\vec{a}_y + 10\vec{a}_z$ and $\vec{B} = -5\vec{a}_\rho + \vec{a}_\phi - 3\vec{a}_z$ find:

a) the angle between $\vec{A}$ and $\vec{B}$ at $P(0,2,-5)$,

b) the scalar component of $\vec{A}$ along $\vec{B}$ at P.

#3) Let $\vec{F} = x^2\vec{a}_x + y\vec{a}_y + z\vec{a}_z$. Compute

a) $\oint_S \vec{F} \cdot d\vec{S}$

b) $\int_v (\nabla \cdot \vec{F})\, dv$

where S is the surface of the cubical volume v bounded by planes $x=0, x=1,\ y=0, y=1,\ z=0,$ and $z=1$.

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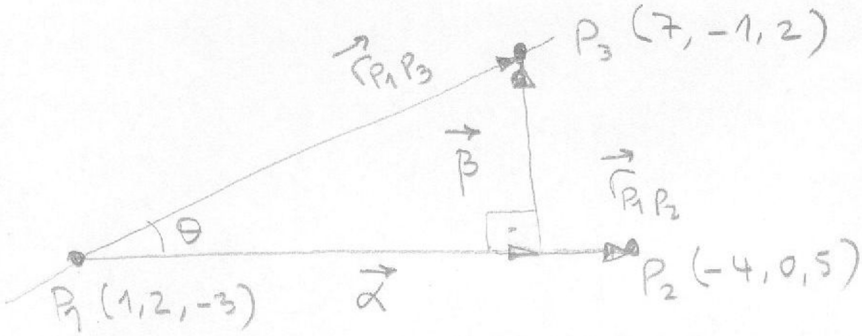
#1) $P_1(1,2,-3)$ $\quad P_2(-4,0,5)$

a) $d = \vec{r}_{P_1P_2} = |\vec{r}_{P_2} - \vec{r}_{P_1}| = \left[(-4-1)^2 + (0-2)^2 + (5+3)^2\right]^{1/2}$

$= \left[25 + 4 + 64\right]^{1/2} = \sqrt{93} = 9.643$ unit

b)



$\vec{r}_{P_1P_3} = \vec{r}_{P_3} - \vec{r}_{P_1} = (7,-1,2) - (1,2,-3) = (6,-3,5)$

$\vec{r}_{P_1P_2} = \vec{r}_{P_2} - \vec{r}_{P_1} = (-4,0,5) - (1,2,-3) = (-5,-2,8)$

$|\vec{\alpha}|$

$\vec{r}_{P_1P_3} \cdot \vec{r}_{P_1P_2} = -30 + 6 + 40 = 16 = |\vec{r}_{P_1P_3}|\,|\vec{r}_{P_1P_2}|\cos\theta$

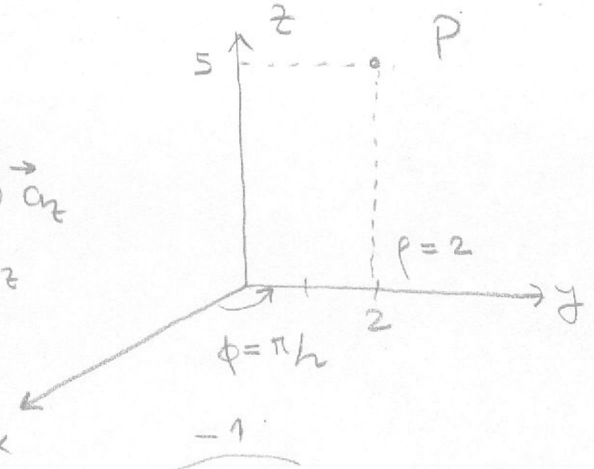
$= \sqrt{93}\,|\vec{\alpha}|$

$|\vec{\alpha}| = \dfrac{16}{\sqrt{93}}$

$|\vec{\beta}| = d = \left[|\vec{r}_{P_1P_3}|^2 - |\vec{\alpha}|^2\right]^{1/2} = \left[(36+9+25) - \left(\dfrac{16}{\sqrt{93}}\right)^2\right]^{1/2}$

$= \left[70 - \left(\dfrac{16}{\sqrt{93}}\right)^2\right]^{1/2} = 8.2$ unit

#2) $P(0, 2, 5)$



$\rho = 2$

$\phi = \pi/2$

$z = 5$

a)

$\vec{A} = 2\vec{a}_x + 4\vec{a}_y + 10\vec{a}_z$

$\vec{B} = -5\vec{a}_\rho + \vec{a}_\phi - 3\vec{a}_z$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \overset{0}{\cos(\pi/2)} & \overset{-1}{-\sin(\pi/2)} & 0 \\ \underset{1}{\sin(\pi/2)} & \underset{0}{\cos(\pi/2)} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \\ -5 \\ -3 \end{bmatrix}$$

$\vec{B} = -\vec{a}_x - 5\vec{a}_y - 3\vec{a}_z$, $\vec{A} = 2\vec{a}_x + 4\vec{a}_y + 10\vec{a}_z$

$\vec{A}\cdot\vec{B} = -2 - 20 - 30 = -52 = |\vec{A}|\,|\vec{B}|\cos\theta$

$= [4 + 16 + 100]^{1/2}\,[1 + 25 + 9]^{1/2}\cos\theta$

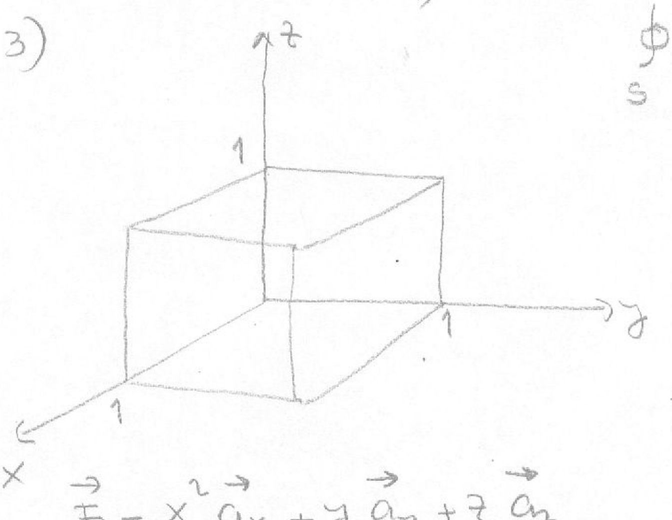
$-52 = \sqrt{120}\,\sqrt{35}$ $\qquad \theta = \cos^{-1}\left(\dfrac{-52}{\sqrt{120\times 35}}\right) = \underline{\underline{143.35^\circ}}$

b) The scalar component of $\vec{A}$ along $\vec{B}$ at P

$|\vec{A}_B| = |\vec{A}|\cos\theta = \dfrac{-52}{\sqrt{35}} = -8.7896$ unit

#3)

a)



$\vec{F} = x^2\vec{a}_x + y\,\vec{a}_y + z\,\vec{a}_z$

$$\oint_S \vec{F}\cdot d\vec{S} = \Big(\underset{\text{front}}{\int} + \underset{\text{back}}{\int} + \underset{\text{top}}{\int} + \underset{\text{bottom}}{\int} + \underset{\text{left}}{\int} + \underset{\text{right}}{\int}\Big)\vec{F}\cdot d\vec{S}$$

Front : $d\vec{S} = dy\,dz\,\vec{a}_x$, $x = 1$

$$\int_{\text{front}} \vec{F}\cdot d\vec{S} = \int_{\text{front}} x^2\,dy\,dz = y\Big|_0^1\, z\Big|_0^1 \cdot 1$$

$= 1.$

<u>Back</u>: $\vec{dS} = dy\, dz\, (-\vec{a}_x)\ ,\ x=0$

$$\int_{back} \vec{F}.\vec{dS} = -\int_{back} x^2\, dy\, dz = 0$$

<u>Top</u>: $\vec{dS} = dy\, dx\, \vec{a}_z\ ,\ z=1$

$$\int_{top} \vec{F}.\vec{dS} = 1 \int_{top} dy\, dx = y\Big|_0^1\, x\Big|_0^1 = 1.$$

<u>Bottom</u>: $\vec{dS} = dy\, dx\, (-\vec{a}_z)\ ,\ z=0$

$$-z\int_{bottom} dy\, dx = 0$$

<u>Left</u>: $\vec{dS} = dx\, dz\, (-\vec{a}_y)\ ,\ y=0$

$$\int_{left} \vec{F}.\vec{dS} = -y\int_{left} dz\, dx = 0$$

Right: $\vec{dS} = dx\, dz\, \vec{a}_y\ ,\ y=1$

$$\int_{right} \vec{F}.\vec{dS} = y\int_{right} dx\, dz = 1\ x\Big|_0^1\, z\Big|_0^1 = 1.$$

$$\oint_S \vec{F}.\vec{dS} = 3$$

b) $\nabla.\vec{F} = 2x + 1 + 1 = 2x + 2 = 2(x+1)$

$$\int_v (\nabla.\vec{F})\, dv = 2\int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1} (x+1)\, dx\, dy\, dz = 2\left\{ \frac{(x+1)^2}{2}\Big|_0^1\, y\Big|_0^1\, z\Big|_0^1 \right\}$$

$$= (4-1)\ 1 \cdot 1 = \underline{\underline{3}}$$

So, divergence theorem is satisfied

$$\oint_S \vec{F}.\vec{dS} = \int_v (\nabla.\vec{F})\, dv$$

volume v is defined
the closed surface S.