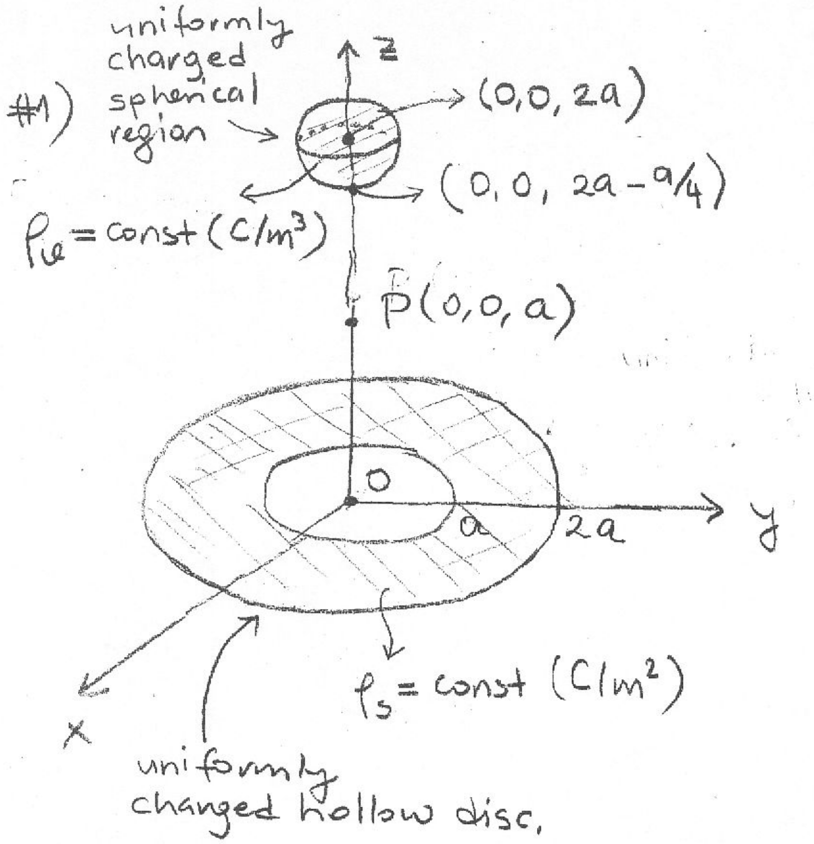
ELECTROMAGNETICS - I SECOND MIDTERM EXAM

Dr. Salih FADIL April 29, 2010

#1)



If electric field intensity at point P, shown in the figure on the left hand side, is zero, find ρ_v in terms of ρ_s.

#2) Assuming that the electric field intensity is $\vec{E} = (100x)\vec{a}_x$ V/m, find the total electric charge contained inside

a) a cubical volume 100 mm on a side centered symmetrically at the origin.

b) a cylindrical volume around z-axis having a radius 50 mm and a height 100 mm centered at the origin.

#3) A 10 nC point charge is located at the origin. Calculate the the work done in moving +2 nC point charge from (10 cm, 10 cm, 10 cm) to (1 cm, 1 cm, 1 cm).
Is this work done by the field? Why?

GOOD LUCK.. ☺

ELECTROMAGNETICS - I SECOND MIDTERM EXAM SOLUTION MANUAL

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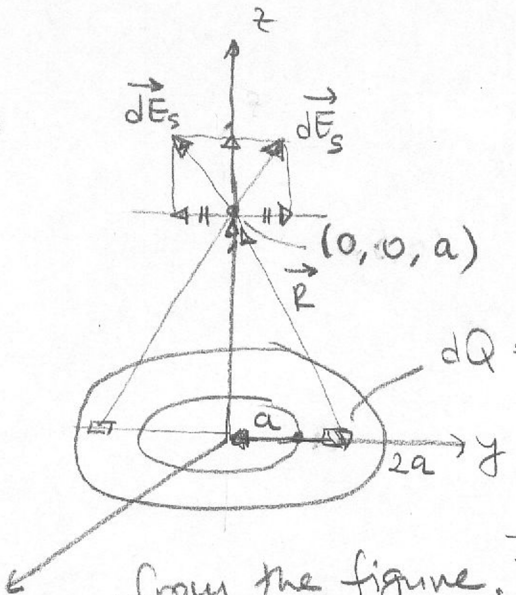
#1)



$$dQ = \rho\, d\rho\, d\phi\, \rho_s$$

$$\vec{R} = -\rho\,\vec{a}_\rho + a\,\vec{a}_z$$

$$R = (\rho^2 + a^2)^{1/2}$$

$$\vec{dE_s} = \frac{\rho\, d\rho\, d\phi\, \rho_s}{4\pi\varepsilon_0 (\rho^2+a^2)^{3/2}}(-\rho\,\vec{a}_\rho + a\,\vec{a}_z)$$

from the figure, $\vec{E}$ at point P is going to have only z component due to symmetry

$$\vec{dE_s} = \vec{dE}_{z,s} = \frac{\rho_s\, \rho\, d\rho\, d\phi}{4\pi\varepsilon_0(\rho^2+a^2)^{3/2}}\, a\,\vec{a}_z = \frac{a\,\rho_s\,\vec{a}_z}{4\pi\varepsilon_0}\cdot(\rho^2+a^2)^{-3/2}\rho\, d\rho\, d\phi$$

$$\vec{E}_s = \frac{a\,\rho_s}{4\pi\varepsilon_0}\vec{a}_z\left\{\int_0^{2\pi} d\phi \cdot \cancel{\frac{1}{2}}\frac{(\rho^2+a^2)^{-1/2}}{\cancel{(-1/2)}}\Bigg|_{\rho=a}^{\rho=2a}\right\}$$

$$\vec{E}_s = \frac{a\,\rho_s}{\cancel{4\pi}_2\,\varepsilon_0}\vec{a}_z\,\cancel{2\pi}\left[\frac{1}{\sqrt{2a^2}} - \frac{1}{\sqrt{5a^2}}\right] = \frac{\cancel{a}\,\rho_s}{2\varepsilon_0}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{5}}\right]\frac{1}{\cancel{a}}\vec{a}_z$$

$$\underline{\underline{\vec{E}_s = \frac{\rho_s}{2\varepsilon_0}(0,25989)\,\vec{a}_z \text{ V/m.}}}$$

$$\vec{E}_v = \frac{\frac{4}{3}\pi\,(a/4)^3\,\rho_v}{4\pi\varepsilon_0\, a^2}(-\vec{a}_z)\ \text{V/m.} = -\frac{\frac{\cancel{4\pi}}{3}\frac{\cancel{a^3}^{\,a}}{64}\rho_v}{\cancel{4\pi}\,\varepsilon_0\,\cancel{a^2}}\vec{a}_z = \frac{-a\,\rho_v}{192\,\varepsilon_0}\vec{a}_z$$

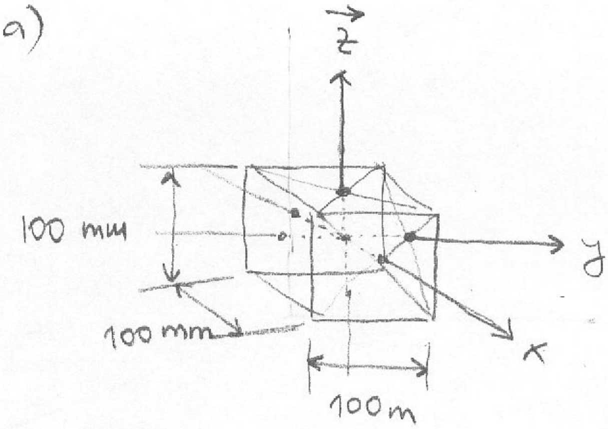
$$\vec{E}_{total} = \underbrace{\left[\frac{\rho_s}{2\varepsilon_0}(0,25989) - \frac{a\,\rho_v}{192\,\varepsilon_0}\right]}_{=0}\vec{a}_z = 0$$

$$\frac{a\,\rho_v}{192\,\cancel{\varepsilon_0}} = \frac{\rho_s}{2\cancel{\varepsilon_0}}(0,25989) \qquad \rho_v = \frac{\rho_s}{a}\,\frac{192\times 0,25989}{2}$$

$$\boxed{\rho_v = \frac{\rho_s}{a}\ 24.95\ \text{C/m}^3}$$

#2)

a)



$$\vec{E} = (100x)\,\vec{a}_x \ \text{V/m}.$$

$$Q_{enc} = \oint_S \vec{D}\cdot d\vec{S} = \Psi$$

$$\vec{D} = \varepsilon_0\, 100\, x\ \vec{a}_x$$

$$\Psi = \Big(\underbrace{\int}_{\text{front}} + \underbrace{\int}_{\text{rear}} + \underbrace{\underbrace{\int}_{\text{top}} + \underbrace{\int}_{\text{bottom}} + \underbrace{\int}_{\text{left}} + \underbrace{\int}_{\text{rigt}}}_{\text{gives zero, since } \vec{D}\perp d\vec{S} \text{ on those surfaces.}}\Big)\vec{D}\cdot d\vec{S}$$

$$\int_{\text{front}} \vec{D}\cdot d\vec{S} = \int_{\text{front}} \varepsilon_0\, 100\cdot(50\, 10^{-3})\,\vec{a}_x \cdot dy\, dz\, \vec{a}_x$$

$$= \varepsilon_0\, 5\, \cancel{10^{3}}\, \cancel{10^{-3}} \int_{-50\,10^{-3}}^{50\,10^{-3}} dy \int_{-50\,10^{-3}}^{50\,10^{-3}} dz = \varepsilon_0\, 5\ 100\, 10^{-3}\ 100\, 10^{-3}$$

$$= 5\,\varepsilon_0\, 10^{4}\, 10^{-6} = 5\, 10^{-2}\,\varepsilon_0$$

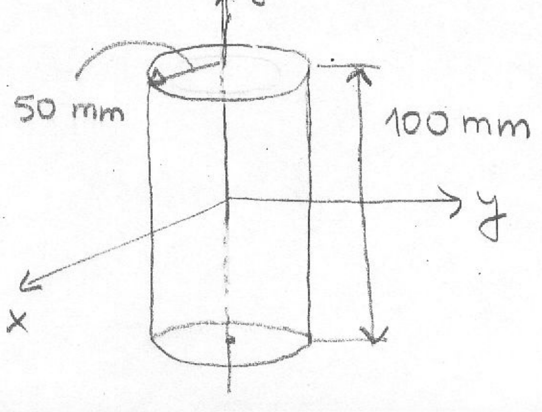
$$= 0{,}05\,\varepsilon_0 \ C$$

$$\int_{\text{rear}} \vec{D}\cdot d\vec{S} = \int_{\text{rear}} \varepsilon_0\, 100\,(-50\, 10^{-3})\,\vec{a}_x\ dy\, dz\,(-\vec{a}_x)$$

$$= \varepsilon_0 \int_{\text{rear}} 5\, 10^{-3}\, dy\, dz = 0{,}05\,\varepsilon_0\ C$$

$$Q_{enc} = 0{,}1\,\varepsilon_0 = 0{,}1\,\frac{10^{-9}}{36\pi} = 8{,}8419\, 10^{-13}\ C$$

b)



$$\Psi = \Big(\underbrace{\underbrace{\int}_{\text{top}} + \underbrace{\int}_{\text{bottom}}}_{\substack{=0 \\ \text{since } \vec{E}\perp d\vec{S}}} + \underbrace{\int}_{\text{curved}}\Big)\vec{D}\cdot d\vec{S} = Q_{enc}$$

$\vec{E} = 100\, x\, \vec{a}_x \qquad x = \rho \cos\phi \qquad \vec{a}_x = \cos\phi\, \vec{a}_\rho - \sin\phi\, \vec{a}_\phi$

$\vec{E} = 100\, \rho \cos\phi \left[\cos\phi\, \vec{a}_\rho - \sin\phi\, \vec{a}_\phi\right]$

$\Psi = \int_{\text{curved}} \vec{D} \cdot d\vec{S} \qquad d\vec{S} = \rho \cdot d\phi\, dz\, \vec{a}_\rho \;, \quad \rho = 50\, 10^{-3}$

$$\Psi = \int_{\text{curved}} \varepsilon_0\, \rho \cos^2\phi \cdot \rho\, d\phi\, dz \,.\, 100 = \varepsilon_0 \left(50\, 10^{-3}\right)^2 100 \int_{\phi=0}^{2\pi} \cos^2\phi\, d\phi \;.\; z\Big|_{-50\, 10^{-3}}^{50\, 10^{-3}}$$

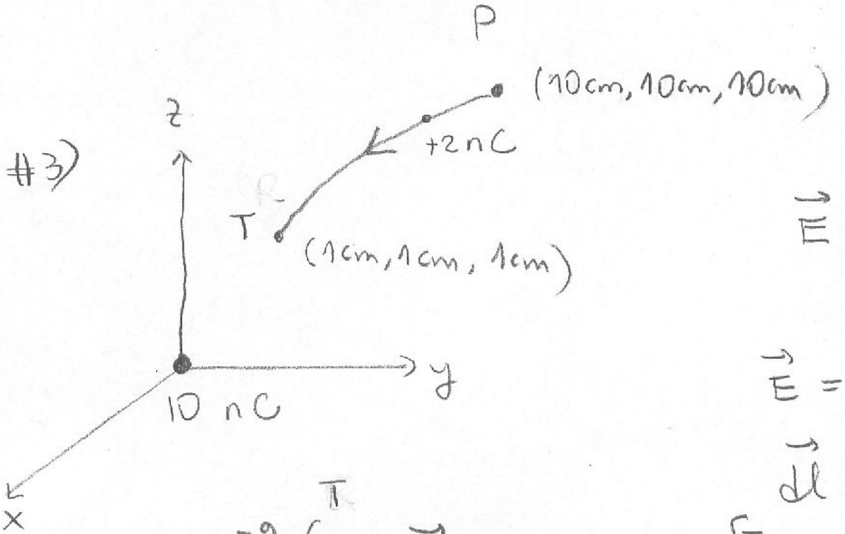
$$\Psi = \varepsilon_0\, 2500\, 10^2\, 10^{-6} \left\{ \int_0^{2\pi} \frac{1}{2}(1 + \cos 2\phi)\, d\phi \right\} 100\, 10^{-3}$$

$$\Psi = \varepsilon_0\, 0{,}025 \left[\frac{1}{2}\phi \Big|_0^{2\pi} + \cancel{\frac{1}{2} \sin 2\phi \Big|_0^{2\pi}} \right\} = \varepsilon_0\, 0{,}025\, \pi \text{ C}$$

$$= \frac{10^{-9}}{36\pi}\, 0{,}025\, \pi \text{ C}$$

$$= 6{,}94\, 10^{-13} \text{ C}$$

#3)



$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\, \vec{a}_r = \frac{10\, \cancel{10^{-9}}}{4\pi \frac{\cancel{10^{-9}}}{36\pi}\, r^2}\, \vec{a}_r \text{ V/m}$$

$$\vec{E} = \frac{90}{r^2}\, \vec{a}_r \text{ V/m.}$$

$$\vec{dl} = dr\, \vec{a}_r + r\, d\theta\, \vec{a}_\theta + r \sin\theta\, d\phi\, \vec{a}_\phi$$

$$W = -2\, 10^{-9} \int_P^T \vec{E} \cdot \vec{dl} = -2\, 10^{-9} \int_{r_P}^{r_T} \frac{90}{r^2}\, \vec{a}_r \cdot dr\, \vec{a}_r$$

$r_T = \sqrt{3\, 10^{-4}} = \sqrt{3}\, 10^{-2}\, m$

$r_P = \sqrt{300\, 10^{-4}} = \sqrt{3}\, 10^{-1}\, m$

$$= 180 \,.\, 10^{-9} \int_{r_P}^{r_T} -\frac{dr}{r^2} = 180\, 10^{-9}\, \frac{1}{r}\Big|_{r_P}^{r_T}$$

$$= 180\, 10^{-9} \left(\frac{1}{r_T} - \frac{1}{r_P} \right) = 180\, 10^{-9} \left(\frac{1}{\sqrt{3}\, 10^{-2}} - \frac{1}{\sqrt{3}\, 10^{-1}} \right)$$

$$= \frac{180\, 10^{-9}}{\sqrt{3}} \left(100 - 10\right) \text{J}$$

$$W = 9.353\ \mu\text{J} > 0$$

Since $W > 0$, field gains energy. Therefore external agent does the work.